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PATTERN FORMATION IN CONVECTIVE MEDIA

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The several models of convection in a thin layer of liquid (gas) with poorly heat conducting boundaries are considered. These models demonstrate a rich dynamics of pattern formation and structural phase transitions. The primary analysis of pattern formation in such a system is performed with using of the well-studied Swift-Hohenberg model. The more advanced Proctor-Sivashinsky model is examined in order to study the second-order structural phase transitions both between patterns with translational invariance and between structures with broken translational invariance but keeping a long-range order. The spatial spectrum of arising structures and visual estimation of the number of defects are analyzed. The relation between the density of defects and the spectral characteristics of the structure is found. We also discuss the effect of noise on the formation of structural defects. It is shown that within the framework of the Proctor-Sivashinsky model with additional term, taking into account the inertial effects, the large-scale vortex structures arise as a result of the secondary modulation instability.
KEY WORDS: Rayleigh-Bénard convection, mathematical modeling, dissipative structures, structural phase transitions, structural defects

ФОРМИРОВАНИЕ СТРУКТУР В КОНВЕКТИВНЫХ СРЕДАХ

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Рассмотрено несколько моделей конвекции в тонком слое жидкости (газа) в условиях слабой теплопроводности на его границах. Эти модели демонстрируют разнообразную динамику формирования пространственных структур и структурно-фазовых переходов между ними. Первоначальный анализ формирования ячеек в таких системах был представлен при использовании модели Свифта-Хоенберга. Более развитая и корректная модель Проктора-Сивашинского исследована для нескольких фазовых переходов между структурами с трансляционной инвариантностью и структурами с нарушенной трансляционной инвариантностью, но с сохраненным дальним порядком. Изучается связь между пространственным спектром структур и количеством дефектов. Найдено соотношение между плотностью дефектов и спектральными характеристиками структуры. Обсуждается эффект влияние шума на развитие фазовых переходов. Показано, что обобщенная модель Проктора-Сивашинского, учитывающая инерциальные эффекты, способна описывать формирование крупномасштабных вихревых структур, как результат вторичной модуляционной неустойчивости.
КЛЮЧЕВЫЕ СЛОВА: конвекция Релея-Бенара, математическое моделирование, диссипативные структуры, структурно-фазовые переходы, структурные дефекты

ФОРМУВАННЯ СТРУКТУР У КОНВЕКТИВНИХ СЕРЕДОВИЩАХ

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Розглянуто декілька моделей конвекції у тонкому шарі рідини (газу) в умовах слабкої теплопровідності на його межах. Ці моделі демонструють різнобарвну динаміку формування просторових структур та структурово - фазових переходів між ними. Попередній аналіз формування чарунок в таких системах було розроблено при використанні моделі Свіфта-Хоенберга. Більш розвинута та коректна модель Проктора-Сівашинського вивчена для декількох фазових переходів між структурами з трансляційною інваріантністю та структурами, які мали порушену трансляційну інваріантність, та дальній порядок. Вивчається зв'язок між просторовим спектром та кількістю дефектів. Знайдено відношення між густиной дефектів та характеристиками просторового спектру. Розглянуто ефект впливу шуму на розвиток фазових переходів. Показано, що більш загальна модель Проктора-Сівашинського, яка враховує інерціальні ефекти, дозволяє розглядати формування крупно масштабних вихорів, я к результат вторинної модуляційної нестійкості.
КЛЮЧОВІ СЛОВА: конвекція Релея-Бенара, математичне моделювання, дисипативні структури, структурно-фазові переходи, структурні дефекти

CONTENTS

1. INTRODUCTION

Considering the various processes in continuous media, we need to take into account the dynamics of perturbations with not only different spatial and temporal scales but also different spatial orientation [1-12]. The last one is responsible in the common geometric sense for symmetry of the spatial structures, which possess not only short-range but also a long-range order [13-17].

The nonlinearity of the medium manifests itself in certain mechanisms of interaction between these perturbations. Different approaches to description of such interactions in nonequilibrium media are presented in [18-25]. The processes involving a large number of perturbations of all scales and orientation are often called multi-wave or multimode, in the case of wave media or periodic systems.

1.1. The problems of description of structure formation processes

Currently, the problem of most interest is the elucidation of the nature of spatial structures appearance, the search for physically transparent mechanisms of these processes, and then the formulation of adequate (which have a clear physical background) mathematical models for description of these phenomena.

Considering the behavior of multimode or multiwave spatial structures formation processes, we can see the appearance of their specific features, such as a change in dynamics of the instability (i.e. delay or even suppression [26]) during the formation of unstable nonlinear structures. The number of degrees of freedom plays the significant role, such as the number of spectrum modes (which leads to appearance of a small parameter $\eta \propto 1/N$ inversely proportional to the number of spectrum modes N). The existence of a dense spectrum of perturbations can form long-lived quasi-stable nonlinear states and can delay the development of transient processes and structural phase transitions between these states [27, 28].

The issues of structural transformations, structural second-order phase transitions, resulting in the changes of the symmetry and some characteristic scales of spatial structures always be of great interest to researchers and developers of technologies.

In particular, one of the main problems of radiative study of materials is the problem of occurrence of a complex system of defects and phase transformations caused by irradiation. The authors of [29-32] drew attention to the collective character of the macro-scale processes in such materials. The formation of spatio-temporal dislocation inhomogeneities, dislocation channels, the moving Chernov-Luders lines, the dynamic self-wave structures (the Danilov-Zuev relaxation waves, the phase transformation front in the dislocation-vacancy ensemble etc.) may be caused namely by macroscopic processes. The self-organization of structural transformation under the action of external factors demonstrates the nonlocal properties caused most likely by the large scale instabilities. Note that in some cases the experimental and calculated data also point to the fact that local defects and disorders may be a result of imperfection in a large scale packing, occurring in particular when the system selects the characteristic scaling with broken geometric orientation on the structural elements.

Developed phenomenological approaches which allows to describe the dynamics of macroscopic characteristics with acceptable degree of accuracy but nevertheless qualitatively (see, e.g. [33,34]) could not adequately represent the microscopic description of the processes. The statistical models, which use the probabilistic characteristics of the process of interaction between the individual elements (particles of dust, fragments-crystallites, etc.) of the medium [35-37] allowed to estimate the time to process equilibrium only qualitatively. The dynamics of non-stationary processes could not be always identified under such consideration. In order to calculate the stable states, the collection of techniques is commonly used known as renormalization procedure. This procedure allows reduction of the large number of interactions (diagrams) to an integral or a small number of interactions. However, some assumption and simplifications should be used at this for classic (see the so-called, S-theory [22, 38-39]) as well as for quantum systems [40-42].

With the development of the computing power the direct methods of simulation became more popular such as descriptions of structurization dynamics on microscopic level, the use of computing procedures for detailed description of structural transformations [43-44]. However, the weak point of such direct computational methods is a necessity to use a very large number of interacting elements with a set of short-range and long-range interactions between them. Besides, the temporal and spatial horizons of interaction are often not fully specified. This leads to the large number of the dynamic equations, and the accounting of interactions increases the dimension of the task.

It is therefore understandable the wish of researchers to find such mathematical models of structurization based on simple physically transparent principles which would be able to select the most essential types of interaction, i.e. select the dominant symmetries of the system and to simplify the model for its practical use in simulation modeling. The models of spatial structure formation were considered by many researches, which main ideas can be found in monographs [45-48]. However, of main interest, as it was pointed in [49], are the dynamical models, which could be described by differential equations in partial derivatives, the mathematical apparatus of the analysis of which is well developed. The special attention should be attended to the models that are capable to describe the imperfect quasi-periodic systems, quasi-crystals (that is characterized by a long-range order and symmetry inadmissible in a classical crystallography [50,51]).

In the opinion of the authors [49], the rational procedure of design of new materials should start with “selection or construction of basic nonlinear models relevant to phenomena of different physical or other origins and, moreover, design of key experiments for verification of *a priori* hypotheses that these models are universal”. Developing the simulation models, it is useful to bring changes into the well-known universal equations and generalize their representations. Even H. Poincaré noted that “equations must teach us, primarily, what we can and what we should change in them”. For specific implementation of these tasks, it is useful to apply the approximate methods based on small parameters, and to use actively the numerical modeling for investigation not only the particular solutions, but mainly in order to discover the nature of the considered phenomena itself, for a formulation of constructive prompts to experimentalists and technologists.

1.2. Choice of models and research methods

Often quite sufficient effective method of the description of quasiperiodic structures is the expansion of perturbations in terms of the orthogonal spatial eigenmodes interacting with each other. All spatial patterns arising in this system will be a result of the linear interference of these modes and under the action of non-equilibrium (pumping) the interference may be considered as stimulated [52]. The nonlinearity of the medium manifests itself in slow evolution of these eigenmodes due to interaction between themselves and with the pumping. The summation over the spectrum of the eigenmodes with appropriate weight gives a possibility to study the process of formation of spatial structures and the diverse forms of spatial derangements (structure imperfections or defects) and even cardinal structure rearrangement (structural-phase transformations). In particular, it was observed a periodic track of defects [53] when considering the periodic spatial perturbations with the spectrum consisting of a small number of modes (for instance, see [54]). The quasi-crystalline spatial structure in turn is often appears due to presence of several incommensurate harmonicas in the spatial spectrum or in non-one-dimensional case due to existence of harmonicas, angles between wave vectors of which are rather random or they correspond to the irrational values (for instance, see, [55]). This fact brings up an association with the Landau – Hopf model of turbulence, that represents a development of a disorder via an excitation of a large number of freedom degrees [56,57] (that is, quasiperiodic "winding" on the multidimensional torus [58]).

The quadratic nonlinearity together with a certain choice of initial conditions often brings into existence the propagating fronts. It is possible when the perturbations lie within a sufficiently broad annular spectrum zone of some radius in $\vec{k}$ -space (the reverse character scale). The existence of minima in the perturbation spectrum intermodal potential makes possible the formation of stable or metastable structures. The narrower the annular the more sharp the spatial structure that demonstrates a long-range order. By the way, the small width of the spectrum of growing spatial modes provides a long-range order even in the absence of minima of the interaction potential in real space, at least on a scale inversely proportional to the spectrum width. The selection of initial conditions can provide a formation of a spatial structure firstly in some bounded space and its further expansion over the periphery of this area. For description of these phenomena that were found at first in numerical simulation of models similar to the Swift – Hohenberg model [59] (for example see also, [60]) there are enough methods of the standard analysis of the equations of mathematical physics. But using of other methods and approaches seems to be also useful.

So, in particular, the imperfect structures can be characterized by various structural and scaling parameters, for example by Lyapunov, Hausdorff or fractal dimension [61]. Generally speaking, any signal which is generated by a real source or simulative dynamic system has a finite set of objects with finite dimension. At the same time, the signals generated by fluctuations in the systems with very large number of constituting elements, can be characterized by a very high dimension (in this sense the dimension of the idealized white noise is infinite). The dimension of the structure is capable to change in time during its evolution [49], and only the approaching to the attractor allows to see the long-living scales and configuration state of the system. It must be kept in mind however that dimensions and scales in different spaces (including phase spaces) where the dynamic system is considered can be different. Therefore it is rationally to discuss the real space only where the structure (a straight or curved line, a plane or surface, a three-dimensional formation) is implemented. As for slowly evolving structures, the observer can face a necessity of studying of the intermediate quasiperiodic metastable states, which in most cases can be similar to quasicrystals or other structures with similar properties [50,51].

Until recently, in models based of the differential and integration-differential equations, the main attention was paid to structurally phase transitions of the first order which result in formation of the regular structures from previously amorphous state [49]. The authors [27] have drawn attention to a possibility of structural-phase transitions of second order (structural transformations) in such models, as was proved afterwards in [28].

1.3. The Proctor-Sivashinsky model for description of pattern formation in a thin layer of liquid or gas

The Proctor-Sivashinsky model is found to be very attractive [62,63] for studying the processes of pattern formation in systems which possess a preferred characteristic spatial scale of interaction between quasi-particles or elements of future structure. This model was developed for description of the convection in a thin layer of liquid with poorly conducting heat boundaries. Authors of [64] have found the stationary solutions with a small number of the spatial modes one of which (convective cells) was steady and the second one (convective rolls) turned out to be unstable. A special future of the model is that it forces a preferred spatial scale of interaction, leaving for the system’s evolution an opportunity to select the symmetry. It was found the type of symmetry and hence the characteristics of the structure are determined by the minima of the potential of interaction between modes with equal absolute wavenumber value. Modifying the structure of intermodal coupling potential within the framework of the generalized Proctor-Sivashinsky model, it is possible to change the symmetry of steady solutions [64] with changing the number of the potential minima. Below, we show that changing the amplitude of the minima also effects on the dynamics of structure and phase transformations in this system. The authors of [64] restricted themselves by studying of stationary states and the analysis of their linear stability (i.e. stability to small perturbations) and didn't consider the dynamics of structural transitions. Therefore, they didn’t discuss a possibility to change a value of a minimum of interactional potential.

The Proctor-Sivashinsky model allows further development and a short time later the generalized Proctor-Sivashinsky-Pismen model [65] was formulated which includes the inertial effects and consider the poloidal vortices inside a thin layer. This model, as was shown by further researches, allows to describe correctly a process of transformation of the energy of toroidal Proctor-Sivashinsky vortices (which forms the periodic structure) into the energy of large-scale poloidal vortex motion [66, 67]. This phenomenon of the "hydrodynamic dynamo" is may be responsible for formation of large-scale vortices in convective layers, in particular in the atmosphere of planets. However even in the Proctor-Sivashinsky model not all processes and the phenomena were studied.

The detailed analysis of instability leading to formation of the quasi-stationary structure – convective rolls will be presented below. The model [27] with use of the multimode description allowed to find out that at first the quasi-stable long-living state (the curved quasi-one-dimensional convective rolls) arises. And later after a lapse of time (which is considerably greater than the reverse linear increment of the process), the system transforms to the stable state (square convective cells). The detailed treatment of the Proctor – Sivashinsky model [28,68] presented below have shown that this structural transition demonstrates all the characteristics of second order phase transition (the continuity of the sum of squared mode amplitudes over the spectrum $I = \sum_j a_j^2 \equiv \sum_{k_j} |a_{k_j}|^2$ or that the same, the continuity of density of this value and discontinuity of its time derivative $\partial I / \partial t$). The important problem discussed in this paper is the determination of level of imperfection of originating regular structures and also the searching for a correlation between integral spectral characteristics and a fraction of defective cells in the originating structure. The deficiency of structures appears, in particular, in the intermediate, transient regimes and is caused by stimulated (due to non-equilibrium) interference of growing modes [52]. In the case of the external influence, the noise is able to support the number of weak spatial modes which were suppressed before and which interference with dominating modes is also capable to provide the interference pattern corresponding to the imperfect spatial lattice. The understanding of processes which lead to violations of spatial periodicity of structures, would allow estimating the level of structures imperfection by their spatial spectrum that can be quite possible measured experimentally. Especially, it should be clarified the influence of external noise on stability of states and structural-phase transitions.

It has been just the existence of the preferred scale (the distances between the regular spatial perturbations) and a possibility of selection of the required steady symmetry (the regular spatial configuration) motivate the interest to this physical model, especially for description of processes in solid state physics where the characteristic distance between elements of spatial structures (atoms, molecules) in their condensed state is almost invariable. This model, as it turned out, is capable to describe not only the nature of pattern formation including with local spatial defects, but also simulate the dynamics of second order phase transitions in a two-dimensional case.

It is also shown below that the intermediate states with broken short-range order, but saving a long-range order can be a result of structural-phase transitions (second order phase transitions) and demonstrate the same formation dynamics, as the regular spatial structures.

Of some interest is the evolution of the secondary modulation instability of the convective cells, which results in formation of a self-similar structure - the convective cells of various scales [27], and also in generation of large-scale poloidal vortices [66, 67]. This phenomenon, which was previously investigated for irregular models (see detailed review [25]), come about from modulation instability of the regular convective structure of finite amplitude, as was predicted earlier by S.S. Moiseev.

The objective in this work is to understand the mechanisms of formation of spatial convection structures. The dynamics and nature of structural phase transitions between structures of different topology are considered in details. We study the development of secondary instability, which leads to generation of large-scale patterns, known as the hydrodynamic dynamo effect. Besides the regular periodic structures, we also analyze the imperfect patterns i.e. the structures with implemented spatial defects.

2. RAYLEIGH-BENARD CONVECTION

2.1. Main equations

Thermal convection in a thin horizontal layer of fluid heated from below can be represented by equations of hydrodynamics and thermal conduction in the Boussinesq approximation [69]

$$\Pr^{-1}\left(\frac{\partial\vec{u}}{\partial t}+\left(\vec{u}\nabla\right)\vec{u}\right)=-\frac{\nabla p}{p}+\Theta\vec{z}+\nabla^2 u \tag{2.1}$$

$$\left\{\frac{\partial}{\partial t}+(\vec{u}\nabla)\right\}\Theta=Ra\left(\vec{z}\vec{u}\right)+\nabla^2\Theta \tag{2.2}$$

$$\nabla\vec{u}=0 \tag{2.3}$$

with the boundary conditions, for example, at the solid boundary $u_z=\partial u_z/\partial z=0$. Here $\vec{z}$ is the unit vector directed against the gravity force, $\vec{u}$ is the velocity vector, p and Θ are the pressure and the temperature deviation from equilibrium (varies linearly) correspondingly, g is the gravity acceleration.

Consider the derivation of the Proctor-Sivashinsky equation, which describes the convection in a thin liquid layer with a weakly conductive heat the walls, following to [63]. Here the value of Prandtl number $\Pr=\nu/\chi$ is taken to be infinity. Due to buoyancy convection in a liquid or a gas (at the appropriate scales of processes) can be represented by the following equations for the dimensionless temperature θ and stream function ψ [63].

$$\frac{\partial\theta}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}+\frac{\partial\psi}{\partial x}=\nabla^2\theta \tag{2.4}$$

$$Ra\frac{\partial\theta}{\partial x}=\nabla^4\psi \tag{2.5}$$

The range of the spatial parameters

$$-\infty<x<\infty\,,\quad 0<y<1\,. \tag{2.6}$$

For a layer between two planes boundary conditions take the form

$$\psi\,|_{y=0}=0\,,\ \frac{\partial\psi}{\partial y}|_{y=0}\equiv\psi_y\,|_{y=0}=0\,,\ \frac{\partial\theta}{\partial y}|_{y=0}-b\theta\,|_{y=0}\equiv\theta_y\,|_{y=0}-b\theta\,|_{y=0}=0\,, \tag{2.7}$$

$$\psi\,|_{y=1}=0\,,\ \frac{\partial\psi}{\partial y}|_{y=1}\equiv\psi_y\,|_{y=1}=0\,,\ \frac{\partial\theta}{\partial y}|_{y=1}+b\theta\,|_{y=1}\equiv\theta_y\,|_{y=1}+b\theta\,|_{y=1}=0\,, \tag{2.8}$$

where θ is the dimensionless temperature in terms of the difference between fixed temperature of the bottom plane T_d and the upper surface T_u in the absence of convection; ψ is the dimensionless velocity in units of the thermal diffusivity χ (is equal to the coefficient of thermal conductivity λ, divided by the density of ρ and thermal conductivity C_p) of fluid; coordinates in terms of the layer thickness d, time interval d^2/χ, the Prandtl number $\Pr=\nu/\chi$ is one of the criteria of similarity; the Rayleigh number $\mathrm{Ra}=\sigma g(T_d-T_u)d^3/\nu\chi$ – the number that determines the behavior of the fluid under the action of a temperature gradient (convection currents arise when this parameter exceeds the threshold value [70]); b is the dimensionless coefficient characterizing the heat transfer between the fluid and the boundary, here it is taken equal to the Bio number $\mathrm{Bi} = \alpha l/\lambda$ – the similarity coefficient for stationary heat exchange between hot or cold body and the environment that is the same on the lower and upper boundaries of the layer; where ν is the kinematic viscosity (dynamic viscosity of the kinematic, multiplied by the density), σ here is the coefficient of thermal expansion of the liquid, α is the coefficient of heat transfer from the surface to the environment, λ is the thermal conductivity, l is the characteristic distance.

The convection flows arise when the Rayleigh number Ra_c exceeds some threshold value

$$Ra = Ra_c(1+\varepsilon), \tag{2.9}$$

where we assume

$$\varepsilon << 1 \tag{2.10}$$

We assume also the small heat loss through the walls, that is

$$b = \varepsilon^2 \cdot \beta, \tag{2.11}$$

and use the following traditional [63] scaling for spatial and temporal variables

$$\xi = x\sqrt{\varepsilon}, \tag{2.12}$$

$$\eta = y, \tag{2.13}$$

$$\tau = \varepsilon^2 t. \tag{2.14}$$

2.2. Selection of scaling

The choice of scaling (2.12) - (2.14) is based on the analysis of satisfying the boundary conditions of the linear problem (2.4) - (2.8) and can be argued as follows. Let seek the solution in the form

$$\theta = h(y)\cdot \exp\{\Omega t + iKx\} \tag{2.15}$$

$$\psi_x = v(y)\cdot \exp\{\Omega t + iKx\} \tag{2.16}$$

Now, the boundary value problem for these functions takes the form

$$\Omega h = h_{yy} + K^2 h - v = 0, \tag{2.17}$$

$$v_{yyyy} + 2K^2 v_{yy} + K^4 v - Ra\cdot K^2 h = 0, \tag{2.18}$$

$$h_y|_{y=0} = bh|_{y=0}, \quad v_y|_{y=0} = v|_{y=0} = 0, \tag{2.19}$$

$$h_y|_{y=1} = -bh|_{y=1}, \quad v_y|_{y=1} = v|_{y=1} = 0. \tag{2.20}$$

In order to find the dependence $\Omega = \Omega(K)$ we use the Bubnov–Galerkin method, for which propose it is necessary to determine the basis functions satisfying the boundary value problem. For example, the authors [63] recommend the using of

$$v(y) = Ay^2(1-y)^2, \tag{2.21}$$

$$h(y) = B(by^2 - by - 1), \tag{2.22}$$

Then we must substitute (2.21) and (2.22) into the left side of (2.17) (2.18). Result of the substitution (discrepancy) must be orthogonal basis functions, that is received two expressions must be multiplied respectively by the functions $(by^2 - by - 1)$ and $y^2(1-y)^2$ and integrate over y from zero to one and put result of integration equal to zero, which gives a system of two equations

$$A\left\{\frac{4}{5} + \frac{4}{105}K^2 + \frac{1}{610}K^4\right\} + B\left\{Ra\cdot K^2\left(\frac{b}{140} + \frac{1}{30}\right)\right\} = 0 \tag{2.23}$$

$$A\left\{\frac{b}{140}+\frac{1}{30}\right\}+B\left\{2b+\frac{b^2}{3}+\left(\Omega+K^2\right)\left(\frac{b^2}{30}+\frac{b}{3}+1\right)\right\}=0 \tag{2.24}$$

At first, we find the stability threshold for steady state, assuming $\Omega=0$ and keeping only the terms proportional to K^2. Then, one obtains for Ra

$$Ra=720+\frac{240}{7}K^2+1440\frac{b}{K^2}. \tag{2.25}$$

In the absence of convection $\mathrm{Ra}_c=720$. For the case of $\Omega\neq 0$, the dispersion equation $\Omega=\Omega(K)$ can be obtained from the requirement on non-triviality of solution of the system (2.23) - (2.24)

$$\Omega=\left(\frac{Ra-Ra_c}{Ra_c}\right)K^2-\frac{1}{21}K^4+2b. \tag{2.26}$$

As $\left(\mathrm{Ra}-\mathrm{Ra}_c\right)/\mathrm{Ra}_c\propto\varepsilon$, $b\propto\varepsilon^2$, the dependences of other variables from ε can be easily restored, e.g. $\Omega\propto\varepsilon^2$, $K\propto\sqrt{\varepsilon}$ that allows to select the desired scaling. Let assume in following that the nonlinear problem has the same scaling.

Note that $\psi\propto\theta\sqrt{\varepsilon}$ as follows from (2.5), and then we assume for convenience

$$\Theta(\xi,\eta,\tau,\varepsilon)=\theta(x,y,t,\varepsilon), \tag{2.27}$$

$$\Psi(\xi,\eta,\tau,\varepsilon)=\sqrt{\varepsilon}\cdot\psi(x,y,t,\varepsilon). \tag{2.28}$$

Now, in the new variables and notation the problem can be written as

$$\varepsilon^2\frac{\partial\Theta}{\partial\tau}+\varepsilon\frac{\partial\Psi}{\partial\eta}\frac{\partial\Theta}{\partial\xi}-\varepsilon\frac{\partial\Psi}{\partial\xi}\frac{\partial\Theta}{\partial\eta}+\varepsilon\frac{\partial\Psi}{\partial\xi}=\varepsilon\frac{\partial^2\Theta}{\partial\xi^2}+\frac{\partial^2\Theta}{\partial\eta^2}, \tag{2.29}$$

$$Ra_c\cdot(1+\varepsilon)\frac{\partial\Theta}{\partial\xi}=\varepsilon^2\frac{\partial^2\Psi}{\partial\xi^2}+2\varepsilon\frac{\partial^2\Psi}{\partial\xi\cdot\partial\eta}+\frac{\partial^2\Psi}{\partial\eta^2}, \tag{2.30}$$

$$\Psi|_{\eta=0}=0,\qquad \frac{\partial\Psi}{\partial\eta}|_{\eta=0}\equiv\Psi_\eta|_{\eta=0}=0,..\frac{\partial\Theta}{\partial\eta}|_{\eta=0}-\varepsilon^2\beta\cdot\Theta|_{\eta=0}\equiv\Theta_\eta|_{\eta=0}-\varepsilon^2\beta\cdot\Theta|_{\eta=0}=0 \tag{2.31}$$

$$\Psi|_{\eta=1}=0,\ \frac{\partial\Psi}{\partial\eta}|_{\eta=1}\equiv\Psi_\eta|_{\eta=1}=0,..\frac{\partial\Theta}{\partial\eta}|_{\eta=1}-\varepsilon^2\beta\cdot\Theta|_{\eta=1}\equiv\Theta_\eta|_{\eta=1}-\varepsilon^2\beta\cdot\Theta|_{\eta=1}=0 \tag{2.32}$$

Let's integrate (2.29), using the identity

$$\Theta_\eta\Psi_\xi-\Theta_\xi\Psi_\eta=\left(\Theta\Psi\right)_{\xi\eta}-\left(\Theta_\eta\Psi\right)_\xi-\left(\Theta\Psi_\xi\right)_\eta \tag{2.33}$$

and the boundary conditions (2.31) and (2.32).

$$\varepsilon\frac{\partial}{\partial\tau}\int_0^1 d\eta\cdot\Theta-\frac{\partial}{\partial\xi}\int_0^1 d\eta\cdot\Theta_\eta\Psi+\frac{\partial}{\partial\xi}\int_0^1 d\eta\cdot\Psi=\frac{\partial^2}{\partial\xi^2}\int_0^1 d\eta\cdot\Theta-\beta\varepsilon\left(\Theta|_{\eta=0}+\Theta|_{\eta=1}\right), \tag{2.34}$$

2.3. The hierarchy of approximations

It is appropriate to seek the solution of (2.29) - (2.32) in the form of series

$$\Psi=\Psi^{(0)}+\varepsilon\cdot\Psi^{(1)}+\varepsilon^2\cdot\Psi^{(2)}+\dots, \tag{2.35}$$

$$\Theta=\Theta^{(0)}+\varepsilon\cdot\Theta^{(1)}+\varepsilon^2\cdot\Theta^{(2)}+\dots. \tag{2.36}$$

In the zero-order approximation

$$\Theta_{\eta\eta}^{(0)}=0, \tag{2.37}$$

$$\Psi_{\eta\eta\eta\eta}{}^{(0)} = Ra_c \cdot \Theta_\xi{}^{(0)}, \tag{2.38}$$

$$\Psi^{(0)}|_{\eta=0} = 0, \ \Psi_\eta{}^{(0)}|_{\eta=0} = 0, \ \Theta^{(0)}{}_\eta|_{\eta=0} = 0, \tag{2.39}$$

$$\Psi^{(0)}|_{\eta=1} = 0, \ \Psi_\eta{}^{(0)}|_{\eta=1} = 0, \ \Theta^{(0)}{}_\eta|_{\eta=1} = 0 \tag{2.40}$$

Thus,

$$\Theta^{(0)} = F(\xi, \tau), \tag{2.41}$$

$$\Psi^{(0)} = \frac{1}{24} Ra_c \cdot F_\xi \left(\eta^4 - 2\eta^3 + \eta^2\right). \tag{2.42}$$

For the zero-order approximation Eq. (2.34) takes the form

$$-\frac{\partial}{\partial\xi}\int_0^1 d\eta \cdot \Theta^{(0)}{}_\eta \Psi^{(0)} + \frac{\partial}{\partial\xi}\int_0^1 d\eta \cdot \Psi^{(0)} = \frac{\partial^2}{\partial\xi^2}\int_0^1 d\eta \cdot \Theta^{(0)}, \tag{2.43}$$

Substituting (2.41) (2.42) to (2.43) we can see that a certain critical value of the Rayleigh number

$$\left(Ra_c - 720\right) \cdot F_{\eta\eta} = 0. \tag{2.44}$$

The next approximation gives following

$$\frac{\partial\Psi^{(0)}}{\partial\eta}\frac{\partial\Theta^{(0)}}{\partial\xi} - \frac{\partial\Psi^{(0)}}{\partial\xi}\frac{\partial\Theta^{(0)}}{\partial\eta} + \frac{\partial\Psi^{(0)}}{\partial\xi} = \frac{\partial^2\Theta^{(0)}}{\partial\xi^2} + \frac{\partial^2\Theta^{(1)}}{\partial\eta^2}, \tag{2.45}$$

$$Ra_c \cdot \frac{\partial\Theta^{(0)}}{\partial\xi} + Ra_c \cdot \frac{\partial\Theta^{(1)}}{\partial\xi} = 2\varepsilon \frac{\partial^4\Psi^{(0)}}{\partial\xi^2 \cdot \partial\eta^2} + \frac{\partial^4\Psi^{(1)}}{\partial\eta^4}, \tag{2.46}$$

$$\Psi^{(1)}|_{\eta=0} = 0, \ \Psi_\eta{}^{(1)}|_{\eta=0} = 0, \ \Theta^{(1)}{}_\eta|_{\eta=0} = 0, \tag{2.47}$$

$$\Psi^{(1)}|_{\eta=1} = 0, \ \Psi_\eta{}^{(1)}|_{\eta=1} = 0, \ \Theta^{(1)}{}_\eta|_{\eta=1} = 0, \tag{2.48}$$

Using the zero-order approximation, for the first approximation we obtain expressions

$$\Theta^{(1)} = G(\xi, \eta) + F_\xi^2\left(6\eta^5 - 15\eta^4 + 10\eta^3\right) + \frac{1}{2}F_{\xi\xi}\left(2\eta^6 - 6\eta^5 + 5\eta^4 - \eta^2\right), \tag{2.49}$$

$$\begin{aligned} \Psi^{(1)} &= \frac{10}{7}F_\xi F_{\xi\xi}\left(2\eta^9 - 9\eta^8 + 12\eta^7 - 20\eta^3 + 15\eta^2\right) + \\ &+ \frac{1}{14}F_{\xi\xi\xi\xi}\left(2\eta^{10} - 9\eta^9 + 15\eta^8 - 42\eta^6 + 84\eta^5 - 70\eta^4 + 20\eta^3 + \eta^2\right) + \\ &+ 30F_\xi\left(\eta^4 - 2\eta^3 + \eta^2\right) + 30G_\xi\left(\eta^4 - 2\eta^3 + \eta^2\right). \end{aligned} \tag{2.50}$$

Meanwhile, the integral (2.43)

$$\begin{aligned} &\frac{\partial}{\partial\tau}\int_0^1 d\eta \cdot \Theta^{(0)} - \frac{\partial}{\partial\xi}\int_0^1 d\eta \cdot \left(\Theta^{(1)}{}_\eta \Psi^{(0)} + \Theta^{(0)}{}_\eta \Psi^{(1)}\right) + \frac{\partial}{\partial\xi}\int_0^1 d\eta \cdot \Psi^{(1)} = \\ &= \frac{\partial^2}{\partial\xi^2}\int_0^1 d\eta \cdot \Theta^{(1)} - \beta\left(\Theta^{(0)}|_{\eta=0} + \Theta^{(0)}|_{\eta=1}\right) \end{aligned} \tag{2.51}$$

Substituting the expressions for zero and first approximation, we can find the equation for $F(\xi, \eta)$

$$\frac{\partial F}{\partial\tau} + \frac{17}{462}\frac{\partial^4 F}{\partial\xi^4} + \frac{\partial}{\partial\xi}\left[\left(1 - \frac{10}{7}\left(\frac{\partial F}{\partial\xi}\right)^2\right)\frac{\partial F}{\partial\xi}\right] + 2\beta F = 0 \tag{2.52}$$

which is symmetrical with respect to change the sign F (By the way, the equations (2.1) - (2.5) has symmetry while simultaneously changing the sign y θ,ψ,y). In addition, the equation has not diffusion term $\propto \partial^2 F/\partial\xi^2$.

Table 1.

The correspondence of used variables and their real physical values

Physical quantity	Representation of explicit view
Temperature $T\left(x\sqrt{\varepsilon},y\right)$	$T_d+\left(T_d-T_u\right)\left(-y+F(x\sqrt{\varepsilon},y\right)$
Horizontal velocity ψ_y	$60\sqrt{\varepsilon}\cdot F_{x\sqrt{\varepsilon}}\cdot\left(2y^3-3y^2+y\right)$
Vertical velocity $-\psi_x$	$-30\varepsilon\cdot\left(F_{x\sqrt{\varepsilon}}\right)_{x\sqrt{\varepsilon}}\cdot\left(y^4-2y^3+y^2\right)$

where $\xi=x\sqrt{\varepsilon}$, $\eta=y$, $Ra=Ra_c\left(1+\varepsilon\right)$.

2.4. Accounting for temperature dependence of viscosity

Equation (2.4) in this case takes the form

$$Ra\frac{\partial\theta}{\partial x}=\left(\nu\psi_{xx}\right)_{xx}+2\left(\nu\psi_{xy}\right)_{xy}+\left(\nu\psi_{yy}\right)_{yy}, \tag{2.53}$$

where ν is the temperature-dependent viscosity, normalized, as noted above. Moreover, to simplify the model, we set this dependence is sufficiently weak

$$\nu=1+\varepsilon\cdot\mu\cdot\left(0.5-\eta+\theta\right) \tag{2.54}$$

where the terms $-\eta+\theta$ takes into account the temperature perturbations in the vertical direction and μ is the numerical coefficient of the order of one. Equation (2.52) will undergo some changes

$$\frac{\partial F}{\partial\tau}+\frac{17}{462}\frac{\partial^4 F}{\partial\xi^4}+\frac{\partial}{\partial\xi}\left[\left(1-\mu F-\frac{10}{7}\left(\frac{\partial F}{\partial\xi}\right)^2\right)\frac{\partial F}{\partial\xi}\right]+2\beta F=0\,. \tag{2.55}$$

2.5. The Proctor-Sivashinsky equation

Applying the following notation

$$F=\varphi\cdot\sqrt{17/660}\,,\ \beta=a\cdot\sqrt{231/68}\,,\ \mu=\gamma\cdot\sqrt{165/17}\,,\ \xi=\zeta\cdot\sqrt{17/231}\,,\ \tau=\mathrm{T}\cdot\sqrt{34/231}\,, \tag{2.56}$$

we obtain the Proctor-Sivashinsky equation having regard the temperature dependence of the viscosity in the one-dimensional case

$$\frac{\partial\varphi}{\partial T}+\frac{\partial^4\varphi}{\partial\zeta^4}+\frac{\partial}{\partial\zeta}\left[\left(2-\gamma\varphi-\left(\frac{\partial\varphi}{\partial\zeta}\right)^2\right)\frac{\partial\varphi}{\partial\zeta}\right]+a\varphi=0\,. \tag{2.57}$$

It should be noted, that the equation contains the nonlocal quadratic nonlinearity $-\gamma\frac{\partial}{\partial\zeta}\left(\varphi\frac{\partial\varphi}{\partial\zeta}\right)$ due to the dependence of viscosity on temperature with the layer height, and the nonlocal cubic nonlinearity in the form of $\frac{\partial}{\partial\zeta}\left[\left(\frac{\partial\varphi}{\partial\zeta}\right)^2\frac{\partial\varphi}{\partial\zeta}\right]$.

In two-dimensional geometry, Eq. (2.57) takes the form

$$\frac{\partial\varphi}{\partial T}+\nabla^4\varphi+\nabla\left[\left(2-\gamma\varphi-\left(\nabla\varphi\right)^2\right)\nabla\varphi\right]+a\varphi=0\,, \tag{2.58}$$

where the two-dimensional operator $\nabla\varphi=\vec{i}\cdot\frac{\partial\varphi}{\partial\zeta}+\vec{j}\cdot\frac{\partial\varphi}{\partial\vartheta}$ with unitary orthogonal vectors $\vec{i}$ and $\vec{j}$ oriented in the plane of the medium division (ζ,ϑ).

If the temperature dependence of the viscosity can be neglected $\gamma = 0$, the model, commonly referred as the Proctor-Sivashinsky model, contains just a nonlocal cubic nonlinearity $\nabla\left[\left(\nabla\varphi\right)^2\nabla\varphi\right]$

$$\frac{\partial\varphi}{\partial T}+\nabla^4\varphi+\nabla\left[\left(2-\left(\nabla\varphi\right)^2\right)\nabla\varphi\right]+a\varphi=0\,. \tag{2.59}$$

This equation can be written as

$$\frac{\partial\varphi}{\partial T}=-\frac{\delta F\left[\varphi\right]}{\delta\varphi} \tag{2.60}$$

where $\delta F[\varphi]/\delta\varphi$ is the variation derivative of the functional

$$F\left[\varphi\right]=-\int d\zeta\cdot d\vartheta\cdot\left\{\left(\nabla\varphi\right)^2-\frac{1}{4}\left(\nabla\varphi\right)^4-\frac{1}{2}\left(\nabla^2\varphi\right)^2-\frac{a}{2}\varphi^2\right\} \tag{2.61}$$

Th particular feature of this model is that it describes as the Marangoni convection and convection in a layer with one free boundary [63].

3. ANALYSIS OF THE SIMPLIFIED SWIFT-HOHENBERG MODEL

3.1. Instability regimes

Consider the Proctor-Sivashinsky equation with the temperature dependence of viscosity in one dimension case. Equation (2.58) can be rewritten as

$$\frac{\partial\varphi}{\partial T}+\left(a-1\right)\varphi+\left(1+\frac{\partial^2}{\partial\zeta^2}\right)^2\varphi-\gamma\frac{\partial}{\partial\zeta}\left(\varphi\frac{\partial\varphi}{\partial\zeta}\right)-\frac{\partial}{\partial\zeta}\left[\left(\frac{\partial\varphi}{\partial\zeta}\right)^2\frac{\partial\varphi}{\partial\zeta}\right]=0\,. \tag{3.1}$$

Let assume the dependence of φ on coordinates as $\varphi\propto\exp\{ik_0\zeta\}$ and $k_0\approx 1$, $e=(1-a)$ since we consider only the weak above-threshold case $|e|<<1$. Indeed, for any deviation from unity of the wave number perturbation amplitude decrease rapidly. The equation (3.1) takes the form

$$\frac{\partial\varphi}{\partial T}=e\varphi-\left(1+\frac{\partial^2}{\partial\zeta^2}\right)^2\varphi-2\gamma\varphi^2+3\varphi^3 \tag{3.2}$$

The growth of the instability $\varphi\propto\exp\left(\operatorname{Im}\omega T\right)$ happens with the growth rate $\operatorname{Im}\omega\approx e-(k^2-1)^2$. For $\gamma>0$ the gas (this corresponds to the gas convection) flows up to the center of the cell, for $\gamma<0$ (which corresponds to the movement of the liquid) the liquid flows outside and down from the center of the cell.

The perturbations with wave vectors in the vicinity of the unit are unstable, that is,

$$1-e<|k|<1+e \tag{3.3}$$

3.2. Swift-Hohenberg model

Equation (3.2) is called the Swift – Hohenberg equation [59].

The parameter $\gamma<0$ determines for convection the instability threshold. So when $e<0$, the subthreshold (subcritical) growth of perturbations is possible only for perturbation amplitudes exceeding the threshold

$$\varphi_{thr}=-\frac{2|\gamma|}{6}+\sqrt{\frac{4\gamma^2}{36}+\frac{|e|}{3}} \tag{3.4}$$

Despite of some unacceptable (from the point of view of strict approach) assumptions, it has been possible to obtain from this equation a number of interesting and relevant effects observed in experiments.

In the one dimension case, (3.2) can be written in gradient form with the Lyapunov potential (free energy functional):

$$F\left[\varphi\right]=-\int d\zeta\cdot\left\{-\frac{2|\gamma|}{\sqrt{3}}\varphi^3+\frac{1}{4}\varphi^4+\frac{1}{2}\left(1+\frac{\partial^2}{\partial\xi^2}\right)^2\varphi^2-\frac{\varepsilon}{2}\varphi^2\right\}. \tag{3.5}$$

Note that

$$\frac{\partial}{\partial T}F[\varphi]=-\int d\zeta\cdot\frac{\partial}{\partial T}\left\{-\frac{2|\gamma|}{\sqrt{3}}\varphi^3+\frac{1}{4}\varphi^4+\frac{1}{2}(1+\frac{\partial^2}{\partial\xi^2})^2\varphi^2-\frac{\varepsilon}{2}\varphi^2\right\}=-\int d\zeta\left(\frac{\partial\varphi}{\partial T}\right)^2<0, \tag{3.6}$$

If the free energy functional has no minima, then the front propagation will be observed as, for example, in the equations describing the reaction of burning. In this case, the free energy functional will be continuosly decreasing until the front approaches the boundary of the medium if it is bounded. An alternative possibility is realized when the free energy functional has minima. There may be many such minima. Each minimum corresponds to an equilibium state in time (multistability).

The Swift-Hohenberg equation can be generalized to two-dimensional case in the following way

$$\frac{\partial\varphi}{\partial T}=e\varphi-\left(1+\nabla^2\right)^2\varphi-2\gamma\varphi^2+3\varphi^3, \tag{3.7}$$

or in the gradient form

$$\frac{\partial\varphi}{\partial T}=-\frac{\delta F[\varphi]}{\delta\varphi}, \tag{3.8}$$

$$F[\varphi]=-\int d\zeta\cdot\left\{-\frac{2|\gamma|}{\sqrt{3}}\varphi^3+\frac{1}{4}\varphi^4+\frac{1}{2}\left(1+\nabla^2\right)^2\varphi^2-\frac{\varepsilon}{2}\varphi^2\right\}. \tag{3.9}$$

Position of the functional minima determines the stable or quasistable state. Note that extension on two-dimensional case like that of the Swift-Hohenberg equation is incorrect, and justifies this is only a good qualitative agreement between the simulation results and experimental data. Incorrectness seen in the fact, that basic and well-received equation of convection in this case is a multidimensional the Proctor-Sivashinsky equation, which has a non-local quadratic (occurring only with accounting of the temperature dependence of viscosity) and cubic nonlinearity. The transition to local dependence in this case is not formally justified. However, as noted in the introduction to this paper for simulation this modification of equations is useful, because they retain a number of important physical features of the systems and quite simple to describe.

3.3. Dynamics of spatial structure defects

During the formation of stable or quasi-stable structures occurs the violations of regularity, defects. More often occurs point and one-dimensional defects, and last one correspond to the dislocation in imperfect crystal. The dynamics of these defects has been well studied [71, 72]. The mobility of defects has come to the end when a stable state is reached. As this takes place, the defects are comes to rest or disappear. The force acting on the defects associated with the change in the functional (3.9) in the vicinity of its action.

3.4. Development of instabilities

In the above-threshold regime $e>0$, the perturbations of arbitrarily small amplitude become unstable. The bell-shaped initial perturbation begins expansion from its center (Fig.1). If the functional has a minimum that we can observe the formation of a periodic structure behind the propagating front which can be considered as the structurization front. Similar processes of structure formation is typical for the first-order phase transitions.

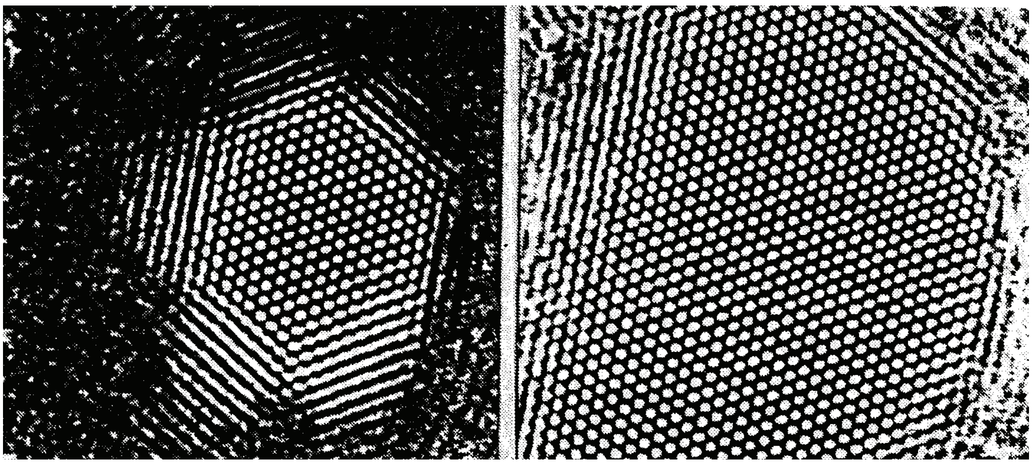


Fig. 1. Experiment. Thermal convection in a thin layer of gaseous CO_2 [73,49]. Growth of hexagonal crystal lattice in the above-threshold mode (figure on the right corresponds to a later time)

In the subthreshold regime $e < 0$ only perturbations of finite amplitude (3.4) grow, which leads either to formation of a localized perturbation region, if the functional has no minimum, or to the formation of structurization region of finite size, if the functional has minima (Fig.2).

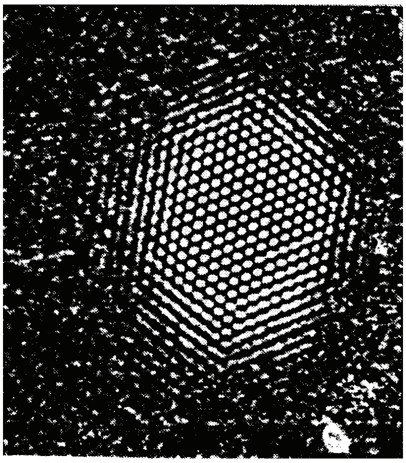


Fig. 2. Experiment. Thermal convection in a thin layer of gaseous CO_2 [73].
Stable localized state in the subthreshold state.

Of particular interest is the growth dynamic of perturbations in the above-threshold regime $e > 0$ for given periodic boundary conditions. In this case, the functional has several minima and it results in arising of expanding areas containing hexagonal cells that simulates Benard-Marangoni convection in a plane layer of liquid heated from below. That is convective rolls (poorly defined metastable structure) quickly decays into hexagonal cells. By the way, the formation of hexagonal structure is caused by existence of fixed scale that requires an equal distance between the maxima and minima of the field correspondingly. In two dimension case, this topology only is possible in an equilateral triangle, set of which generates a hexagon.

It should be noted that results obtained by simulation of the Swift-Hohenberg model and experimental results are demonstrate qualitative agreement despite the fact that after generalization of this model to the two-dimensional case, the non-local nonlinearity in quadratic and cubic term has been replaced by a local one that results in the loss of anisotropy in the cubic nonlinearity. However, these problems were solved in the below discussed Proctor-Sivashinsky model, where the cubic nonlinearity retained the anisotropy.

So, we can conclude that the models of this type demonstrate the behavior, which very close to experimental observations. This is indicative of the physical adequacy of the models, based on the differential and integro-differential equations and their applicability for analysis of a number of physical processes which take place in technical devices.

4. PROCTOR-SIVASHINSKY MODEL TO DESCRIBING INVISCID CONVECTION

4.1. Convection equation including the noise influence

More correct description of convection in a thin layer of liquid or gas with poorly heat conducting boundaries can be carried out within the framework of the Proctor-Sivashinsky model.

As shown below, this model allows detecting the structural-phase transitions which can be identified as second order phase transitions. In addition, if we define the quasi-crystal as a spatial structure with long-range order and broken translational symmetry than one of the metastable states corresponds to this definition. In this case, the structural transitions leading to this state can be also considered as second-order transitions. Namely the analysis of phase structure transitions is being the objective of this work, because the transformation of the spatial structures of all scales (from planetary to microscopic) in the hydro- and gas dynamics is the key problem in the subject area of physical technologies.

Restrict the consideration to the case of the lack of dependence of viscosity on the temperature ($\gamma = 0$). The equation for the temperature field in the horizontal plane (*x,y*) has the form:

$$\frac{\partial \Phi}{\partial T} = \varepsilon^2 \Phi - \left(1 - \nabla^2\right)^2 \Phi + \frac{1}{3} \nabla \left(\nabla \Phi \left| \Phi \right|^2 \right) + \varepsilon^2 f \ , \tag{4.1}$$

where $\Phi = \theta / \sqrt{3}$, $\varepsilon^2 = e$, *f* is the random function describing the external noise, and the quantity ε that determines the convection threshold overriding is assumed to be sufficiently small ($0 < \varepsilon < 1$).

We shall search for solution of Eq. (4.1) in the form of series

$$\Phi = \varepsilon \sum_j A_j \exp\left(i \vec{k}_j \vec{r} \right) \tag{4.2}$$

where $|\vec{k}_j|=1$. Substituting $T\varepsilon^2 = t$, we get the mathematical expression of the Proctor-Sivashinsky model for slow amplitudes A_j (with additional term corresponding to the noise):

$$\frac{\partial A_j}{\partial t} = A_j - \sum_{i=1}^{N} V_{ij} \, |A_i|^2 \, A_j + f \tag{4.3}$$

where the interaction coefficients V_{ij} are defined as follows

$$V_{jj} = 1, \tag{4.4}$$

$$V_{ij} = (2/3)\left(1 - 2\left(\vec{k}_i \vec{k}_j\right)^2\right) = (2/3)\left(1 + 2\cos^2\vartheta\right), \tag{4.5}$$

and ϑ is the angle between vectors $\vec{k}_i$ and $\vec{k}_j$. Here we should note the difference between Eq. (4.3) with interaction potential (4.4)-(4.5) from the two-dimensional Swift-Hohenberg equation, where the cubic nonlinearity has the isotropic but not a vector form which was made from the qualitative considerations.

Expressions (4.1) - (4.5) should be supplemented by the initial values of the amplitudes A_j

$$A_j \,|_{t=0} = A_{j0}. \tag{4.6}$$

The instability interval in *k*-space represents a ring with average radius equal to unit and the width is order of relative above-threshold parameter ϵ, i.e. much less than unity. During the development of the instability, the effective growth rate of modes that lies outside of the very small neighborhood near the unit circle will decrease due to the growth of the nonlinear terms and can change sign which will lead to a narrowing of the spectrum to the unit circle in the *k*-space. Since the purpose of further research will be the study of stability of spatial structures with characteristic size of order $2\pi / k \propto 2\pi$ and the important characteristic for visualization of simulation results will be evidence of these structures, so we restrict ourselves by considering some idealized model of the phenomenon, assuming that the oscillation spectrum is already located on the unit circle in the *k*-space.

From the results of preliminary studies [64] it is clear that at least two stationary solutions can exist in the system: the roll structure (Fig. 3a), and the field of square cells (Fig. 3b).

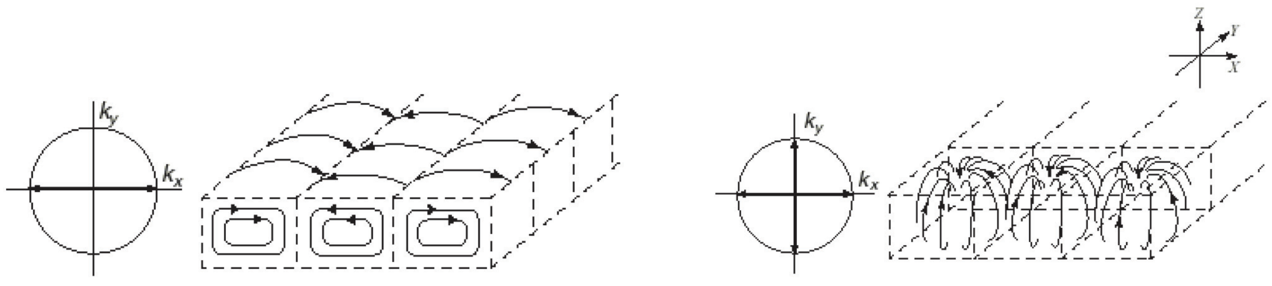


Fig. 3. Convective structures: rolls (a) and square cells (b).

4.2. Linear stability

It follows from Eqs. (4.3)-(4.5) with the initial conditions of the form (4.6) that initial development of the process will be determined by the exponential growth of the spectrum modes with the identical linear growth rate equal to unit in the conventional time scale.

Further growth of unstable modes will slow down due nonlinear terms in Eq.(4.3). The nonlinear growth rate can be written as

$$\left(\mathrm{Im}\,\omega\right)_{NL} = 1 - \langle \sum_{i=1}^{N} V_{ij} \, |A_i|^2 \rangle \propto 1 - \langle V \rangle \sum_{i=1}^{N} A_i^2 \,, \tag{4.7}$$

and the averaged interaction potential takes the value of

$$\langle V \rangle \propto \frac{4}{3} \tag{4.8}$$

Thus, we can obtain the energy density[1] of perturbation for this «*amorphous state»* at the active stage of the interaction between modes when they are approximately equal in amplitude

$$I = \frac{1}{N}\sum_{i=1}^{N} A_i^2 \approx I_{amor} = \frac{3}{4}. \tag{4.9}$$

4.3. The qualitative Proctor-Sivashinsky model for inviscid convection

We can qualitatively estimate further dynamics of the system. Let introduce the amplitude of the fundamental mode $A_1 = A(\vartheta = 0)$ (assuming it is large enough), as well the intensity of the spectrum near $\vartheta = \pi/2$ in the form of $A_{sp} = \sum_{\vartheta_i \neq \pi/2} A^2(\vartheta_i)$ excluding from this sum the central mode $A_2 = A(\vartheta = \pi/2)$. Thus, we can get from (4.3)-(4.5) the system of equations for these quantities

$$\dot{A}_1^2 = 2A_1^2\left[1 - A_1^2 - \frac{2}{3}A_2^2 - \frac{2}{3}A_{sp}\right], \tag{4.10}$$

$$\dot{A}_2^2 = 2A_2^2\left[1 - \frac{2}{3}A_1^2 - A_2^2 - 2A_{sp}\right] \tag{4.11}$$

$$\dot{A}_{sp} \approx 2A_{sp}\left[1 - \frac{2}{3}A_1^2 - 2A_2^2 - 2A_{sp}\right] \tag{4.12}$$

Dynamics of the spectrum without the central mode $A_2 = A(\vartheta = \pi/2)$ and dynamics of this mode can be described by equations

$$Y' = Y\left(1 - \frac{14}{5}X - Y\right), \quad X' = X\left(1 - X - Y\right), \tag{4.13}$$

where $Y = 14A_{sp}/3$, $X = 5A_2^2/3$, $X' \equiv dX/d\tau$, $\tau = 1.5t$, and for the quantities $I = \frac{1}{N}\sum_j A_j^2$ and $Z = 5A_1^2/3$, is valid relation of $I = \frac{3}{5}\left[X + Z + 5Y/4\right]$ and $Z = \frac{5}{3} - \frac{2}{3}X - \frac{5}{21}Y$, respectively. The result of numerical solution of Eqs.(4.13) is represented in Fig. 4.

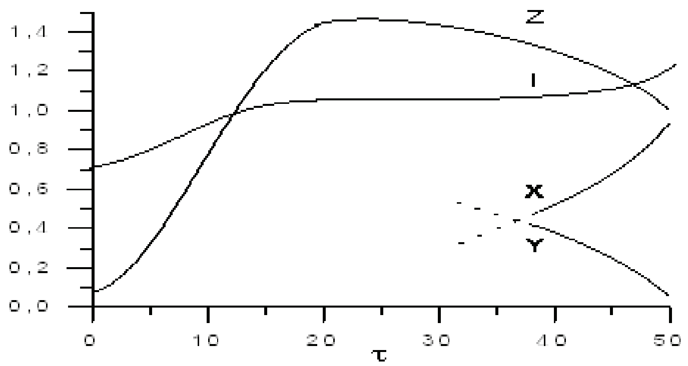


Fig. 4. Dynamics of some parameters during the secondary instability: the spectrum of without the central mode Y, the squared amplitude of the central mode of this spectrum X, squared amplitude of the fundamental mode Z and the integral value $I = \frac{1}{N}\sum_j A_j^2$.

4.4. The mechanism of mode competition

Let us consider the nonlinear growth rate (or damping factor, which depends on the sign of the expression) of each

[1] As $\sum_k |A(k)|^2 = \frac{1}{L}\int_{-L/2}^{L/2} |A(r)|^2\, dr$, where $A(k)$ is the Fourier transform of the function $A(r)$.

mode (obviously $\left(\operatorname{Im}\omega\right)_L = 1$)

$$\left(\operatorname{Im}\omega\right)_{NL} = 1 - \sum_{i=1}^{N} V_{ij} \mid A_i \mid^2 . \tag{4.14}$$

At the end of the stage of rapid linear growth, the so-called "amorphous" state is established when the nonlinear growth rates become much less than unity, and the process slows down essentially. One of the modes begins to grow due to fluctuation, while suppressing the other modes which nonlinear growth rates become negative. As a result of this competition only one mode from the initial spectrum survives. Then, it turns out that this state is also unstable, that leads to the growth of side spectrum in vicinity of $\vartheta = \pi/2$. This results in the competition among the side modes, which develops in the same scenario and only one leader mode $A_2 = A(\vartheta = \pi/2)$ survives. The fundamental mode $A_1 = A(\vartheta = 0)$ slightly reduces its amplitude, and the mode $A_2 = A(\vartheta = \pi/2)$ increases its amplitude until they align. Each state has different intensity values and has a different topology.

4.5. Accounting of the external noise

In the case of a sufficiently high level of noise, both additive ($f \neq 0$) and multiplicative (a random component proportional to ε^2 in the first term in r.h.s of Eq.(4.1)), the level of modes amplitudes may be rather large from the start of the process. The initial conditions may also provide the starting system state can be considered as highly irregularity “amorphous”, i.e. the perturbation amplitudes are large enough and randomly different from each other. This state can be maintained in the future by random noise. It is important to find out in what noise levels it is possible the "amorphous" state, characterized by a large number of spatial modes can exist for a long time.

Apparently, the very intensive noise is able to keep the system from the formation of convective structures, however, preliminary estimates suggest that the noise of lesser intensity cannot prevent the successive transition to metastable (rolls) and stable (square cells) states. When the noise intensity falls down, the transition from a metastable to stable state can slow down and the system stays (“freezes”) for a long time in the metastable state.

5. RESULTS OF NUMERICAL MODELING

5.1. The mathematical Proctor-Sivashinsky model for inviscid convection

The basic Proctor-Sivashinsky model (4.3)–(4.5) can be represented by the equation

$$\frac{\partial A_j}{\partial t} = A_j - \sum_{i=1}^{N} V_{ij} \mid A_i \mid^2 A_j + f , \tag{5.1}$$

where the coefficients are defined as following

$$V_{jj} = 1 , \tag{5.2}$$

$$V_{ij} = (2/3)\left(1 - 2\left(\vec{k}_i \vec{k}_j\right)^2\right) = (2/3)\left(1 + 2\cos^2 \vartheta\right), \tag{5.3}$$

where ϑ is the angle between vectors $\vec{k}_i$ and $\vec{k}_j$. Expressions (2.8) - (2.9) should be supplemented by the amplitudes initial values of the spectrum

$$A_j \mid_{t=0} = A_{j0} . \tag{5.4}$$

The width of the instability interval in k-space is the unit circle with a radius $\mid \vec{k}_i \mid = 1$. Let initial values of $\vartheta_s (t = 0)$ are distributed uniformly from zero to 2π for each mode and interval should be divided by N (this is the number of modes). Then, if we impose zero boundary conditions, the spatial dependence of each of n-th mode will be

$$A_{n,m} Sin\left(2\pi nx\right) Sin\left(2\pi my\right), \tag{5.5}$$

where n, m (they can be represented as $n = N\cos\vartheta_s$, $m = N\cos\vartheta_s$) are integers and $N^2 = n^2 + m^2$. In the calculations, in general, it is sufficient to sum over n, as m determined from $m^2 = N^2 - n^2$. Obviously

$$n \leq N , \quad m = \sqrt{N^2 - n^2} \geq 0 . \tag{5.6}$$

That is, in this case (5.5) can be written as

$$A_{n,\sqrt{N^2-n^2}} \sin(2\pi nx)\sin\left(2\pi y\sqrt{N^2-n^2}\right) = A_n \cdot \sin(2\pi nx)\sin\left(2\pi y\sqrt{N^2-n^2}\right), \quad (5.7)$$

5.2. Structural-phase transitions

Development of perturbations in the system, as shown by the numerical analysis of Eq.(5.1) will be as follows [4,6]. Starting from initial fluctuations, the modes over a wide range of ϑ begin grow. The value of the quadratic form of the spectrum $I = \frac{1}{N}\sum_j A_j^2$ can be estimated by equating the r.h.s of Eq. (5.1) to zero and to obtain in result a value close to 0.75. It was shown in [4, 7] that when the number of modes is sufficiently large and calculation proceeds with high precision the system delayed the development while remaining in a dynamic equilibrium. For further development - "crystallization", one of the modes must get a portion of the energy which excesses some threshold value. That is, in these case, it is necessary a certain level of noise (fluctuations). This can be achieved either at finite noise level $f \neq 0$ or by decreasing the accuracy of calculations that is the same as noted in [68]. Similar cases, when the noise can trigger or accelerate instability are reviewed in the book [26].

Researches of this process have found the following dynamics of integral characteristics with time (Fig.5, 6).

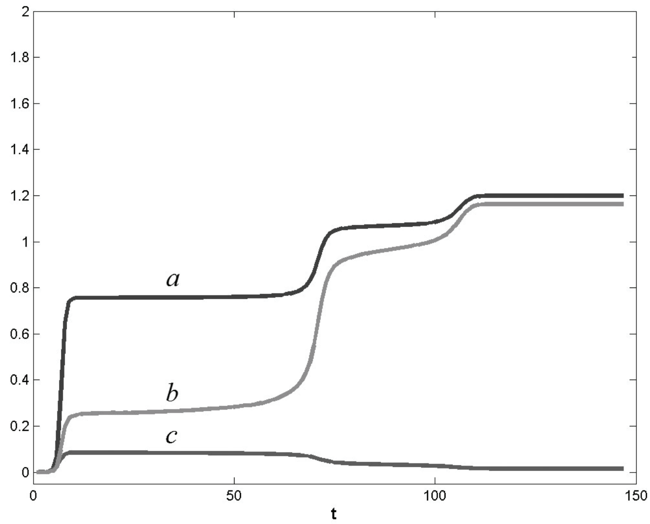


Fig. 5. Time dependence of the integral characteristics of the process.
a) $\frac{1}{N}\sum_i A_i^2$, b) $\frac{1}{N}\sqrt{\sum_i (A_i - \bar{A})^2} = \sigma$, c) $\frac{1}{N}\sum_i A_i = A_{av}$.

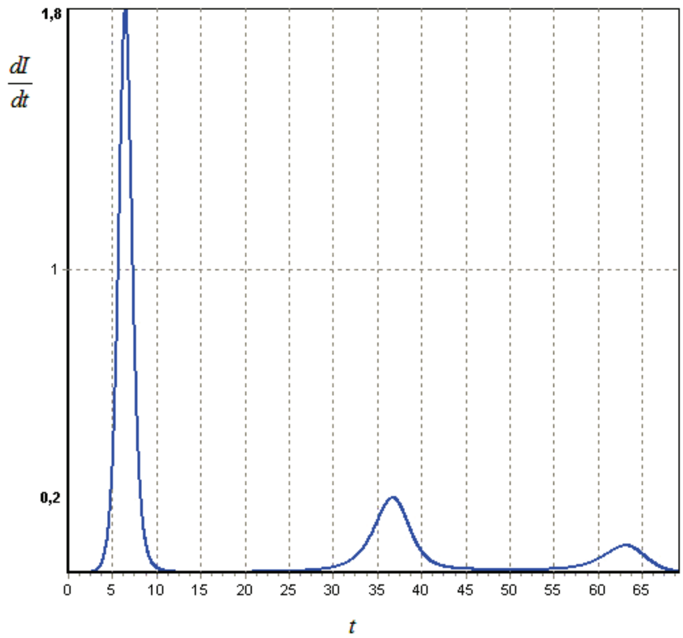


Fig. 6. The evolution of the derivative dI/dt of the integral quadratic form $I = \sum_j a_j^2$.

Exact after the first peak of the derivative, the metastable structure – a system of convective rolls is formed, and up to the moment when the second burst have appeared with value of $I \approx 1$ it has remain unchanged. The next burst of $\partial I / \partial t$ indicates the emergence of a secondary metastable structure with a new value of $I \approx 1.07$. After the second burst of the quadratic form derivative a stable structure of squared convective cells is started to build up (Fig. 7). Such behavior proves the existence of structural-phase transitions in the system.

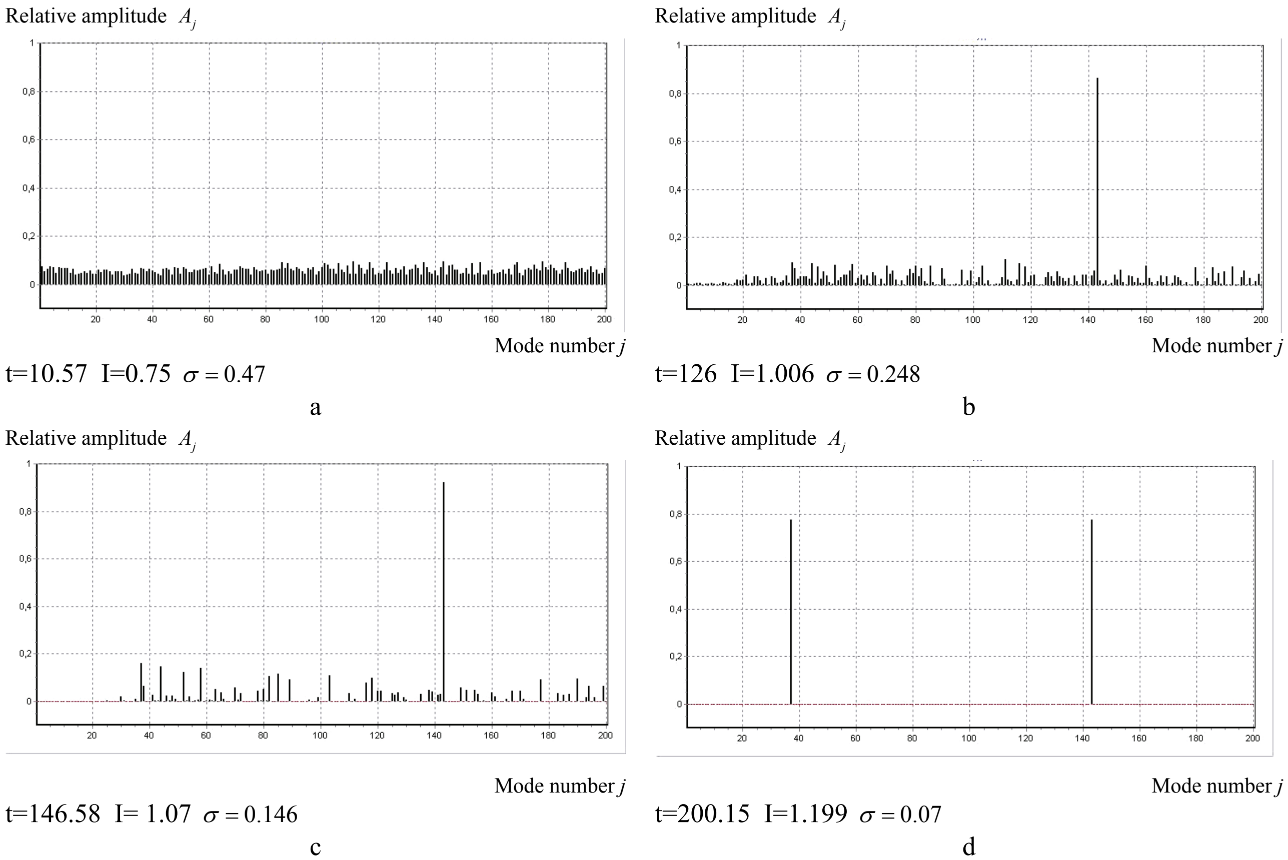


Fig. 7. Regimes of instability
a – the spectrum of the "amorphous" state, *b*,*c* – a short-lived intermediate state, *d* – stable state – convective cells.

If one of the modes gets the proper amount of energy, then the process of formation of a simplest convective structure – rolls begins. Note that in the nature, the thin clouds also can form the roll structure, as shown in Fig. 8

Fig. 8. A fragment of a thin cloud in the form of convective rolls. Circumurban Road, Kharkiv, 09.12.2012

The value of I in this case tends to unity ($I \to 1$). However, this state is not stable and then we can see the next structural transition: convective rolls are modulated along the axis of fluid rotation, and the typical size of this modulation phases down. In this transition state, the system stays for a sufficiently long time (which slightly increases within some limits with increase in the number of modes), and the value $I \approx 1.07$ remains constant during this time.

Further, the growth of side spectrum, rotated on 90^o relative to the fundamental mode (corresponding to the formation of the roll structure) results in rolls modulation (Fig. 9) and over the time there is the second structural-phase transition occurs leading to the formation of the metastable spatial structure with broken short-range order (Fig. 10) but possessing long-range order, that is a result of mode interference.

After a rather long time, ten times more than the inverse linear growth rate of the initial instability only the one mode "survives" from newly formed "side" spectrum, which amplitude is comparable with the amplitude of the primary leading mode. In the end, the stable convective structure –square cells is generated, and the quadratic form I reaches the value of $I = 1.2$.

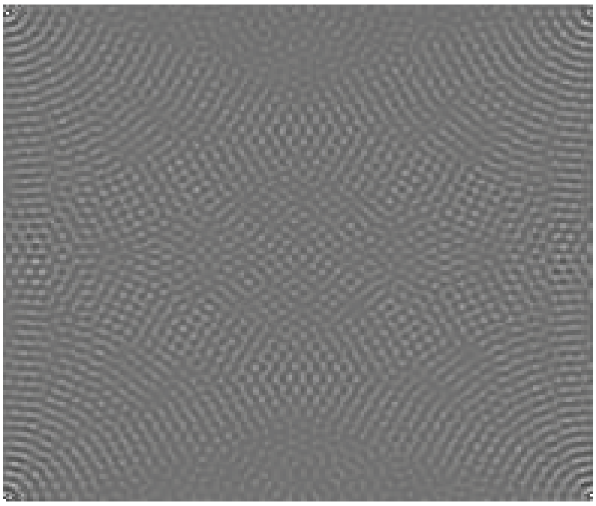


Fig. 9. Spatial short-lived convective structure with long-range order, but violations range order ($I = \sum_j a_j^2 = 15/14$).

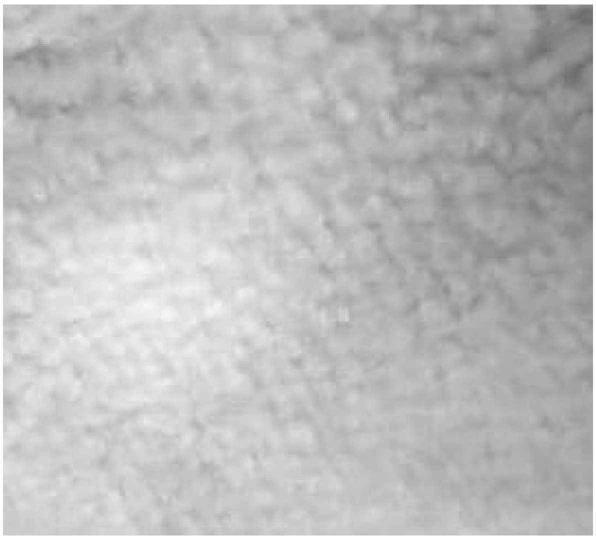


Fig. 10. A fragment of the spatial convection structure in a thin cloud layer. White Lake (Zmiyov), 06.10.2012

5.3. Structure imperfection

Let us consider in more detail the formation of square convective cells. Denote the amplitudes of the modes forming a spatial structure of square convective cells as a_1 and a_2. Consider the dynamics of "spectrum imperfection (defectiveness)" of the structure $D = \sum_{j \neq 1,2} a_j^2 / \sum_j a_j^2$. It is defined as the ratio of the sum of squared mode amplitudes which does not fit the system of square cells to the total sum of modes squares. In addition, let introduce so-called "visual imperfection (defectiveness)" $d = N_{def} / N$, where N_{def} is the number of defective spatial cells (the area of the structure occupied by irregular cells) and N is the number of cells in a perfect regular structure (the total area of the structure). The process of structure rearrangement is observed in the interval between the second and third burst of the derivative quadratic form (Fig. 2).

The criteria by which the cell was considered as regular and the method of calculation the number of these cells are following. The picture for the field is converted to 8-bit image. I.e. the maximum number of colors is reduced to 256. Thus, the formed structure becomes more evident and observable. Increasing this image, one can quite clearly distinguish which of the structural units is the proper cell, and which is not. The proper cell has the correct geometry with uniformly dark center and four lighter hills surrounding the center and of comparable size.

Despite on qualitative character of quantity description characterizing the spectral and visual defectiveness of structure we can note a similarity in its behavior (Fig. 11) near the completion of the structural transition.

Let place the grid mesh over the pattern obtained by simulation of Eq. (5.1) and count the number of picture elements (meshes) for each value of the temperature field Φ. The result can be considered as a field distribution function which characterizes the pattern (Fig. 12).

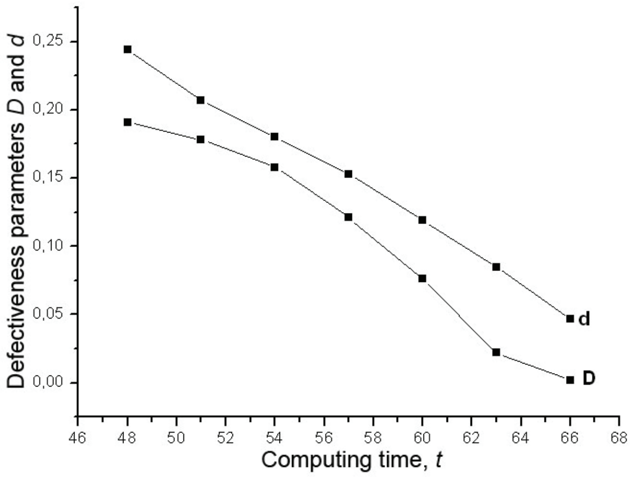
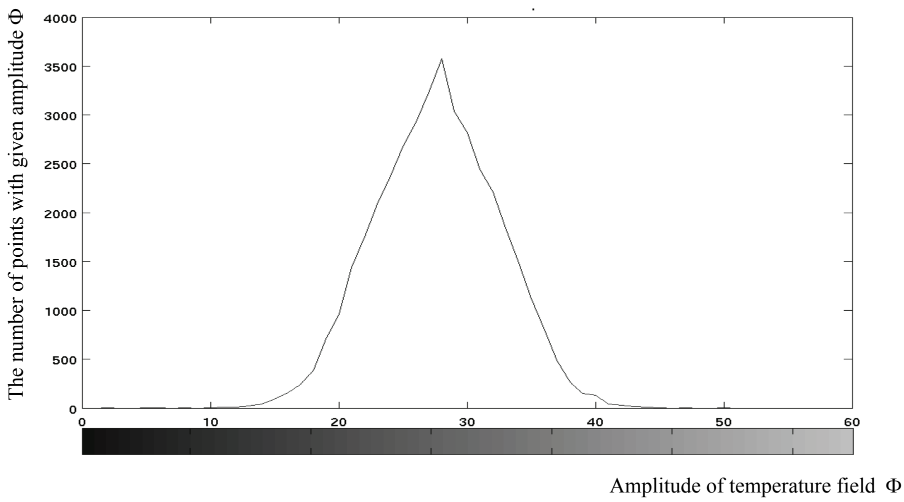
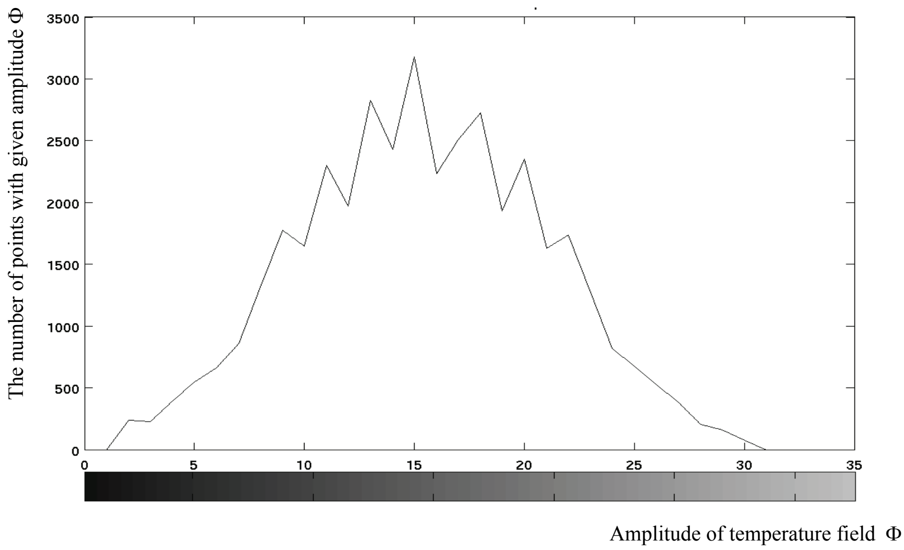


Fig. 11. Comparative analysis of the spectral *D* and visual defectiveness parameters *d*.
The number of modes is 50.



a)



b)

Fig. 12. Distribution of the temperature field *Φ*
a - defective structure, *b* - regular structure

The analysis of this distribution (the presence of local maxima and minima of their position) gives ideas about the nature of the regular structure, the level of its imperfection, and helps to investigate it's topological features.

6. MODULATION INSTABILITY OF CONVECTIVE CELLS IN A THIN LAYER. EFFECT OF HYDRODYNAMIC DYNAMO

6.1. The Proctor-Sivashinsky-Pismen model

The first report about the possibility of the convective cells system modulation instability within framework of the extremely productive Proctor-Sivashinsky-Pismen model [65] was made in [66]. This modulation of the convective cells in a thin layer of liquid between poorly conducting heat horizontal surfaces (which origination was discussed earlier in this paper) is caused by generation of vortices of a different nature than those that form the convective structure. As a result of modulation instability the large flat vortices appear in the system of developed regular convective cells. In other words, this is the effect of hydrodynamic flow (vortex) dynamo [27, 66, 67] which in contrast to the well-known physical models constructed for media with a helical hydrodynamic turbulence (see review [25]) represents a regular process and there is no need in the presence of uncompensated helicity in the system for developing of this effect.

The Proctor-Sivashinsky model updated by author of [65] describes the convection with taking into account the toroidal component of the velocity $\vec{U}_{\mathrm{tor}} = \mathrm{rot}(\vec{e}_z\Psi)$,

$$\dot{\Phi} = \varepsilon^2\Phi - \left(1-\nabla^2\right)^2\Phi + \frac{1}{3}\nabla\left(\nabla\Phi\left|\Phi\right|^2\right) + \gamma\nabla\Phi\times\nabla\Psi\,, \tag{6.1}$$

$$\nabla^2\Psi = \nabla\nabla^2\Phi\times\nabla\Phi\,, \tag{6.2}$$

where γ is the inverse Prandtl number $\mathrm{Pr}^{-1} = \kappa/\nu$, characterizing a non-equilibrium state of the fluid, ν is the kinematic viscosity and κ is the specific temperature conductivity, $\varepsilon << 1$.

Despite the fact that the model of Proctor-Sivashinsky-Pismen [65] was introduced for Prandtl numbers of unity order it remains applicable to description of evolution of developed convective cells structure for which, as shown above, $\Phi \propto \varepsilon$. Moreover, the modulation instability of the structure occurs only at low Prandtl numbers. [12].

It means that deriving Eq. (4.1) we assumed $\gamma\Psi \propto \gamma\Phi^2 \propto \varepsilon^2$, and $\gamma \propto 1$. The analysis of the modulation instability of the developed convective cells structure shows that non-vanishing values of $\Psi \propto \left(\frac{\Delta k}{k}\right)\Phi^2$, where in turn $\left(\frac{\Delta k}{k}\right) \propto \varepsilon$. Thus, the model [65] can be used to describe the modulation instability of the developed convective structure only when $\gamma \propto 1/\varepsilon$, because only in this case the condition $\gamma\Psi \propto \varepsilon^2$ is satisfied.

6.2. Secondary modulation instability of convective cells

The secondary instability threshold is determined by the setting the parameter $\varepsilon_2 = 27b^2\Gamma^2/20 - 1$ to zero, where $\Gamma = \varepsilon\gamma$, $b = \sqrt{5/3}A$ is the renormalized amplitude of perturbations in the primary instability discussed above. When the threshold is exceeded ($\varepsilon_2 > 0$) there are conditions for secondary instability with the maximum growth rate

$$\mathrm{Im}\,\omega_{\max} = 1 - 6b^2/5 + 27\Gamma^2 b^4/200 + 2/27\Gamma^2\,, \tag{6.3}$$

located near the central modes of the primary structure ($k_x = \pm 1$, $k_y = 0$ and $k_x = 0$, $k_y = \pm 1$) and transversely spaced from these points at a range of $\Delta = (\sqrt{2}/b\Gamma)\,|\,\varepsilon_2\,| << 1$.

When the gap between the modes of the secondary structure and the primary structure approaches zero, the growth rate of the modulation instability tends to zero too. The large-scale vortex perturbations, which arise due to the modulation instability, lead to occurrence of shear flows and deform the convective structure on a large scale.

The equation describing the evolution of the spectrum of instability is the following:

$$\dot{b}_j = b_j - \sum_{j}^{N} V_{ij}\,|\,b_i\,|^2\, b_j + \sum_{i,n,m}^{N} W_{jinm} b_i b_n b_m\,, \tag{6.4}$$

where the coefficients are defined by the interaction $V_{jj}=1$, ϑ is the angle between the vectors $\vec{k}_i$ and $\vec{k}_j$,

$$V_{ij} = (2/3)\left(1 - 2\left(\vec{k}_i\vec{k}_j\right)^2\right) = (2/3)\left(1 + 2\cos^2\vartheta\right), \tag{6.5}$$

$$W_{jinm} = \left(\vec{k}_i \times \vec{k}_n\right)\left(\vec{k}_m \times \vec{k}_j\right)\left[\frac{k_i^2 - k_n^2}{\left(\vec{k}_i + \vec{k}_n\right)^2} + \frac{k_i^2 - k_m^2}{\left(\vec{k}_i + \vec{k}_m\right)^2}\right]\delta_{\vec{k}_j,\vec{k}_i+\vec{k}_n+\vec{k}_m}\,. \tag{6.6}$$

It can be shown that if we impose a condition of proper symmetry on the arising perturbations, the equations for the fundamental modes of convection cells (each of which has an amplitude equal to *b*) and for modes b_d having the fastest growing rate of the modulation instability (we suppose that the rest of the spectrum is suppressed due to action of above discussed competition mechanisms) have the form

$$\dot{b} = b\left(1 - b^2 - 4b_d^2\right), \tag{6.7}$$

$$\dot{b}_d = b_d\left(1 - b^2 - b_d^2\right) + \frac{2}{27\Gamma^2} b^2 b_d \theta\left(\varepsilon_2\right), \tag{6.8}$$

where the threshold of the modulation instability is introduced qualitatively by the theta-function $\theta(\varepsilon_2)$. When the modulation instability threshold is exceeded, the amplitudes of primary structure decreases from the values comparable with unity to values of $b_\infty{}^2 = 20/27\Gamma^2$, while the amplitude of the growing modes are reach the value of $b_{d\infty} = 1/2(1-b_\infty^2)^{1/2}$. The intensity (i.e., the value $I = \sum |b_{k_i}|^2$) of the primary structure in the absence of modulation instability (at $\Gamma^2 < 20/27$ the primary structure is stable) and the intensity of the defective structure, which is a result of the development of this instability is found to be equal. The deficiency of the developed structures is equal to ε_2.

6.3. The effect of regular hydrodynamic dynamo

The interaction between the modes determining the modulation (modes of the distributed defect) and the modes of the primary structure is caused by the existence of large-scale vortices which streamlines in the configuration space can be represented as

$$\Psi \approx \varepsilon\varepsilon_2 b^2 \left[\cos(l_0\xi) - \cos(l_0\eta)\right], \tag{6.9}$$

where $l_0 = \varepsilon\Delta$, $\xi = \sqrt{\varepsilon}x$, $\eta = \sqrt{\varepsilon}y$ and the ratio of the characteristic linear size of the characteristic large-scale vortex L_V to the linear size of the convection cell L_C is equal to $L_V/L_C \approx \left(b\varepsilon\varepsilon_2\right)^{-1}$ (Fig. 13). The occurrence of such large-scale vortices is one of the possible realizations of the hydrodynamic dynamo effect [66, 67].

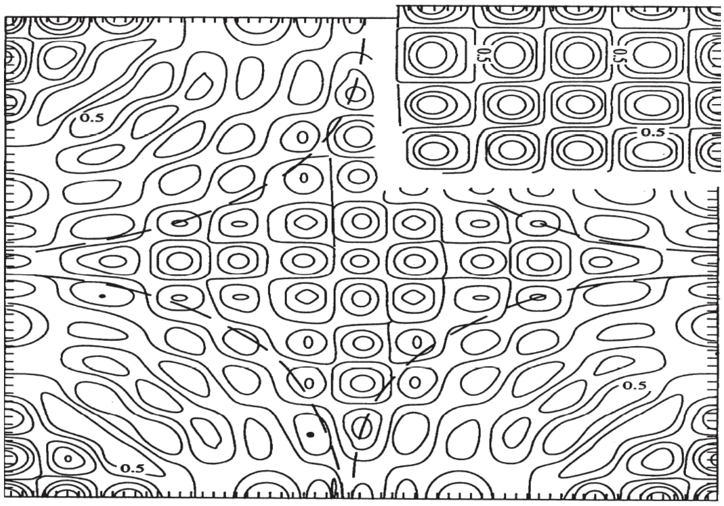


Fig. 13. Regular defect in the convective structure. The fragment of unperturbed primary structure is shown in the upper right corner. The dotted line shows the characteristic streamlines of large-scale vortices.

7. CONCLUSION

The special feature of the Proctor-Sivashinsky model is the existence of three possible metastable states. The times of structural transitions between these metastable states are much less than the time of their existence.

The characteristic size of the convective structures in the regime of advanced instability is of order $2\pi / k \propto 2\pi$ and the length of the wave vectors is of order unity (in conventional dimensionless units). The potential of interaction between spatial modes $V_{ij} = (2/3)\left(1 + 2\cos^2\vartheta_{ij}\right)$ has a deep minimum for angles $\vartheta_{ij} = \vartheta_i - \vartheta_j = \pm\pi/2$ between vectors $\vec{k}_i$ and $\vec{k}_j$. Namely these minima are the reason of the instability of convective rolls [27, 28], because the existence of a minimum V_{ij} for modes with relatively low amplitudes allows them to continue growth, while suppressing the perturbations occurred before.

When approaching to the stable state, the spatial structure is getting rid of many defects. There is a correlation between the relative fraction of visually (geometrically) observed structural defects and the imperfection parameter, defined as the ratio of the squares of the spectrum mode amplitudes which does not fit the system of square cells to the total sum of squared modes.

The modulation instability of the developed convective cells results not only in appearing of the self-similar system – convective cells of different sizes [12] but also in formation of the large-scale poloidal vortices [66, 67]. This phenomenon, which was previously investigated for irregular models (see detailed review [25]), as was conjectured by S.S. Moiseev, can take place through the modulation instability of regular spatial convective structure of finite amplitude.

The model discussed above can be used after some small modification for purpose of simulation modeling of structures and second-order structural-phase transitions. With increase of the number of minima of the interaction potential $V_{ij}(\vartheta)$ (for example, three), the structures with another type of symmetry can arise (for example, hexagonal cells). Governing the spatial structure of the potential' local minima and their depth, one can form the structure of any specified symmetry type. It is possible in so doing to observe the formation of intermediate metastable states and all stages of structural transitions. This allows to use this model (appropriately modified if necessary) for a qualitative description of pattern formation in systems which possess the preferred spatial scale, particularly in condensed media.

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