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Modelling internal erosion with random geometric graph

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Abstract

This paper introduces the basic definition and characteristics of porous media, focusing on the application of stochastic geometric map theory in 2 D porous media simulation. This paper first summarizes the pore structure and connectivity of porous media, and then describes the basic concept, generating method and algorithm implementation of random geometric graph. Then, the application of random geometry in porous medium simulation, including flow and transmission simulation, erosion process, simulation. Through constructing mathematical model and numerical simulation method, the paper analyzes the internal erosion process in 2 D porous medium, and realizes the visual display of the erosion process and result data analysis. Based on the simulation results, this paper discusses the influencing factors of the erosion process, verifies the effectiveness of the model, and puts forward useful enlightenment for practical application fields such as petroleum industry and environmental protection. Finally, the paper discusses the future research directions, including the deep exploration of the erosion mechanism, expanding the application scope of the model and the comprehensive consideration of various influencing factors, which provides new ideas and methods for the further study of the internal erosion in porous media.

Keywords: porous media; mathematical model; random geometric graph; internal erosion

Table of Contents

1. Introduction	3
1. Research background and significance.....	3
2. The purpose and innovation points of this study.....	5
2. Definition and characterization of the porous media	6
1. Porous medium definition and properties.....	6
2. Porous media models.....	7
3. Pore structure and connectivity description.....	8
3. Theory and application of random geometric graphs	9
1. Basic concepts of a random geometric graph.....	9
2. Generation method and implementation of the algorithm.....	10
3. Application in porous media simulations.....	12
4. Simulation method of internal erosion process	13
1. Analysis of the erosion mechanism.....	13
2. Construction of the mathematical model of the erosion process	15
3. Numerical simulation methods and steps.....	16
5. Simulation Results and Analysis	17
1. Simulated scenario settings and parameter selection.....	17
2. Visualized presentation of the erosion process.....	18
6. Conclusions	19
7. References	21

1. Introduction

1. Research background and significance

In the fields of geology, civil engineering and environmental engineering, the study of 2 D porous media has important practical significance. The porous medium consists of a large number of interconnected pores and solid skeletons, and this special structure makes the flow and transport of matter complicated and difficult to predict [1]. And the internal erosion, as an important phenomenon in the porous media, has a significant impact on the overall structure and hydrodynamic characteristics of the porous media [2].

The internal erosion study in two-dimensional porous media focuses on the flow characteristics of the fluid in the pore and the erosion mechanism of the solid particles. These studies are crucial for a thorough understanding of the mechanism of fluid flow and matter transport in porous media. In the field of soil science, internal erosion is one of the main causes of soil structure destruction and the decline of soil fertility. The simulation and study of internal erosion in 2 D porous media can reveal the process and mechanism of soil erosion, and provide a scientific basis for soil protection and sustainable development of agriculture.

In the field of groundwater literature, the study of internal erosion in 2 D porous media is also of great significance. Groundwater is one of the important water sources for human life, and the flow and pollution of groundwater are affected by the fluid flow and material transmission in the porous media. Through the simulation and study of internal erosion in 2 D porous media, the flow path and pollution diffusion range of groundwater can be predicted and evaluated, so as to provide a scientific basis for the protection and management of groundwater resources.

The study of internal erosion in 2 D porous media is also of great significance to environmental protection and civil engineering. For example, in the field of environmental engineering, internal erosion can cause the migration and diffusion of soil and pollutants, causing pollution and damage to the environment. Through the simulation and study of internal erosion in 2 D porous media, the migration path

and diffusion range of pollutants can be predicted, providing a scientific basis for environmental protection and pollution treatment.

The internal erosion simulation study based on random geometric graphs in 2 D porous media has important academic and practical significance. By studying the flow characteristics of fluid in the pore and the erosion mechanism of solid particles, we can deeply understand the mechanism of fluid flow and material transport in the porous medium, and provide a theoretical basis for engineering practice and scientific research in related fields.

In recent years, domestic researchers have deeply studied the problem of internal erosion in 2 D porous media and achieved certain results. In the numerical simulation of erosion process, the researchers simulated fluid flow and particle erosion in porous media through numerical simulation methods, such as finite difference method, finite element method, lattice Boltzmann method and lattice method. These studies contribute to a deeper understanding of erosion processes and provide important evidence for predicting erosion behavior. In the analysis of erosion mechanism, the researchers revealed the interaction mechanism between fluid and particles through the microscopic observation of the internal pore structure of the porous medium, which provided an important idea for the in-depth exploration of erosion mechanism. However, existing studies still have deficiencies in simulating complex pore structures, considering the combined effect of multiple factors, and experimental validation. Especially in the modeling and simulation of complex pore structures, the existing numerical methods are often difficult to accurately describe the geometric characteristics of the pore and the dynamic process of fluid flow, resulting in a large difference between the simulation results and the actual situation.

Foreign researchers have studied the internal erosion in 2 D porous media relatively early, and the research results are relatively rich. In the analysis of erosion process and mechanism, foreign researchers also used the combination of numerical simulation and experimental research to explore the interaction mechanism between fluid flow, such as flow, particle erosion, and chemical reaction. They improved the

accuracy and reliability of the model to simulate more complex erosion processes. Foreign researchers also pay attention to experimental verification and model optimization, and verify and correct the model through a large number of experimental data to make it more in line with the actual situation. Foreign researchers have also actively explored the problem of internal erosion in multidimensional porous media and the coupling effect of erosion and other physical processes, such as fluid flow, material transmission, chemical reaction, etc., which provides a broader perspective for comprehensively understanding the erosion mechanism and predicting the erosion behavior [3].

2. The purpose and innovation points of this study

The core goal of this study is to deeply analyze and simulate the effects of the internal erosion process on pore structure, hydrodynamic characteristics, and material transport by constructing a two-dimensional porous medium model based on random geometric graphs. The process of fluid flow and solute transport in pore media is complex and changeable, influenced by many factors, and the traditional empirical formulas and simplified models are often difficult to accurately describe their internal laws. Therefore, this study is dedicated to develop a new simulation method to improve the understanding and predictive power of the internal erosion process in porous media.

In terms of research purposes, this study aims to reveal the influence of the internal erosion process on pore structure, hydrodynamic characteristics, and material transport through simulation analysis. By constructing the two-dimensional porous medium model based on a random geometric map, we can simulate the complexity and randomness of pore structure and reflect the characteristics of actual porous media more accurately. At the same time, this study will also focus on the hydrodynamic characteristics and the change law of material transmission in the process of internal erosion, so as to provide new ideas and methods for the research in related fields.

In terms of innovation points, this research is somewhat innovative in simulation methods, model construction and analytical means. This study uses random geometric graphs to construct two-dimensional porous medium models, which can describe the complexity and stochasticity of pore structure more accurately than conventional mesh methods. This study focuses on the coupling effect of internal erosion process with hydrodynamic characteristics and material transmission, and reveals the erosion mechanism and influencing factors through multi-factor comprehensive simulation. Finally, this study optimized the model parameters and improved the accuracy and applicability of the model by comparing experimental data and simulation results. These research methods and means have high innovative and practical value in related fields.

2. Definition and characterization of the porous media

1. Porous medium definition and properties

As an important material, porous medium is widely used in petroleum, chemical industry, environmental protection, agriculture and other fields. Porous media refers to those solid materials containing a large number of pores, which can be natural or artificially formed and can allow fluids (such as gases, liquids) to flow through these pores. To understand the properties and applications of porous media, we need to understand the definition of porous media and its main properties.

The porosity of the porous media is the ratio of the pore volume to the total volume of the porous media. Porosity is one of the most important physical properties of porous media, which directly affects the storage capacity of porous media and the seepage speed of porous media. The larger the porosity, the stronger the liquid storage capacity of the porous media, and the faster the fluid seepage speed in it. The size of the porosity is related to the particle size, particle shape, particle arrangement, and the pore structure of the porous media.

The permeability of the porous medium refers to the flow rate of the fluid passing through the porous medium under the unit pressure gradient when the fluid flows through the porous medium. Permeability is an important indicator reflecting

the permeability performance of porous media, which determines the penetration speed and penetration ability of fluid in porous media. The size of the permeability is related to the porosity, pore structure, particle size and distribution, and particle shape of the porous media. The greater the permeability, the better the permeability of the porous media, and the smoother the fluid flow in it.

The specific surface area of the porous media is the ratio of the pore inner wall area to the volume of the porous medium. The larger the specific surface area, the larger the contact area between the porous medium and the fluid, and the easier the heat transfer and mass transfer processes in the porous medium are to occur. Therefore, the specific surface area of porous media is one of the important factors affecting the heat transfer and mass transfer properties of porous media.

2. Porous media models

The porous media model is an important tool to study the characteristics of fluid flow and mass transfer in porous media. The porous media model is widely used in geology, petroleum, chemical industry, environmental protection and other fields. Several common porous media models are presented in this section.

The spherical particle model

The spherical particle model is a classical model in the porous media model. The model assumes that the porous medium is composed of spherical particles and the voids between the particles form pores. In the spherical particle model, the geometry and size of the pore can be described by the diameter and arrangement of the particles. Due to its simplicity and ease of solution, the model is widely used in some regular-arranged porous media. However, the spherical particle model is too simplified to accurately describe the complex pore structure and connectivity of actual porous media.

The cube model

The cube model is another common model of the porous media. The model assumes that the porous medium consists of cubic units, and the connections between the units form pores. In the cube model, the geometry and size of the pore can be

described by the side length and connection mode of the cube. The cube model is regular and easy to construct and solve. However, it similarly fails to accurately describe the pore structure and connectivity of actual porous media, especially in cases of irregular pore shape and distribution.

Random geometric graph model

The random geometric graph model is a model capable of describing the complex pore structure and connectivity of a porous medium. The model describes the pore structure and connectivity of a porous medium by randomly generating a geometry. In the stochastic geometric graph model, the geometry and size of the pore are random, consistent with certain statistical regularity. By adjusting the model parameters, porous media with different pore structures and connectivity can be generated. The stochastic geometric graph model can truly reflect the complex characteristics of porous media, so it has been widely used in geology, petroleum and chemical fields. However, due to the complexity and computational amount of random geometric graph models, there are some limitations in practical applications.

3. Pore structure and connectivity description

The pore structure in porous media is an important factor affecting the fluid flow and transport properties. The pore structure includes the shape, size, distribution, and connectivity of the pore, which jointly determine the permeability of the porous medium and the way the fluid flows in it.

The pore structure has a direct influence on the fluid properties of the porous media. The shape and size of the pore determine the permeability path, which then affects the permeability velocity and permeability. Larger pores often have higher permeability, where fluids flow faster. However, the shape and distribution of the pores can also affect the flow path, thus affecting the penetration efficiency of the fluid. For example, a porous medium with irregular pore shape and uneven distribution, in which the fluid flows, encounters more resistance and eddy current, resulting in a decrease in penetration efficiency.

Connectivity is another important property in the pore structure, which determines the flow ability of a fluid in a porous medium. A porous medium with

good connectivity, a high degree of connection between pores, the fluid can freely flow from one pore to another pore, small flow resistance, easy flow. This porous medium is an ideal fluid channel with its high permeability and penetration efficiency during fluid transport. However, in some porous media with poor connectivity, there are fewer connections between pores, fluid flow is limited and high flow resistance, making fluid difficult to flow in them.

To accurately describe the pore structure and connectivity of porous media, a series of experimental methods and mathematical models are needed. These methods can provide detailed information on the pore shape, size, distribution, and connectivity of porous media, providing basic data for the study of fluid flow and transport properties. Meanwhile, the intensive study of pore structure and connectivity can also help to understand the physical and chemical properties of porous media and provide strong support for their development and application.

3. Theory and application of random geometric graphs

1. Basic concepts of a random geometric graph

As a product of modern probability theory and geometry, stochastic geometric graphs provide a powerful tool to describe and analyze geometric phenomena with randomness. Its core is to use stochastic process theory to construct geometric figures and to explore the statistical properties of these figures. This section details the basic definition, properties, and construction of random geometric graphs.

Definition and nature

A random geometric graph, as the name suggests, is a geometric graph based on a stochastic process. In a random geometric graph, the elements of figures, such as points, lines, planes and so on, all have certain randomness. This stochastic geometry allows for more flexibility to describe and simulate complex phenomena in practice, such as flow and transmission processes in two-dimensional porous media.

The random geometric graph has several remarkable properties:

1. Randomness: The position and number of elements in random geometric graphs are random, which allows it to describe uncertainty.

2. Statistics: Random geometric graphs can be analyzed by statistical methods, so as to obtain their statistical properties, such as average edge length, connectivity, etc.

3. Applicability: Random geometric graphs can be widely used in physics, chemistry, biology and other fields, for the simulation and analysis of practical phenomena.

Build a way

The construction of random geometric graphs is usually based on certain probability distributions and stochastic processes. Here are several common builds:

1. Poisson distribution: randomly scattered points on the plane, each point is independent and equally likely to fall in any position. Then, draw the circle with each point as the center and a distance as the radius. These circles overlap each other to form a random geometric graph.

2. Normal distribution: points are scattered on the plane, but the position of each point follows a normal distribution. Then, using these points as nodes, we connect the points within a certain range to form a random geometric graph.

There are many other ways of construction, such as random polygons, random networks, etc. These construction methods can be selected and adjusted to actual requirements to obtain random geometric graphs consistent with specific statistical properties.

2. Generation method and implementation of the algorithm

One of the key steps in simulating a 2 D porous medium. This process requires that the generated graphs should be both random and can reflect the characteristics of the actual porous media. To achieve this goal, a series of generative algorithms and algorithmic implementation techniques are usually employed.

In terms of generation algorithm, Monte Carlo method and cellular automaton are two common methods. The Monte Carlo method gradually constructs a random geometry through random sampling and probability calculation. It relies on a large

number of random numbers and probability distribution functions, and is able to simulate the randomness and irregularity in the porous media. However, the Monte Carlo method has large computational quantities, and the generated graphs may have statistical errors. Therefore, we need to balance computational accuracy against computational efficiency in practical applications.

The cellular automata method simulates the interaction and evolution process between the cells by defining the state and state transfer rules, so as to generate the geometry with randomness. The cellular automata method has the advantages of simple calculation, easy implementation and parallel calculation. However, the cellular automata method has many model parameters and a strong dependence on the model, which requires appropriate model construction and parameter adjustment for specific problems.

In terms of algorithmic implementation details, it is necessary to set appropriate probability distribution functions and stochastic processes according to the characteristics of the simulated two-dimensional porous medium. For example, when simulating the pore distribution of a porous medium, a probability distribution function such as a Poisson distribution or a Markov chain can be used to describe the distribution and size of the pore. At the same time, the efficiency, accuracy and stability of the algorithm also need to be considered to ensure that the generated random geometric graph can accurately reflect the actual physical phenomena. In order to improve the efficiency of the algorithm, some optimization techniques can be adopted, such as fast iterative algorithm, parallel computing, etc.

Some technical details also need to be paid attention to during the implementation of the algorithm. For example, the generation of random numbers is an important part of the algorithm, and an appropriate random number generation algorithm is needed to ensure that the generated random numbers have uniformity and independence. At the same time, the algorithm also needs to be verified and verified to ensure the correctness and reliability of the algorithm.

Generation method and algorithm implementation play an important role in the simulation of random geometric graphs in 2 D porous media. By selecting suitable

generating algorithms and algorithm implementation techniques, geometries with randomness and authenticity can be generated, providing strong support for the study of porous media.

3. Application in porous media simulations

Porous media is a complex structure widespread in the natural and engineering fields, including soil, rock, porous ceramics, fiber materials, etc. The pore structure, shape, and connectivity within these media are very complex and difficult to describe using traditional mathematical methods. Therefore, numerical simulation becomes an important means to study the process of fluid flow and material transport in porous media. While random geometric graphs, as an effective tool to describe complex structures, play an important role in porous medium simulations.

In porous media, the flow of fluid and the transport of matter are influenced by the internal pore structure, particle shape, and pore size of the medium. These factors are highly stochastic and complex, and are difficult to describe with regular geometries. Random geometric graphs can generate geometries with randomness, which can simulate the internal structure of porous media very well. By constructing a porous medium model based on random geometric graphs, the fluid flow path and the velocity distribution in the medium can be simulated more accurately. Random geometric graphs can also consider the specificity within the medium, such as differences in permeability and pore connectivity, thus improving the accuracy and accuracy of simulation.

In practice, random geometric graphs are widely used in porous medium simulation. For example, in oil and gas field development, the spatial distribution and connectivity of the reservoir can be simulated to optimize the water injection and oil recovery schemes. In groundwater research, random geometric graphs can simulate the permeability and water level changes of aquifers, providing a scientific maps environmental protection, random geometric graphs can simulate the diffusion process of pollutants in the soil and assess the extent and extent of pollution.

In addition to flow and transmission simulations, random geometric graphs have important applications in the simulation of internal erosion processes in porous media. Internal erosion is a common destructive phenomenon in the porous medium, which causes changes in the permeability, mechanical properties and overall stability of the medium. The occurrence and development of the internal erosion process is closely related to the internal structure of the medium, the fluid flow state, the particle erosion and other factors. While random geometric graphs can simulate these factors well to more accurately predict the development and behavior of internal erosion processes.

In internal erosion simulations, random geometric graphs can generate collections of particles with different shapes, sizes, and distributions to simulate the internal structure of a porous medium. Then, through the hydrodynamic simulations, the flow state of the fluid in the medium and the scouring process of the particles can be simulated. Random geometric graphs can reflect the interactions between particles and changes in the pore structure, thus more accurately modeling the development of internal erosion processes. By simulating the internal erosion process under different conditions, the stability and erosion resistance of the porous media can be evaluated, providing a scientific basis for related engineering design and application.

3. Simulation method of internal erosion process

1. Analysis of the erosion mechanism

When exploring the erosion mechanism of solid particles in 2 D porous media, it is necessary to deeply understand the interaction of their movement rules, pore structure changes and hydrodynamic characteristics. Together, these factors determine the degree and manner of solid particles eroded in porous media.

The movement of solid particles in 2 D porous media is a comprehensive result of the fluid force and the interaction force between particles. Fluid action force mainly includes drag force, lift force and fluid viscosity force, which act together on the particle surface to produce its displacement. The interaction forces between the

particles include van der Waals force, electrostatic force and friction between the particles, which hinder or promote the movement of the particles. Under the joint action of fluid force and interaction force between particles, solid particles show complex motion states in porous media, such as rolling, sliding, jumping and so on.

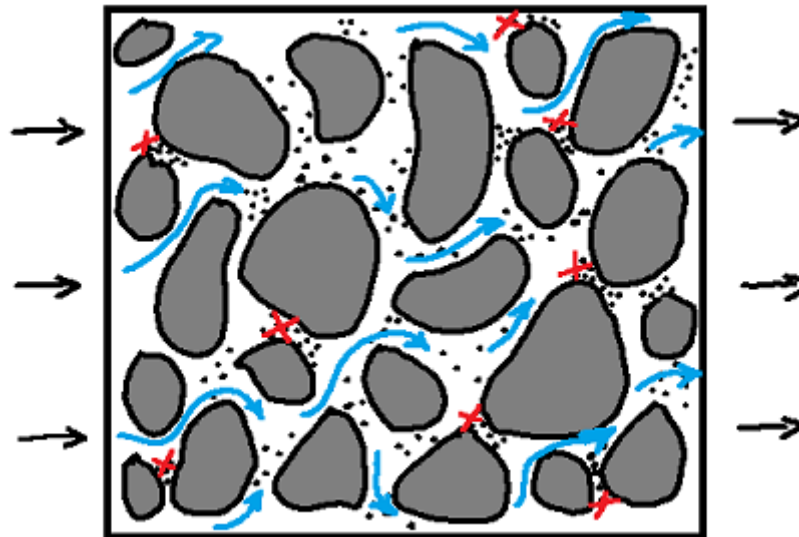


Figure 1. Diagram of the internal erosion. Red crosses mean closed pores. Black dots indicate mobile solid particles. Blue arrows show the direction of flow inside the pores

The change in pore structure is a direct result of solid particle motion and deposition (See Figure 1). With the movement of the particles, the originally stable pore structure is destroyed, and the pore size, shape and connectivity all change. These changes have a significant impact on the flow of the fluid, such as the change of the flow rate distribution, the generation of the pressure gradient, and the change of the fluid viscosity. At the same time, the change of pore structure will also affect the deposition and blockage of particles, thus further changing the permeability of porous media.

Hydrodynamic properties play a key role in the erosion process of solid particles. The flow of the fluid in a two-dimensional porous medium is influenced by the structure of the medium and exhibits different kinetic properties. For example, in regions with smaller pores, the fluid flow rate is slower and the pressure is higher,

and the opposite is observed in regions with larger pores. This inhomogeneity of the flow velocity distribution leads to the uneven force of the particles in the pore, which produces the shear force and pressure difference, prompting the particle movement. Fluid viscosity also has an important effect on the movement of the particles, which hinders and stabilizes them.

2. Construction of the mathematical model of the erosion process

In the study of the erosion process, the construction of mathematical models is crucial. It can help us understand the flow law of fluids in porous media, predict the migration path of solid particles, and the evolution of pore structure. This chapter will elaborate on three core mathematical models: fluid dynamics, equations of particle motion and evolution of pore structure.

Hydrodynamic equations

Hydrodynamic equations are the mathematical basis for describing fluid flow in porous media. In 2 D porous media, we usually adopt Darcy's law to describe the flow of a fluid. The law takes into account the fluid viscosity force and the permeability of the porous medium, and can accurately describe the fluid flow law in the porous medium. We also need to consider the continuity equation of the fluid, the principle of the mass conservation during the flow. By combining these two equations, we can obtain the flow equations of the fluid in two-dimensional porous media, which provides the basis for the subsequent simulation of the erosion process.

Equations of particle motion

In a fluid, the movement of solid particles is affected by various forces, including the drag force of the fluid, the gravity of the particle itself, and the collision force between the particles. In order to describe the motion state of the solid particles in the fluid, we need to establish the equation of particle motion. The equation takes into account the force situation, velocity and acceleration, and can accurately describe the trajectory of the particle in the fluid. Through the equation of particle motion, we can understand the dispersion, deposition and migration law of particles

in the fluid, which can provide important data support for the simulation of the erosion process.

The evolution equation of the pore structure

Pore structure is one of the important features of porous media, which directly affects the fluid flow and particle transport. During erosion, the pore structure changes as the fluid washes and transports the particles. To describe this variation, we need to establish the pore structure evolution equations. This equation considers the variation law of porosity and permeability parameters, which can accurately describe the evolution process of pore structure. Through the pore structure evolution equation, we can understand the permeability change of the porous media, and then predict the development trend of the erosion process.

3. Numerical simulation methods and steps

Numerical simulation method plays an important role in studying the internal erosion process of 2 D porous media. This section details the methods and steps of numerical simulation to provide guidance for related studies.

Mesh generation and partition are the basis of the numerical simulation. Before performing the numerical simulation, it is necessary to generate a suitable mesh based on the actual structure of the two-dimensional porous medium. The division of the grid should follow certain principles, such as moderate grid size, grid shape rules, grid boundary consistent with the actual boundary. Reasonable meshing can improve computational accuracy and reduce computation time. In practice, grid division can be made by finite difference method, finite element method or finite volume method. In the grid division process, the physical characteristics of porous media, such as porosity, permeability, should be fully considered to ensure the accuracy and applicability of the mesh.

The initialization condition setting is the starting state of the numerical simulation. Before numerical simulation, the initial hydrodynamic parameters, solid particle distribution, pore structure, etc. The choice of these parameters should be based on actual situations and considering the purpose and accuracy requirements of the simulation. For example, the hydrodynamic parameters can include fluid

velocity, pressure, density, etc. the distribution of solid particles can be described by parameters such as particle grading and particle shape, and the pore structure can be characterized by parameters such as porosity and pore size distribution. Reasonable initialization conditions can improve the accuracy and confidence of the simulation.

Iterative calculation and result analysis are the core links of numerical simulation. After the initialization conditions, the fluid dynamic parameters, particle distribution and pore structure should be updated by iterative calculation. In the process of iterative calculation, the calculation parameters and the iterative step size need to be constantly adjusted to ensure the stability and convergence of the calculation. During the iterative calculation process, the calculation results shall be regularly output and the results analyzed. By comparing the calculation results in different time periods, the evolution law of the internal erosion process in 2 D porous medium can be revealed. Meanwhile, the sensitivity results can be analyzed to evaluate the influence of different parameters on the simulation results.

Numerical simulation methods and steps play an important role in the study of internal erosion in 2 D porous media. Through reasonable grid generation and division, initialization conditions, iterative calculation and result analysis, accurate and reliable simulation results can be obtained, providing strong support for related studies.

5. Simulation Results and Analysis

1. Simulated scene setting and parameter selection

Constructing a porous medium model in 2 D plane is an important method to simulate fluid flow and mass transfer process in porous medium. Porous media are widely found in nature and engineering fields, such as soil, rocks, ceramics, fibers, etc. To accurately simulate the fluid flow in the porous media, it is necessary to rationally set the simulation scenario and select the parameters.

In the simulated scenario setting, it is first necessary to construct a two-dimensional plane model to distinguish between the pore and the solid skeleton by different colors or patterns. The pore is the channel of fluid flow, while the solid

skeleton is the support for the pore. In the model, the shape, size and distribution of pores have an important influence on fluid flow. To simulate the actual situation, random generation or based on real porous medium image reconstruction is usually used to generate pore structures.

Parameter selection is an important link in the simulation process. In the porous medium model, the parameters such as pore size distribution, porosity and permeability significantly influence the fluid flow and mass transfer process. The pore size distribution describes the size distribution of the pore, which affects the fluid permeability and flow speed. Porosity is the ratio of the pore volume to the total volume, which determines the storage capacity of the fluid in the medium. Permeation is a parameter describing the ability of fluid flow in porous media, which is influenced by various factors including pore structure, particle shape and size, and fluid properties.

In the actual simulation, the flow rate, flow rate and viscosity need also be considered. The flow rate determines the flow velocity of the fluid in the medium, and the flow rate is the product of the flow rate and the cross-sectional area. The viscosity is the internal friction coefficient of the fluid, which determines the resistance size of the fluid during the flow process. The choice of these parameters requires appropriate adjustments according to the simulation purpose and actual requirements to obtain accurate simulation results.

2. Visualized presentation of the erosion process

Visualization is a crucial link in the study of the erosion process. Through the visualization technology, we can present the internal erosion process in the 2 D porous media in an intuitive way, so as to have a deeper understanding of the erosion mechanism, master the erosion law, and provide a scientific basis for the subsequent engineering design and protection.

In terms of visualization strategy, we have adopted advanced visualization techniques, including numerical simulation, image processing, etc. These techniques can effectively transform the internal erosion processes in two-dimensional porous

media into graphics or images, enabling a dynamic display of the erosion process. When choosing the visualization strategy, we fully consider the complexity and dynamics of the erosion process, and the availability and accuracy of the data. Through repeated attempts and comparisons, we finally chose the visualization strategy suitable for this study, which can accurately present the changes and trends of the erosion process.

In terms of presentation content, we mainly present three aspects: the initial state, the evolution of erosion process and the final erosion morphology. We show the initial state of the medium, including its pore structure, particle distribution, etc., to provide a benchmark for the comparison of subsequent erosion processes. We show in detail the evolution of the erosion process, including the flow path of the water flow in the medium, the expansion of the erosion region, etc. By comparing with the different time points, we can clearly see the dynamic changes of the erosion process. Finally, we present the final erosion pattern, including the shape and size of the erosion area, to provide a direct basis for engineering design and protection.

Through the visual display, we can intuitively see the influence of the erosion process on the porous media, and reveal the changing trend and influencing factors of the erosion process. This is of great significance for us to deeply understand the mechanism of erosion, grasp the rules of erosion, and develop effective protective measures. At the same time, the visual display also provides convenience for the subsequent data analysis and processing, which can mine the information in the data more efficiently and provide strong support for scientific research.

6. Conclusions

After completing the simulation of internal erosion based on random geometric graphs in 2-D porous media, data analysis and result discussion are crucial links. This section will conduct a detailed statistics and analysis of the data extracted during the simulation process to reveal the laws of internal erosion and its influencing factors.

Data extraction is the basis of data analysis. During the simulation, we recorded geometric parameters such as erosion depth, width, and volume, as well as physical parameters such as fluid flow velocity and pressure. These data are automatically generated by simulation software and are obtained with high accuracy and reliability. During the data extraction process, we performed rigorous data screening and verification to ensure the accuracy and integrity of the data.

Data analysis is the key to reveal the law of erosion. We first performed descriptive statistics on the extracted data, calculating the mean, standard deviation, maximum and minimum values of each parameter to understand the distribution characteristics and the degree of dispersion of the data. We then performed a correlation analysis to explore the intrinsic connections and relationships between the parameters. Through the correlation analysis, we found a significant positive correlation between fluid flow velocity and erosion depth, width and volume, while fluid pressure was negatively associated with erosion depth. These findings provide an important basis for our deep understanding of the mechanism of internal erosion.

In the results discussion, we combine the actual situation and theoretical knowledge. We find that the fluid flow rate is the main factor affecting the internal erosion. When the fluid flow rate is large, the scouring effect of the fluid on the porous medium is enhanced, resulting in increased erosion depth and width. At the same time, the increase of the fluid flow rate will also lead to the decrease of the fluid pressure, which makes it easier for the fluid to penetrate into the porous medium and accelerate the process of internal erosion. The geometry and physical properties of porous media also have effects on internal erosion. For example, the porosity and particle size of the porous medium will affect the flow speed and pressure distribution of the fluid, thus affecting the occurrence and development of internal erosion.

Through this simulation and analysis, we reveal the laws and influencing factors of internal erosion based on random geometric graphs in two-dimensional porous media. These findings provide a useful reference and basis for erosion prediction and prevention in practical engineering. At the same time, we also realize

that there are still some deficiencies and limitations in this simulation, such as the type of fluid simulation and the complexity of porous media, which need further research and improvement.

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