

A method for 3D computer tomography using blending approximation

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Summary: The new method of restoration of internal structure (density, absorption or easing factor) of a three-dimensional body using information on it in the form of the tomograms set on some system of planes, crossing object of research is offered. This method is based on use of operators of blending approximation of functions of three variables. It has high accuracy and thus allows to smooth the inexact task of experimental data.

Keywords: computer tomography, tomograms, blending approximation, interflation.

1. Introduction

The problem of restoration of internal structure (density, absorption or easing factor) a three-dimensional body by information on it in the form of the tomograms set on some system of planes, crossing object of research is considered. This problem arises in practice when among the planes entering into experimental data, there is no the plane containing this or that set of points, interesting researcher. For example, such problem can arise after the patient has passed research on a medical tomograph. And after the analysis of the received tomograms, there is a necessity to find by them one or several tomograms in the planes crossing a body and in coincident from one of set planes. In existing tomographs this problem dares only in a case when received tomograms lay on system of parallel planes. The same lack have also known 3D packages, for example, 3D Max.

One of the major theoretical and practical problems of a computer tomography is the problem of the account of errors in experimental data. These errors can be in the inexact task of planes on which tomograms lay, and also in the image located on tomograms.

The general solution method of a problem of restoration of internal structure of a three-dimensional body by the information on it in the form of the tomograms set on system of three groups of crossed planes, in each of which plane are parallel, is proposed in works (Lytvyn and Pershina, 2005; Lytvyn et al., 2007). It has been solved using interflation functions operators of three variables (look definition of interflation below). This method has very high accuracy. However this accuracy is theoretically proved only for cases when experimental data are set without errors. In a case when experimental data have an errors, we are offer in the given work approximation based on

use of approximation's operators of three variables functions. The offered method also has very high accuracy. This method allows smoothing the inexact the set experimental data by polynomials, rational functions, splines, trigonometrical polynomials, wavelets.

In the given work we offer new method of restoration of internal structure of a three-dimensional body. In a method smoothing of experimental data are offer in the form of the tomograms set on three groups of crossed planes by operators of the blending approximation (Lytvyn, 2002; Lytvyn, 2006).

The basic theoretical statements of a method are given below. Also results of its realisation using the blending approximation by Bernstein's polynomial are analyzed.

2. The basic definitions

Let function of three variables $f(x, y, z)$ represents density of a three-dimensional body (or factor of absorption, easing). Let a body completely placed in unit cube E^3 , $E = [0, 1]$ and the system of planes

$\Pi_k : \omega_k(x, y, z) = 0$, $k = \overline{1, n}$, crosses a body, are given.

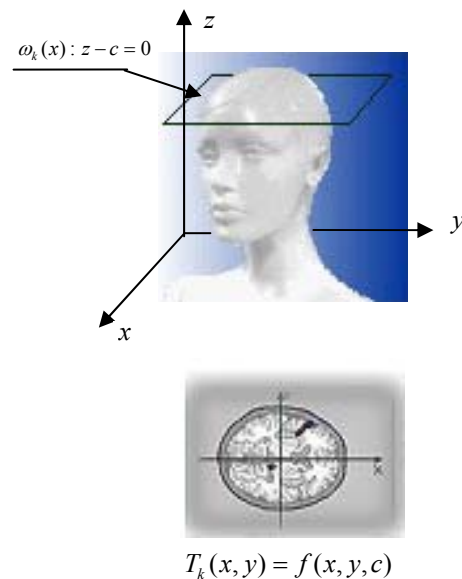


Fig. 1. Tomogram $T_k(x, y)$ represents function of two variables - a trace of function $f(x, y, z)$ on plane $\omega_k(x) = 0$.

Definition 1. Traces function $f(x, y, z)$ on plane $\Pi_k : \omega_k(x, y, z) = 0$ we will name function of two variables $\varphi_k(x, y)$ or $\varphi_k(x, z)$, or $\varphi_k(y, z)$ which in each point of this plane Π_k accepts the same values, as function $f(x, y, z)$

$$f|_{\Pi_k} = \varphi_k|_{\Pi_k}, \quad k = \overline{1, n} \quad (1)$$

Definition 2. Restoration (probably, approximately) functions $f(x, y, z)$ in points between planes

$\Pi_k : \omega_k(x, y, z) = 0$ by its traces (1) on these planes is called interflation of functions $f(x, y, z)$.

Definition 3. The tomogram (traces function $f(x, y, z)$) $T_k(\bar{x})$ on plane $\omega_k(x) = 0$ we will name one of three functions:

$$T_k(\bar{x}) = \begin{cases} f(x_k(y, z), y, z) \\ f(x, y_k(x, z), z) \\ f(x, y, z_k(x, y)) \end{cases}; \bar{x} = \begin{cases} (x, y) \\ (x, z) \\ (y, z) \end{cases} \quad (2)$$

3. Operators of approximation using tomograms

Let set three systems of tomograms on mutually perpendicular planes which are received using a computer tomograph. Not reducing a generality we will consider, that these three groups of planes are parallel to co-ordinate planes, i.e. there are such tomograms:

- $T1_k(y, z) = f(x_k, y, z)$, $k = \overline{1, n}$;
- $T2_l(x, z) = f(x, y_l, z)$, $l = \overline{1, m}$;
- $T3_p(x, y) = f(x, y, z_p)$, $p = \overline{1, s}$.

Formulas for operators of approximation on each of a variable we will represent in a kind:

$$\begin{aligned} L_{x,n}f(x, y, z) &= \sum_{k=1}^n g1_k(x) \cdot T1_k(y, z), \\ L_{y,m}f(x, y, z) &= \sum_{l=1}^m g2_l(y) \cdot T2_l(x, z), \\ L_{z,s}f(x, y, z) &= \sum_{p=1}^s g3_p(z) \cdot T3_p(x, y), \end{aligned} \quad (3)$$

where $g1_k(x)$, $g2_l(y)$, $g3_p(z)$ – basic approximational functions (the generalised polynomials, algebraic and trigonometrical splines). Operators $L_{x,n}f$, $L_{y,m}f$, $L_{z,s}f$ operate on function $f(x, y, z)$ on variable x, y, z accordingly. The operator of the blending approximation is:

$$Of(x, y, z) = (L_{x,n} + L_{y,m} + L_{z,s} - L_{x,n}L_{y,m} - L_{x,n}L_{z,s} - L_{y,m}L_{z,s} + L_{x,n}L_{y,m}L_{z,s})f(x, y, z) \quad (4)$$

Theorem 1. For an error of approximation $Rf(x, y, z) = (I - O)f(x, y, z)$ valid operational equality: $Rf(x, y, z) = r_{x,n}r_{y,m}r_{z,s}f(x, y, z)$, where $r_{x,n} = I - L_{x,n}$, $r_{y,m} = I - L_{y,m}$, $r_{z,s} = I - L_{z,s}$, I – the unit operator.

The proof follows from operational identity:

$$\begin{aligned} (I - L_{x,n} - L_{y,m} - L_{z,s} + L_{x,n}L_{y,m} + \\ + L_{x,n}L_{z,s} + L_{y,m}L_{z,s} - L_{x,n}L_{y,m}L_{z,s})f = \\ = (I - L_{x,n})(I - L_{y,m})(I - L_{z,s})f \end{aligned} \quad (5)$$

Theorem 2. For an estimation of an error of approximation of function $f(x, y, z)$ operator

$Of(x, y, z)$ carries out an inequality:

$$\|f - Of\|_{C([0,1]^3)} = O(\varepsilon_1 \cdot \varepsilon_2 \cdot \varepsilon_3), \quad \varepsilon_k \rightarrow 0, \quad k = \overline{1, 3},$$

where

$$\varepsilon_1 = \|(I - L_{x,n})f\|_{C([0,1]^3)}, \quad \varepsilon_2 = \|(I - L_{y,m})f\|_{C([0,1]^3)}, \quad \varepsilon_3 = \|(I - L_{z,s})f\|_{C([0,1]^3)}.$$

4. Operators of the blending approximation by Bernstein's polynomials.

Let's consider above stated theory on an example of the blending approximation by Bernstein's polynom.

Definition 5. Bernstein's polynomial (or Bernstein's operator of degree n for function $g(t) \in C[0,1]$) one variable is called:

$$B_n g(t) = \sum_{k=0}^n C_n^k t^k (1-t)^{n-k} g\left(\frac{k}{n}\right) \quad (6)$$

where C_n^k – binomial coefficients.

Definition 6. The blending approximation of Bernstein's operator is called:

$$Of(x) = (B1_n + B2_m + B3_s - B1_n B2_m - B1_n B3_s - B2_m B3_s + B1_n B2_m B3_s)f(x) \quad (7)$$

where

$$\begin{aligned} B1_n(x, y, z) &= \sum_{k=0}^n C_n^k x^k (1-x)^{n-k} T1_k(y, z), \\ B2_m(x, y, z) &= \sum_{l=0}^m C_m^l y^l (1-y)^{m-l} T2_l(x, z), \\ B3_s(x, y, z) &= \sum_{p=0}^s C_s^p z^p (1-z)^{s-p} T3_p(x, y) \end{aligned}$$

$n+1, m+1, p+1$ – quantity of the tomograms located on planes which are perpendicular to axes Ox, Oy, Oz accordingly.

Theorem 3. Let $R1_n f = (I - B1_n)f$, $R2_m f = (I - B2_m)f$, $R3_s f = (I - B3_s)f$ – errors of approximation by operators $B1_n f$, $B2_m f$, $B3_s f$ accordingly. Then error $Rf = (I - O)f$ of approximation f by operator $Of(x)$ is defined by a parity: $Rf(x, y, z) = R1_n R2_m R3_s f(x, y, z)$.

Operator $Of(x)$ defined by equation (7) and constructed on set system of tomograms approaches each function $f(x, y, z) \in C^{2,2,2}(I^3)$, $E = [0,1]$ with a margin error

$$\|f - Of\| = O((nms)^{-1}) \quad (8)$$

For the proof of this theorem we are used following approximational properties of polynomials of Bernstein.

Statement. For each continuous function $g(t) \in C(E)$ and any number $\varepsilon > 0$ there will be such natural number n , that $|g(t) - B_n g(t)| \leq \varepsilon, \forall t \in [0,1]$.

Bernstein's polynomials converge in regular intervals to everyone continuous on E functions $g(t)$. If function $g(t)$ is limited in point c , $a < c < b$ and has rupture of the first sort in this point, then

$$\lim_{x \rightarrow c} B_n g(t) = \frac{g(c-0) + g(c+0)}{2}, \quad n \rightarrow \infty \quad (9)$$

If function $g(t)$ is twice differentiated in point $c \in [a, b]$ then

$$B_n g(c) - g(c) = \frac{g''(c)(c-a)(b-c)}{2n} + o\left(\frac{1}{n}\right) \quad (10)$$

Thus, theoretically at use of the operator of the blending approximation by Bernstein's polynomial for restoration of internal structure of a three-dimensional body with the same quality enough smaller quantity of the tomograms made in system of mutually perpendicular planes.

In work (Backman, 1998) for restoration of three-dimensional model of a head of the person at use of techniques of visualization *isosurface* it has been used 167 tomograms laying on planes, parallel to one plane (fig. 2)



Fig. 2. One of 167 tomograms used in (Backman, 1998)

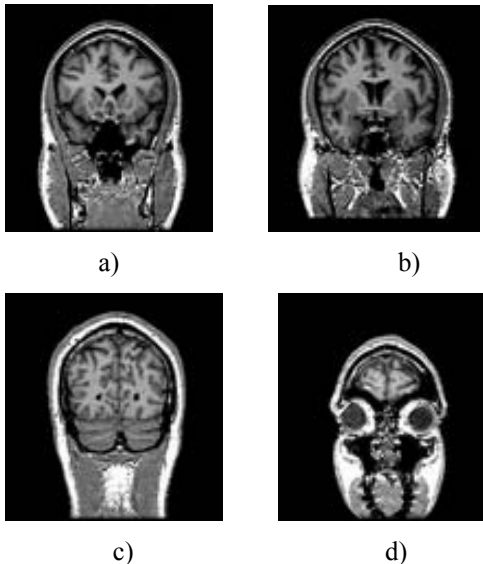


Fig. 3. Examples of sections of a brain by planes: a) $x = 0.5$; b) $x = 0.6$; c) $x = 0.8$; d) $x = 0.9$

One of lacks of this method described in (Backman, 1998) is that it uses only sections in the planes parallel to one plane. We will notice, that though in (Backman,

1998) there are available tomograms in three systems of mutually perpendicular planes, however for diagnostics in (Backman, 1998) were used only one of these systems.

The techniques of visualization *isosurface* cannot use the information on internal structure of a three-dimensional body is set on three mutually perpendicular planes.

5. Test example of restoration of internal structure of a three-dimensional body.

Authors had been made computing experiment for restoration of internal structure of a three-dimensional body under the set tomograms by offered method.

A 30 tomograms laying on system of mutually perpendicular planes (on 10 in everyone) (fig. 3 – 5) are experimental data.

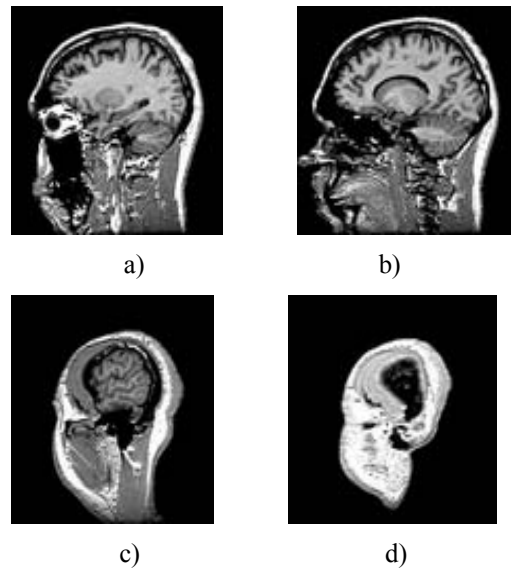


Fig. 4. Examples of sections of a brain by planes: a) $y = 0.3$; b) $y = 0.6$; c) $y = 0.8$; d) $y = 0.9$

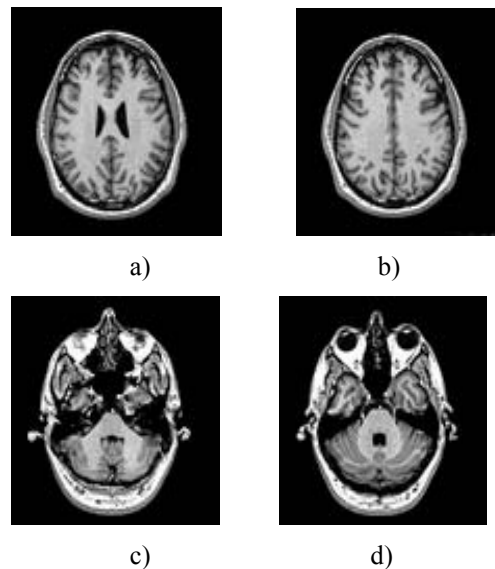


Fig. 5. Examples of sections of a brain by planes: a) $z = 0.2$; b) $z = 0.3$; c) $z = 0.8$; d) $z = 0.9$

Results of work of computing experiment are given on fig. 6-7.

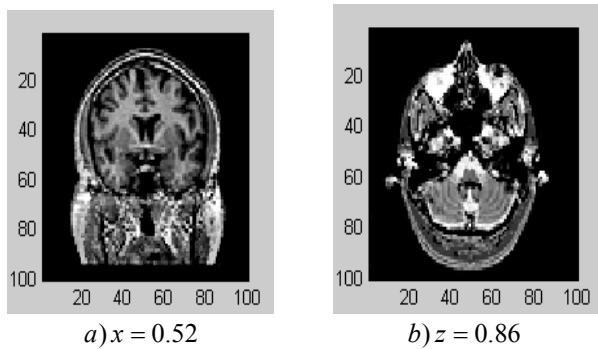


Fig. 6. Results of restoration of density $f(x, y, z)$ by given method in planes a) $x = 0.56$ (between the tomograms located on planes $x = 0.5$, $x = 0.6$ (look fig. 3)); b) $z = 0.86$ (between the tomograms located on planes $z = 0.8$, $z = 0.9$ (look fig. 5)).

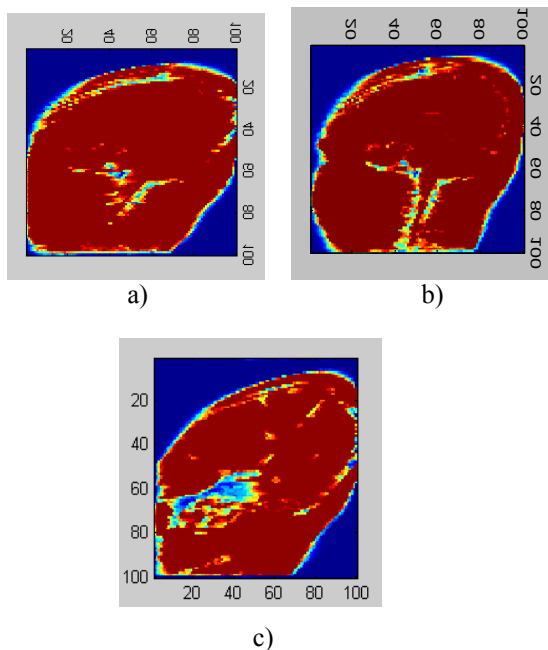


Fig. 7. Results of restoration of density $f(x, y, z)$ by given method in plane $x + y + 10z = 0$ (a) if the experimental tomograms lay on planes, parallel only to axis Oy ; b) if the experimental tomograms lay on planes, parallel only to axis Oz ; c) if the experimental tomograms lay on mutually perpendicular planes.

From fig. 7 it is visible, that the third drawing is more informative.

6. Conclusions

Thus, we are state the new method of restoration of internal structure of a three-dimensional body. This

method is recommended to be used when experimental data (characteristics of tomograms - geometrical parameters of a plane in which the tomograms lays, and also the image on tomograms) are set with error.

Authors convince that after creation of the corresponding software co-ordinates with the software on operating computer tomographs, received above the statements can be used in medical practice, and also at not destroying control, at customs etc.

We invite the firms which are making tomographs to cooperation with the purpose of creation of the software for operating or projected tomographs. The purpose - increase of accuracy of diagnostics.

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