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A random geometric graph as a math model of a porous medium

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Abstract

Porous media play a vital role in many fields such as geology, materials science, petroleum engineering and environmental science. Their complex pore structure and unique seepage characteristics have a profound impact on related processes and applications. However, traditional methods for studying porous media have certain limitations in accurately characterizing their microstructure and dynamic characteristics.

As an emerging mathematical modeling tool, random geometric graphs have shown unique advantages in describing the structure and relationship of complex systems. This study aims to construct random geometric graphs as mathematical models of porous media in order to understand and analyze various characteristics of porous media more deeply and accurately.

In the process of model construction, reasonable assumptions were made based on the actual physical characteristics of porous media, key parameters were set, and the specific algorithm steps for generating the random geometric graph model were elaborated in detail. Through this model, the pore structure of porous media can be analyzed in detail, including pore size, shape and distribution law, and the effectiveness of the model in describing the pore structure was verified by comparing it with the actual porous media microstructure image.

At the same time, the model was used to conduct in-depth research on the connectivity of porous media, calculate indicators such as the number of connected branches and the maximum size of connected branches, and further explore their relationship with seepage characteristics. The seepage model established based on this model, with the help of percolation theory, successfully revealed the law of changes in seepage characteristics such as seepage threshold and seepage velocity with model parameters, and compared and verified with experimental results and existing theories, achieving a good degree of consistency.

By comparing with the collected experimental data related to porous media (such as porosity measurement, seepage experimental data, etc.), and comprehensively comparing with the traditional porous media mathematical model in describing the pore structure, seepage and other characteristics, this study used defined evaluation indicators (such as mean square error, correlation coefficient, etc.) to quantitatively evaluate the random geometric graph model. The results show that the model performs better than traditional models in many aspects and can more accurately capture the complex characteristics of porous media, but there are also some areas that need to be improved, such as some assumptions that may lead to poor adaptability to extremely complex media conditions.

In general, this study provides a new perspective and powerful tool for a deeper understanding of the characteristics of porous media by using random geometric graphs as mathematical models of porous media. It also lays a foundation for its subsequent application expansion in material design, groundwater hydrology and other fields. In the future, the model can be further optimized and improved to better serve the research and practice in related fields.

Keywords : random geometric graph; porous media; mathematical model; pore structure; seepage

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1. Introduction

1. Research background

Porous media are widely present in nature and in many engineering application fields. Their unique physical properties and complex internal structures play a key role in many processes and phenomena, and thus have become an important object of in-depth research in multiple disciplines.

In the field of geology, porous media such as soil and rock constitute the main part of the earth's surface. As the matrix for plant growth, the pore structure of soil affects the storage of water, the transmission of nutrients and the exchange of air [1], which in turn determines the growth of vegetation and the health of the ecosystem. The pores in rocks are closely related to the occurrence and migration of groundwater and the storage of resources such as oil and natural gas. For example, in the process of oil extraction, understanding the pore characteristics of reservoir rocks is crucial for accurately assessing oil reserves and designing reasonable extraction plans, because oil is stored in rock pores and flows through these pore channels.

Thiedmann et al. developed a stochastic network model in the form of a random geometric graph to describe the 3D morphology of the pore space in fibre-based materials. The vertex set was modelled by random point processes, and the edges were determined using graph theory and Markov chain Monte Carlo simulation. The model parameters were fitted to real image data obtained by X-ray synchrotron tomography, specifically ensuring that the distributions of vertex degrees and edge lengths in the real and simulated data were largely consistent. This model was used to introduce a morphology-based concept of pores and their sizes and was validated by considering physical characteristics relevant to transport processes in the pore space, such as geometric tortuosity (the distribution of shortest path lengths through the material relative to its thickness) [2].

Feng and Yin presented methods to directly model the random structure of porous media using Voronoi tessellations. They generated three basic structures corresponding to porous medium geometries with intersecting fractures (granular), interconnected tubes (tubular), and fibres (fibrous). Fluid flow through these models was solved by a massively parallelized lattice Boltzmann code. They established the porosity–permeability relations for these basic geometry models. It was found that for granular and tubular geometries, the specific surface area was a critical structural parameter that could unify their porosity–permeability relations under the Kozeny–Carman equation. Connected fractures increased the dimensionless permeability relative to the equation, while isolated large pores (vugs) decreased it. The equation could not distinguish a heterogeneous structure with an embedded partially penetrating fracture. The porosity–permeability relation for fibrous geometries generally agreed with those of simple-cubic, body-centered cubic, and face-centered cubic models. In the dilute limit, the dependence on the solid fraction was weaker in Voronoi geometries, indicating weaker hydrodynamic interactions among randomly interconnected fibres than in the idealized models [3].

The field of materials science also pays great attention to porous media. Many advanced materials, such as porous ceramics and porous metals, have a wide range of applications in filtration, catalysis, heat exchange, etc. due to their low density, high specific surface area, good permeability and other characteristics [4]. Taking porous ceramics as an example, their pore structure can effectively filter impurity particles while allowing fluids to pass through, playing an important role in fields such as water treatment and air purification [5]. The performance of these materials depends largely on the size, shape, distribution and connectivity of their pores, so in-depth understanding and precise control of the structure of porous media has become a key link in material design and optimization.

In environmental science, porous media play an important role in groundwater hydrology, soil pollution diffusion and other processes. Groundwater flows in the pores of porous media such as soil and rock, and its flow rate, path and material exchange with the surrounding environment are all affected by the pore structure [6,7]. For the problem of soil pollution diffusion, the migration path and diffusion rate of pollutants in porous media are also closely related to factors such as pore connectivity and pore size. Accurately grasping these characteristics will help predict the diffusion range of pollutants, thereby formulating effective pollution prevention and control measures.

However, despite the importance of porous media, traditional research methods face many challenges in fully and accurately describing their characteristics due to the high complexity of their internal structure. Traditional research methods are mainly based on the assumption of continuum mechanics, which regards porous media as a uniform continuum, and can provide a more effective analysis method when dealing with some macroscopic phenomena [8]. However, when it comes to issues such as pore structure, local heterogeneity, and connectivity between pores at the microscopic level, this continuum model is inadequate.

In addition, the network model, as another traditional method for studying porous media, although it takes into account the connectivity between pores to a certain extent, often oversimplifies the actual shape and distribution of pores [9], making it difficult to accurately reflect the true microstructural characteristics of porous media.

In summary, the importance of porous media in many fields is self-evident, but the existing traditional research methods have obvious shortcomings in characterizing their complex characteristics [10]. This urgently requires a new mathematical model that can more comprehensively and accurately describe the characteristics of porous media, so as to provide strong support for the in-depth understanding and effective utilization of porous media.

2. Introduction to Random Geometric Graphs

As a distinctive graphical model in the field of graph theory, random geometric graphs have emerged in the research of many disciplines in recent years, providing a novel and effective way to describe the structure and relationships of complex systems.

From the perspective of its basic composition, random geometric graphs are mainly composed of two parts: nodes and edges. Nodes can usually be regarded as points randomly distributed within a certain spatial range, which represent the basic units in the system under study[11]. For example, when simulating social networks, nodes may represent people; when studying the relationship between biological communities, nodes may correspond to individuals of different species. Edges are connections formed between nodes according to specific rules.

For random geometric graphs, the edge formation rules are often related to the distance between nodes. Specifically, given a connection radius parameter , when the spatial distance between two nodes is less than or equal to , an edge is generated between the two nodes to represent the existence of a certain connection or interaction between the basic units represented by the two nodes[12]. This distance-based edge formation mechanism enables random geometric graphs to naturally capture the relationship between units in the system due to spatial proximity.

Random geometric graphs have shown great potential for application in other fields. In the field of communication networks, they can be used to simulate the communication links between sensor nodes in wireless sensor networks. By properly setting the node distribution and connection radius, key performance indicators such as network connectivity and transmission efficiency can be accurately analyzed. In ecological research, random geometric graphs can be used to construct biological habitat models, treating individuals of different species as nodes, determining the connection of edges based on their spatial distribution in the habitat and the range of interaction, and then studying ecological relationships such as symbiosis and competition between species.

In summary, random geometric graphs, with their unique structural definition and distance-based edge formation rules, as well as successful application examples in other fields, have the characteristics of being an effective modeling tool, which makes them an attractive choice when we explore the construction of mathematical models that can more accurately describe the characteristics of porous media.

3. Research Purpose and Significance

In view of the key role of porous media in many fields and the many limitations of traditional research methods in characterizing their complex properties, this study aims to introduce random geometric graphs as a new mathematical model to describe and analyze porous media, in order to achieve the following important goals and thus highlight the significance of this research.

Purpose of the study:

First, we hope that by constructing a random geometric graph model, we can more accurately and meticulously characterize the pore structure of porous media. Factors such as the pore size, shape, distribution, and connectivity of porous media are crucial to understanding the overall performance of the medium [13]. Traditional models are often unable to handle these microscopic details, and random geometric graph models, with their flexible node and edge settings, have the potential to treat pores as nodes and simulate the connectivity between pores by using the connection relationship between nodes, thereby providing a more accurate mathematical framework for in-depth exploration of pore structure.

Secondly, the model is used to further study the connectivity characteristics of porous media. Connectivity not only affects the transmission path of substances (such as fluids and gases) inside porous media, but is also closely related to important physical processes such as seepage. By analyzing the connectivity of nodes in the random geometric graph model, such as calculating the number of connected branches and the maximum connected branch size, we hope to clearly reveal the connectivity structure inside the porous media, thereby laying the foundation for understanding the transport mechanism of substances in it.

Furthermore, an effective seepage model is established based on the random geometric graph model to accurately predict the seepage characteristics of porous media. The seepage process plays a core role in many fields related to porous media (such as oil extraction, groundwater hydrology, etc.) [14]. Traditional methods have certain uncertainties in predicting key parameters such as seepage threshold and seepage velocity [15]. With the help of the random geometric graph model and combined with relevant knowledge such as percolation theory, we try to accurately simulate the seepage process and understand the law of changes in seepage characteristics with model parameters, so as to provide a more reliable theoretical basis for related engineering practices.

Research significance:

From a theoretical perspective, this research will enrich and expand the mathematical modeling method system of porous media. At present, traditional continuum models and network models have many shortcomings in describing the characteristics of porous media, and the introduction of random geometric graph models has brought new ideas and tools to the field. It will help to gain a deeper understanding of the intrinsic relationship between the microstructure and macroscopic properties of porous media and promote the further development of theoretical research on porous media.

At the application level, its significance is also very significant. For the field of petroleum engineering, accurate understanding of the pore structure and seepage characteristics of reservoir rocks is crucial to improving oil production efficiency and optimizing production plans. The random geometric graph model established in this study can more accurately simulate the relevant characteristics of reservoir rocks, thereby providing more targeted guidance for oil extraction, which is expected to increase oil production and reduce extraction costs.

In the field of materials science, the performance optimization of many porous materials depends on the precise control of their pore structure and connectivity [16]. The research results can be applied to the design and development of porous materials, helping engineers to better understand the relationship between the internal structure and performance of materials, and then to manufacture porous materials with better performance, expanding their application range in filtration, catalysis, heat exchange, etc.

In environmental science, issues such as groundwater hydrology and soil pollution diffusion are closely related to the characteristics of porous media. The use of random geometric graph models can more accurately analyze the flow path and seepage characteristics of groundwater in porous media such as soil and rock, as well as the migration and diffusion laws of pollutants in them, which is of great guiding significance for formulating scientific and effective groundwater protection and soil pollution control measures.

In summary, studying random geometric graphs as mathematical models of porous media is of great significance both in theoretical expansion and practical application, and is expected to bring new opportunities and breakthroughs to the development of related fields.

2. Characteristics of porous media and traditional modeling methods

1. Physical properties of porous media

As a material with complex internal structure, the physical properties of porous media largely determine its performance and application value in various fields. The following is a detailed description of several key physical properties of porous media:

1. Porosity

Porosity is an important indicator to measure the volume ratio of pores in porous media, which directly reflects the looseness of the medium. Its calculation formula is usually the ratio of pore volume to total volume. The size of porosity has a profound impact on many properties of porous media. For example, in soil, higher porosity means more space can be used to store water and air, which is essential for the respiration of plant roots and the absorption of water and nutrients [17]. In the field of building materials, such as porous concrete, appropriate porosity can not only ensure that the material has a certain strength, but also enable it to have good thermal insulation and sound insulation properties. The porosity of different types of porous media varies greatly. For example, the porosity of loose sand may be as high as 40% - 50%, while the porosity of some dense rocks may be as low as 1% - 5%.

2. Pore size distribution

The pore size distribution describes the distribution of pore sizes in porous media. It is a parameter that more carefully describes the pore structure. The pores in porous media are not uniform in size, but present a variety of size ranges [18]. The pore size distribution is usually expressed by a probability distribution function, such as normal distribution, lognormal distribution, etc. The size of the pore size directly affects the medium's ability to filter, adsorb different substances, and allow fluids to pass through. For example, in porous filter materials used for water treatment, smaller pore sizes can effectively intercept tiny impurity particles and ensure the purification of water quality; in oil extraction, the pore size of rock pores determines the ease with which oil can flow through them, and larger pore sizes are relatively more conducive to oil seepage.

3. Connectivity

Connectivity refers to the degree of interconnection between pores in a porous medium, and it is a key factor affecting the transmission of substances within the medium. A well-connected porous medium can provide a smooth transmission channel for fluids, gases and other substances. Conversely, if the connectivity between pores is poor, the transmission of substances will be hindered. In the groundwater system, the connectivity of porous media such as soil and rock determines the flow path and speed of groundwater. If the rock pore connectivity in a certain area underground is poor, it may cause groundwater to accumulate in the area or form local stagnation. Similarly, in the process of oil extraction, the connectivity of reservoir rocks has an important influence on the efficiency of oil extraction. Reservoirs with good connectivity are more conducive to the smooth extraction of oil from oil wells.

4. Pore shape

Pore shape is also an important component of the physical properties of porous media, and it presents a variety of forms in different porous media. Common pore shapes include spherical, cylindrical, lamellar, and various irregular shapes [19]. The pore shape not only affects the volume calculation of the pores (the volume calculation of irregular pores is more complicated than that of regular shapes), but also affects other properties of the medium. For example, in porous catalyst materials, pores of different shapes will affect the diffusion path and residence time of reactant molecules in the pores, thereby affecting the efficiency of the catalytic reaction. In the design of some porous materials with special application requirements, such as lightweight porous materials used in the aerospace field, precise control of pore shape can optimize material properties to meet specific strength, thermal insulation and other requirements.

5. Interactions between characteristics

The above physical properties of porous media do not exist in isolation, but influence and restrict each other. For example, the porosity will affect the pore size distribution. Generally speaking, a higher porosity may be accompanied by a wider pore size distribution because more pore space provides conditions for the formation of pores of different sizes. Connectivity is also closely related to porosity and pore size distribution. A higher porosity and a suitable pore size distribution often help to improve connectivity because more pores and pores of appropriate size are more conducive to the formation of connected channels. Pore shape also affects connectivity. Irregularly shaped pores may lead to poor connectivity in some cases because they may form some local blockages or structures that hinder connectivity.

Through a deeper understanding of these physical properties of porous media, we can better appreciate the complexity of their internal structure and the importance of these properties in different application fields. At the same time, we can also more clearly see the challenges faced by traditional modeling methods in accurately describing these properties.

2. Overview of Traditional Mathematical Models

In the long history of studying porous media, researchers have developed a variety of traditional mathematical models in order to understand their complex physical properties and related physical processes. These models have helped us understand porous media to a certain extent, but they also have their own limitations. The following is an introduction to several common traditional mathematical models:

Table 1: Comparison of Traditional Mathematical Models and Random Geometric Graph Models

Model	Description of Pore Structure	Description of Seepage Characteristics	Advantages	Disadvantages
Continuum Model	Treats the porous medium as a uniform continuum, ignoring microscopic pore structure information, and only roughly characterizes overall characteristics such as porosity through macroscopic parameters.	Describes the seepage of fluids in porous media based on macroscopic theories like Darcy's law, focusing on macroscopic seepage velocity and flow rate, and having difficulty grasping microscopic seepage details.	Simple and easy to use, effective in dealing with macroscopic phenomena.	Cannot accurately describe microscopic pore structure and seepage characteristics.
Network Model	Abstracts pores as nodes and connecting channels as edges, but simplifies pore shapes. Determining node and edge properties requires a lot of assumptions and empirical data.	Takes into account pore connectivity, but due to the simplified treatment of pore structure and inaccurate property determination, it is difficult to accurately predict seepage characteristics.	Takes into account pore connectivity.	Simplifies pore structure and cannot accurately describe the real microstructure and seepage characteristics.
Random Geometric Graph Model	Nodes correspond to pore centers, and edges determine connectivity based on distance. It can accurately reflect pore size, shape, distribution, and connectivity characteristics through attribute settings.	Based on percolation theory, it can deeply analyze microscopic details of the seepage process, such as the specific flow path of the fluid between different pores, seepage threshold, and seepage velocity, and the predicted results are consistent with the actual situation.	Accurately describes pore structure and seepage characteristic s, provides microscopic detail analysis.	The model construction process is complex, with many parameters that need to be flexibly adjusted; assumptions may not be consistent with the actual situation, resulting in deviations.

1. Continuum Model

The continuum model is the earliest and most widely used mathematical model for studying porous media. Based on the assumption of continuum mechanics, the model regards the porous medium as a uniform continuum, ignoring its internal microscopic pore structure. In this model, the porous medium is regarded as composed of continuous substances with equivalent physical properties (such as density, permeability, etc.), and the overall characteristics of the medium are described by defining some macroscopic parameters (such as porosity, permeability and other macroscopic measurable parameters).

For example, when studying the flow of groundwater in soil, the continuum model considers soil as a continuous, isotropic or anisotropic medium, and analyzes the seepage characteristics of groundwater such as velocity and flow rate through Darcy's law (the basic law describing the linear seepage of fluids in porous media). Darcy's law is based on the continuum assumption and considers the permeability of soil as a macroscopic parameter to predict the flow of groundwater.

However, the main limitation of the continuum model is that it cannot accurately describe the pore structure, the connectivity between pores, and local heterogeneity at the microscopic level inside the porous medium. Since the porous medium is regarded as a homogeneous continuum, it loses a lot of information about the pore details. For some situations that require a deep understanding of the impact of pore structure on physical processes, the performance of the model is not satisfactory.

2. Network Model

The network model is another common traditional mathematical model used to study porous media. Unlike the continuous medium model, the network model attempts to consider the connectivity between pores in the porous medium. It abstracts the pores in the porous medium as nodes in the network and the connecting channels between the pores as edges in the network, thus constructing a graph-like network structure to describe the porous medium.

In the network model, the description of porous media can be further refined by defining the properties of nodes (such as pore size, shape, etc.) and edge properties (such as channel permeability, length, etc.). For example, when studying the seepage of oil in reservoir rocks, the pores in the reservoir rocks can be regarded as network nodes, and the connecting channels between the pores can be regarded as network edges [20]. The seepage process of oil in the pores of the rock can be simulated by setting the properties of different nodes and edges.

Although the network model takes into account the connectivity between pores to a certain extent, it also has obvious shortcomings. On the one hand, it tends to oversimplify the actual shape and distribution of pores, abstracting pores into simple nodes, which makes it difficult to accurately reflect the real microstructural characteristics of porous media. On the other hand, when determining the properties of nodes and edges, a large number of assumptions and empirical data are often required, and the way these properties are determined may not be accurate enough, thus affecting the model's accurate description of the characteristics of porous media.

3. Dual medium model

The dual medium model is a mathematical model developed to make up for the shortcomings of the continuous medium model and the network model in some aspects. It regards the porous medium as a composite system composed of two media with different properties, usually divided into two parts: matrix and fracture. The matrix part is generally considered to be a continuous medium with relatively low permeability, while the fracture part is considered to be a connected channel with higher permeability.

In the dual medium model, porous media are described by defining the physical properties of the matrix and fractures (such as porosity, permeability, etc.) and the interaction between them (such as the transfer of fluid between the matrix and fractures). For example, when studying the flow of groundwater in rocks with fractures, the dual medium model describes the matrix and fracture parts of the rock separately, taking into account the transfer of fluid from the matrix to the fracture part and the rapid flow in the fracture part.

However, the dual medium model also has its limitations. Although it takes into account the existence of media with different properties in porous media, it still does not accurately describe the microscopic pore structure inside the matrix and fractures and the precise interaction relationship between them. Moreover, in practical applications, how to accurately divide the matrix and fractures and determine their physical properties and interaction relationships is also a challenging problem.

In summary, these traditional mathematical models have played a certain role in the study of porous media, but they are insufficient in describing certain key characteristics of porous media (such as microscopic pore structure, precise connectivity, etc.). This has prompted us to seek a new mathematical model that can more comprehensively and accurately describe the characteristics of porous media.

3. Construction of random geometric graphs as mathematical models of porous media

1. Model Assumptions

In order to effectively construct the random geometric graph as a mathematical model of porous media, we put forward the following series of reasonable assumptions based on the actual physical properties and related laws of porous media:

1. Correspondence assumption between nodes and pores

We assume that the nodes in the random geometric graph correspond to the center positions of pores in porous media. In actual porous media, pores are a special form of material distribution, and their center positions play a key role in describing the pore structure and the relationship with other pores. Setting nodes as pore centers can more intuitively reflect the distribution of pores in space, and facilitate the subsequent simulation of the connectivity between pores through the connection relationship between nodes. For example, when studying porous media such as soil, the centers of each pore in the soil can be regarded as nodes in the random geometric graph, so that the spatial layout of soil pores and their connectivity can be characterized by the distribution and connection of nodes.

2. Assumptions about edge-pore connectivity

It is assumed that in a random geometric graph, when the distance between two nodes is less than or equal to a certain connection radius, there is a connectivity relationship between the pores corresponding to the two nodes, that is, an edge connecting the two nodes is generated in the random geometric graph. This assumption is based on practical considerations of pore connectivity in porous media. In a real porous media environment, the connectivity between pores often depends on the spatial distance between them and the physical properties of the pores themselves (such as pore size, etc.). When the distance between two pores is close and their own characteristics allow, they are more likely to form a connecting channel [21]. By setting the connection radius, we simulate this pore connectivity mechanism in a simplified but reasonable way. For example, when simulating the connectivity of rock pores, if the distance between the centers of two pores (i.e., the corresponding nodes) is within the set connection radius, the two pores are considered to be connected, and an edge is used to represent this connectivity relationship.

3. Homogeneity Assumption

In some cases, in order to simplify the model analysis process, we can assume that the porous media has a certain degree of uniformity in a certain area. Specifically, it is assumed that the physical properties such as the distribution density of pores, the distribution of pore sizes, and the shape of pores in the porous media area under study are statistically uniform. This means that when constructing a random geometric graph model, a relatively uniform parameter setting can be used to describe the porous media conditions in the area. For example, when studying a relatively flat soil area with a relatively uniform texture, it can be assumed that the distribution, pore size, and shape of the soil pores in the area are roughly the same, making the construction of the random geometric graph model easier. However, it should be noted that this uniformity assumption may not be completely consistent with the actual situation in some complex and diverse porous media environments, so it needs to be carefully selected according to the characteristics of the specific research object in practical applications.

4. Independence Assumption

It is assumed that the generation of each node (i.e., the pores in the corresponding porous media) in the random geometric graph and the formation of the edges between them are independent processes. In other words, the determination of the position of a node and whether it forms an edge with other nodes are not affected by the positions and connections of other nodes that have been determined. This assumption simplifies the model construction and analysis process to a certain extent [22]. Although there may be some mutual influences between pores in actual porous media (such as pore deformation caused by local pressure changes, which in turn affects the connectivity relationship), when initially constructing the model, ignoring these minor influencing factors and treating the formation of nodes and edges as independent processes can enable us to focus more on capturing the main structural characteristics and connectivity relationships of porous media. For example, when simulating a large-scale porous media system, such as a large area of oil reservoir rock, if all possible mutual influence factors are considered, model construction and analysis will become extremely complicated, while the use of the independence assumption can make model construction and analysis relatively simple while ensuring a certain degree of accuracy.

Through these assumptions, we can simplify the construction process of random geometric graphs as mathematical models of porous media to a certain extent, while being able to reasonably simulate the key physical properties of porous media, such as pore structure and connectivity. Of course, these assumptions also have certain limitations, and in subsequent research and analysis, they need to be appropriately adjusted and improved according to specific circumstances.

2. Model parameter setting

In the process of constructing a mathematical model of porous media based on random geometric graphs, it is very important to reasonably set the relevant model parameters, which will directly relate to and reflect various key characteristics of porous media. The following is a detailed description of several main model parameters and their setting methods:

Table 2: Parameter Settings and Significance of the Random Geometric Graph Model

Parameter	**Definition**	**Calculation or Determination Method**	**Impact on the Model**
Node Density (ρ)	The density of nodes (corresponding to pore centers in porous media) in a random geometric graph within a given study area	ρ = N / V (N is the total number of nodes, V is the volume of the study area)	Affects the accuracy of the model's simulation of pore distribution. For porous media with high porosity (such as loose sand), a higher node density is required; for studying local details, a higher node density is needed, while for macroscopic analysis, it can be appropriately reduced.
Connection Radius (r)	A parameter used to determine whether edges are formed between nodes to simulate pore connectivity	Determined based on the actual pore size distribution and connectivity of the porous medium. When the pores are large and the connectivity is good, a larger value can be set; otherwise, a smaller value is set.	Affects the realism of simulating pore connectivity. Reasonable settings can make the formation of edges more in line with the actual pore connection situation.
Node Attribute Parameter - Pore Size (s)	The pore size value corresponding to each node	Determined through experimental measurement, statistical analysis, and other methods based on the actual pore size distribution of the porous medium	Accurately reflects the pore size characteristics and affects the simulation of the medium's filtration, adsorption, and fluid flow capabilities.
Node Attribute Parameter - Pore Shape (p)	The pore shape attribute corresponding to the node	Using simplified classification methods (such as spherical, cylindrical, irregular, etc.), set according to the	Considers the influence of pore shape on the performance of the medium, such as in catalytic reactions.

		proportion of pore shapes in the actual porous medium	
Edge Attribute Parameter - Channel Permeability (k)	The permeability attribute of the edge (the connecting channel between nodes, corresponding to the connecting channel between pores)	Determined through experimental measurement or by referring to existing relevant data	Accurately reflects the permeability of the actual pore connecting channel and affects the simulation of fluid flow in the seepage model.
Edge Attribute Parameter - Edge Length (l)	The length value of the edge	Determined by measurement, estimation, etc. based on the actual structure of the porous medium being studied	Affects the simulation of the flow rate and other characteristics of substances in the connecting channel in the model.

1. Node density (ρ)

Node density refers to the density of nodes (corresponding to pore centers in porous media) in a random geometric graph within a given study area. It largely determines the accuracy of the model's simulation of the pore distribution in porous media.

The formula for calculating node density is usually: $\rho = N / V$, where N represents the total number of nodes and V represents the volume of the area under study.

When setting the node density parameters, it is necessary to comprehensively consider factors such as the porosity of the porous medium itself and the purpose of the study. For example, if the porous medium under study has a high porosity, such as loose sand, then in order to accurately simulate its pore distribution, a higher node density should be set accordingly to ensure that there are enough nodes to represent the numerous pores. Conversely, for porous media such as dense rocks with low porosity, the node density can be appropriately reduced, but it must also be ensured that it can reasonably reflect its pore structure characteristics.

In addition, the purpose of the study will also affect the setting of node density. If you focus on the pore structure and connectivity details of the local area of the porous medium, you may need to set a higher node density to obtain more detailed simulation results; while if you only want to conduct a preliminary analysis of the macroscopic characteristics of the overall medium, a relatively low node density may be sufficient to meet your needs.

2. Connection radius (r)

The connection radius is a critical parameter in the random geometric graph model. It is used to determine whether edges are formed between nodes, thereby simulating the connectivity between pores in porous media.

When the spatial distance between two nodes is less than or equal to the connection radius r, an edge is generated between the two nodes, indicating that the pores corresponding to them are connected.

When determining the value of the connection radius, it is necessary to consider factors such as the actual pore size distribution of the porous medium and the pore connectivity. Generally speaking, if the average pore size of the pores in the porous medium is large and the connectivity between the pores is relatively good, then a larger connection radius can be appropriately set to simulate the connectivity between the pores more realistically [23]. For example, when simulating a porous ceramic material with a large pore size and good connectivity, a larger connection radius can be set so that edges can be formed more reasonably between nodes to reflect the connectivity of the pores.

On the contrary, if the pore size of the porous medium is small and the connectivity is poor, such as the micropore structure in some dense rocks, then a smaller connection radius needs to be set to avoid excessive connection that causes the model to be inconsistent with the actual situation. At the same time, information about the pore connectivity of the porous medium can be obtained through experimental measurements or reference to existing relevant data, thereby assisting in determining the appropriate value of the connection radius.

3. Node attribute parameters

In addition to the two key parameters of node density and connection radius, it is also necessary to set the node attribute parameters to more comprehensively describe the characteristics related to the pores of porous media.

(1) Pore size attribute (s): A value representing the corresponding pore size can be set for each node. This value can be determined based on the distribution of pore sizes in the actual porous medium through experimental measurement, statistical analysis, and other methods. For example, when studying soil pores, the size of each pore can be measured by equipment such as a soil porosity meter, and then these measured values can be assigned to the nodes in the random geometric graph, so that the nodes can more accurately reflect the size characteristics of the actual pores.

(2) Pore shape attribute (p): Considering the diversity of pore shapes in porous media, a pore shape attribute can be set for the node. Although it is difficult to simulate various irregular pore shapes completely and accurately in actual operation, some simplified classification methods can be used, such as classifying pore shapes into spherical, cylindrical, irregular, etc., and assigning corresponding shape attributes to each node. In this way, when analyzing the model, the influence of pore shape on the characteristics of porous media can be considered to a certain extent, such as studying the influence of pores of different shapes on the flow of fluid in porous media.

4. Edge attribute parameters

In order to further improve the simulation of porous media characteristics, it is also necessary to set the attribute parameters of the edge. These parameters are mainly used to describe the relevant characteristics of the connection between nodes (i.e., the connectivity between pores).

(1) Channel permeability (k): A channel permeability attribute can be set for each edge, which represents the permeability of the connecting channel between nodes (pores). The value of this attribute can be determined by experimental measurement or reference to existing relevant data. For example, when studying the seepage of oil in the pores of reservoir rocks, the permeability of the connecting channels between pores can be measured by equipment such as rock permeability

testers, and then these measured values can be assigned to the edges in the random geometric graph, so that the edges can more accurately reflect the permeability of the actual connecting channels between pores.

(2) Edge length (l): Set a value for each edge to represent its length. In actual porous media, the length of the connecting channel between pores affects the flow rate of the material in it and other properties. By setting the edge length attribute, this can be taken into account when analyzing the model [24]. The value of the edge length can be determined by measurement, estimation, etc. based on the actual structure of the porous medium being studied. For example, when studying the flow of groundwater in soil pores, the actual length of the connecting channel between pores in the soil can be measured and assigned to the edge in the random geometric graph, so that the edge can more accurately reflect the length of the actual connecting channel between pores.

By reasonably setting the above model parameters, we can construct a random geometric graph model that can accurately simulate the characteristics of porous media. Of course, the values of these parameters need to be flexibly adjusted according to factors such as the specific porous media type, research purpose, and available experimental data to ensure the effectiveness and applicability of the model.

3. Model Generation Algorithm

1. Specific algorithm steps

Based on the parameters related to the random geometric graph model set previously (such as node density, connection radius, node attribute parameters, edge attribute parameters, etc.), the following are the detailed algorithm steps for generating a random geometric graph as a mathematical model of porous media:

Step 1: Determine the research area and total number of nodes

First, clarify the spatial region range corresponding to the porous medium under study. For example, assuming that we are studying a porous medium sample in the shape of a cuboid, its spatial range can be expressed by length (L), width (W), and height (H), which determines the size of the entire study area. Then, the total number of nodes (N) that should be generated in the area is calculated based on the set node density (P). The calculation formula for node density is P=N/V (where V is the volume of the study area, that is, $V = L \times W \times H$), from which the total number of nodes $N = P \times V$ can be inferred.

```
import numpy as np
import networkx as nx
import matplotlib.pyplot as plt

# Parameters
L = 5  # size of the region
n = 70  # number of pores
r = 0.5  # radius of each pore

# Generate random centers of pores
np.random.seed(42)
centers = np.random.uniform(0, L, size=(n, 2))

# Function to calculate distance between two points
def euclidean_distance(p1, p2):
    return np.sqrt((p1[0] - p2[0])**2 + (p1[1] - p2[1])**2)

# Create a random geometric graph
G = nx.Graph()
for i in range(n):
    G.add_node(i, pos=centers[i])

# Add edges if pores overlap
for i in range(n):
    for j in range(i+1, n):
        if euclidean_distance(centers[i], centers[j]) <= 2 * r:
            G.add_edge(i, j)

# Visualize the random geometric graph
pos = {i: centers[i] for i in range(n)}
plt.figure(figsize=(8, 8))
nx.draw(G, pos, node_size=100, node_color='lightblue', with_labels=False, edge_color='gray')

# Draw circles representing pores
for i in range(n):
    circle = plt.Circle(centers[i], r, color='blue', fill=False)
    plt.gca().add_artist(circle)

plt.xlim(0, L)
plt.ylim(0, L)
plt.title(f"Random Geometric Graph with n={n}, r={r}")
plt.gca().set_aspect('equal', adjustable='box')
plt.show()
```

Step 2: Generate a random distribution of nodes

After determining the total number of nodes, the next step is to randomly generate the locations of each node in the study area. For each node, its coordinates (assuming it is in three-dimensional space) need to be randomly selected within the boundary of the study area.

For example, in a three-dimensional Cartesian coordinate system, for the i -th node (i=1,2,...,N), its coordinates (x_i, y_i, z_i)

x_i in the interval [0, L] can be achieved by using a random number generation function (such as the common random() function in programming languages);

y_i randomly takes values in the interval [0, W] and can use a random number generation function.

z_i randomly takes values in the interval [0, H] and a random number generation function can be used.

In this way, by randomly assigning values to the coordinates of each node, a set of nodes randomly distributed in the study area is obtained, and these nodes will correspond to the pore center positions in the porous medium.

Step 3: Calculate the distance between nodes and determine the connection relationship of edges

After generating a random distribution of nodes, it is necessary to determine which nodes should form edges to simulate the connectivity between pores in porous media.

First, calculate the spatial distance between any two nodes. For node (with coordinates (x_i, y_i, z_i)) and node j (with coordinates (x_j, y_j, z_j)), the distance d_{ij} between them can be calculated according to the distance formula between two points in three-dimensional space:

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}$$

Then, the set connection radius (r) is used to determine whether an edge is formed between the nodes. If the distance d_{ij} between two nodes is less than or equal to the connection radius r , an edge is generated between the two nodes, indicating that the pores corresponding to them are connected.

That is, when $d_{ij} \leq r$, there is a connectivity relationship between node i and node j , and there will be an edge connecting these two nodes in the random geometric graph model. By performing the above distance calculation and edge determination operations on all node pairs, the edge connectivity relationship in the entire random geometric graph model is determined.

Step 4: Generate node attributes and edge attributes

In addition to determining the location of nodes and the connection relationship of edges, it is also necessary to assign corresponding attributes to nodes and edges to more comprehensively simulate the characteristics of porous media.

Node attribute generation:

For each node, its corresponding attribute value is generated according to the set node attribute parameters.

For example, for the pore size attribute (s), a pore size value that conforms to the distribution can be randomly assigned to each node based on a predetermined pore size distribution (such as the pore size distribution law obtained through actual measurement data).

For the pore shape attribute (p), a corresponding pore shape attribute is assigned to each node based on the pre-set pore shape classification and distribution (such as the approximate proportion of spherical, cylindrical, irregular and other pore shapes in a certain porous medium).

In this way, each node has two properties: pore size and pore shape, which more accurately simulates the characteristics of pores in porous media.

Edge attribute generation:

For each edge, the corresponding attribute value is generated according to the set edge attribute parameters.

For example, for the channel permeability attribute (k), a channel permeability value that conforms to the actual situation can be randomly assigned to each edge through experimental measurement or reference to existing relevant data (such as the measurement data of the permeability of the connecting channels between pores in the actual porous media).

For the edge length property (l), a corresponding edge length value is assigned to each edge according to the actual structure of the porous medium under study and the distance between nodes (because the edge length can be approximately regarded as the distance between two connected nodes to a certain extent).

Through the above steps, a random geometric graph is completely generated as a mathematical model of porous media, which includes the random distribution of nodes, the connection relationship of edges, and the corresponding properties of nodes and edges [25]. These elements together constitute a simulation of the characteristics of porous media.

2. Simple examples to demonstrate the model generation process

In order to better understand the above model generation process, a simple two-dimensional example is presented below.

Suppose we want to simulate a simple two-dimensional porous medium. The study area is a square area with a side length of 10 units (i.e. L=W=10), the node density p=0.5 (unit: node/unit area), and the connection radius r=2 units.

Step 1: Determine the total number of nodes

First, calculate the area of the study area $V = L \times W = 10 \times 10 =$ unit area.

According to the node density formula, the total number of nodes is calculated as $N=p \times V=0.5 \times 100=50$ nodes.

Step 2: Generate a random distribution of nodes

In a two-dimensional Cartesian coordinate system, for each node, its coordinates (x_i, y_i) (i=1,2,...,50) are generated as follows:

x_i takes random values in the interval [0,10]. For example, using a random number generator, we may get $x_1=3.5$, $x_2=7.2$, and so on.

y_i takes random values in the interval [0,10]. For example, using a random number generator, we may get $y_1 = 2.1$, $y_2 = 8.9$, and so on.

50 nodes randomly distributed in the square study area are obtained . These nodes can be regarded as the projection of the pore centers in the porous medium on the two-dimensional plane.

Step 3: Calculate the distance between nodes and determine the connection relationship of edges

For any two nodes i (with coordinates (x_i , y_i) and node j (with coordinates (x_j , y_j)), the distance d_{ij} between them is calculated according to the distance formula between two points in two-dimensional space:

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

Then, the connection radius r=2 is used to determine whether an edge is formed between the nodes.

For example, suppose the coordinates of node 1 are (3.5, 2.1) and the coordinates of node 2 are (3.8, 2.3), calculate the distance between them:

$$d_{12} = \sqrt{(3.8 - 3.5)^2 + (2.3 - 2.1)^2} = \sqrt{0.3^2 + 0.2^2} = \sqrt{0.09 + 0.04} = \sqrt{0.13} \approx 0.36$$

Because $0.36 \leq 2$, there is a connectivity relationship between node 1 and node 2, and there will be an edge connecting these two nodes in the random geometric graph model.

By performing such distance calculations and edge determination operations on all node pairs, the connection relationship of the edges in the entire random geometric graph model is determined.

```python
import numpy as np
import networkx as nx
import matplotlib.pyplot as plt

# Parameters
L = 10  # size of the region
n = 70  # number of pores
r = 0.7  # radius of each pore

# Generate random centers of pores
np.random.seed(42)
centers = np.random.uniform(0, L, size=(n, 2))

# Function to calculate distance between two points
def euclidean_distance(p1, p2):
    return np.sqrt((p1[0] - p2[0])**2 + (p1[1] - p2[1])**2)

# Create a random geometric graph
G = nx.Graph()
for i in range(n):
    G.add_node(i, pos=centers[i])

# Add edges if pores overlap
for i in range(n):
    for j in range(i+1, n):
        if euclidean_distance(centers[i], centers[j]) <= 2 * r:
            G.add_edge(i, j)

# Visualize the random geometric graph
pos = {i: centers[i] for i in range(n)}
plt.figure(figsize=(8, 8))
nx.draw(G, pos, node_size=100, node_color='lightblue', with_labels=False, edge_color='gray')

# Draw circles representing pores
for i in range(n):
    circle = plt.Circle(centers[i], r, color='blue', fill=False)
    plt.gca().add_artist(circle)

plt.xlim(0, L)
plt.ylim(0, L)
plt.title(f"Random Geometric Graph with n={n}, r={r}")
plt.gca().set_aspect('equal', adjustable='box')
plt.show()
```

Step 4: Generate node attributes and edge attributes

Suppose we set the pore size attribute of the node to be a value randomly drawn from a normal distribution with a mean of 3 and a standard deviation of 1, and the pore shape attribute is divided into two types: circular and elliptical, with the circular accounting for 70% and the elliptical accounting for 30% .

For each node, generate the attributes as follows:

For node 1 , a pore size value is randomly selected from the normal distribution, assuming that s_1 =2.5 is obtained, and then the pore shape attribute is randomly determined based on the proportion of circle and ellipse, assuming that a circle is obtained, that is, p1 = circle.

For node 2 , the pore size value is also drawn from the normal distribution, assuming that s_2 =3.2 is obtained, and the pore shape attribute is randomly determined, assuming that an ellipse is obtained, that is, p1 = ellipse.

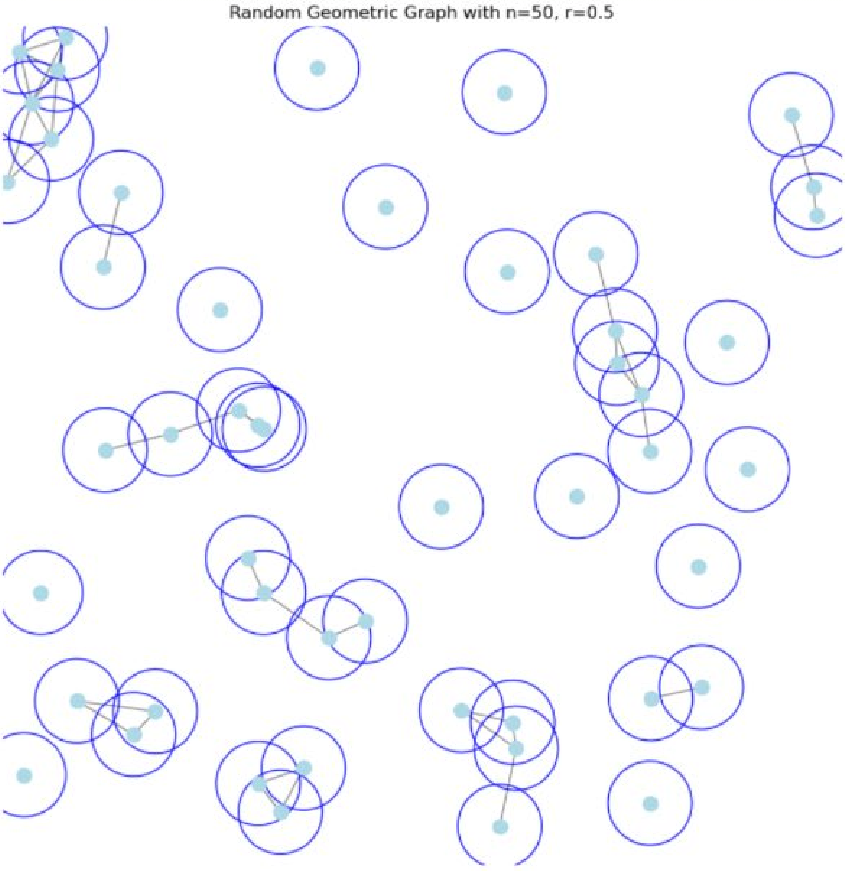
For the attributes of the edge, suppose we set the channel permeability attribute of the edge to be a value randomly drawn from a normal distribution with a mean of 0.5 and a standard deviation of 0.1, and the edge length attribute is the distance between the two connected nodes.

For example, for the edge connecting node 1 and node 2, a channel permeability value is randomly drawn from a normal distribution, assuming that k_{12} =0.45 and the edge length attribute l_{12} =d_{12} =0.36

Through the above steps, a simple two-dimensional random geometric graph is generated as a mathematical model for simulating the two-dimensional porous medium, which includes the random distribution of nodes, the connection relationship of edges, and the corresponding properties of nodes and edges. These elements can be further used to analyze the various characteristics of the simulated porous medium.

```python
import numpy as np
import networkx as nx
import matplotlib.pyplot as plt

# Parameters
L = 10  # size of the region
n = 50  # number of pores
r = 0.5  # radius of each pore

# Generate random centers of pores
np.random.seed(42)
centers = np.random.uniform(0, L, size=(n, 2))

# Function to calculate distance between two points
def euclidean_distance(p1, p2):
    return np.sqrt((p1[0] - p2[0])**2 + (p1[1] - p2[1])**2)

# Create a random geometric graph
G = nx.Graph()
for i in range(n):
    G.add_node(i, pos=centers[i])

# Add edges if pores overlap
for i in range(n):
    for j in range(i+1, n):
        if euclidean_distance(centers[i], centers[j]) <= 2 * r:
            G.add_edge(i, j)

# Visualize the random geometric graph
pos = {i: centers[i] for i in range(n)}
plt.figure(figsize=(8, 8))
nx.draw(G, pos, node_size=100, node_color='lightblue', with_labels=False, edge_color='gray')

# Draw circles representing pores
for i in range(n):
    circle = plt.Circle(centers[i], r, color='blue', fill=False)
    plt.gca().add_artist(circle)

plt.xlim(0, L)
plt.ylim(0, L)
plt.title(f"Random Geometric Graph with n={n}, r={r}")
plt.gca().set_aspect('equal', adjustable='box')
plt.show()
```

Random Geometric Graph with n=50, r=0.5



4. Analysis of porous media characteristics based on random geometric graph model

1. Pore structure analysis

By constructing a random geometric graph model, we can conduct an in-depth and multi-dimensional analysis of the pore structure of porous media, thereby better understanding its internal microscopic characteristics and the impact of these characteristics on the overall performance of porous media.

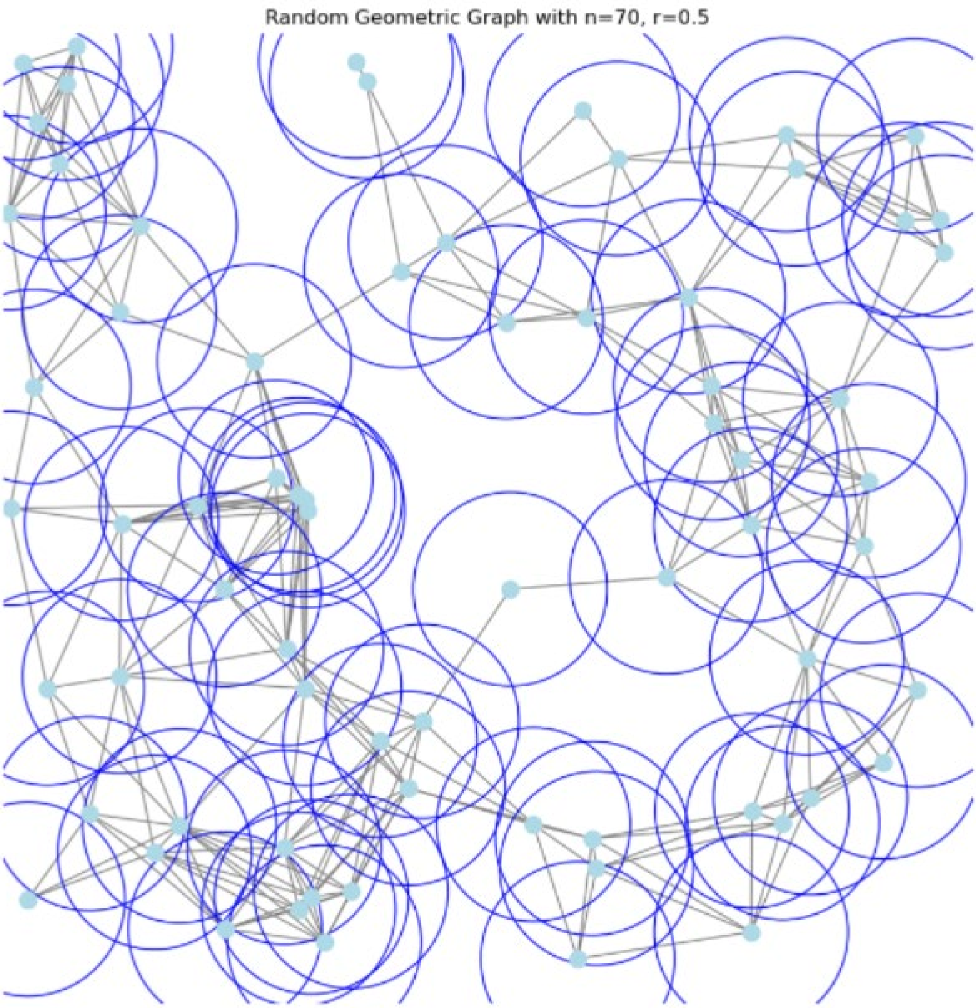
1. Pore size analysis

In the random geometry graph model, each node is assigned an attribute representing the pore size (as set in the model generation algorithm section above). We can obtain detailed information about the pore size of the porous media through statistical analysis of these node attributes.

First, the pore size data of all nodes are collected, and then statistical indicators such as the average pore size and the standard deviation of the pore size can be calculated. The average pore size can intuitively reflect the approximate size of the pores in the porous medium as a whole, which is of great significance for understanding the filtration, adsorption and fluid flow capacity of the medium for different substances [26]. For example, in the field of water treatment, if the average pore size of the porous medium is large, it may not be effective in filtering larger particles of impurities, but it may be more beneficial for allowing fluids to pass quickly; on the contrary, a smaller average pore size is more conducive to intercepting tiny impurities, but may affect the flow rate of the fluid.

The standard deviation of pore size reveals the degree of dispersion of pore size distribution. A larger standard deviation means that the pore sizes are more dispersed in the medium, with significant differences between larger pores and smaller pores; while a smaller standard deviation means that the pore sizes are relatively uniform [27]. This uneven pore size will have an impact on the transport path of materials in porous media, because pores of different sizes may have different resistance to the transport of materials, and uneven pore size distribution may lead to local aggregation of materials during the transport process. or diversion phenomenon.

In addition, we can also draw a probability distribution histogram of pore size to observe the distribution law of pore size more clearly through intuitive graphical display [28]. For example, if the histogram presents a single peak and nearly symmetrical shape, it may indicate that the pore size follows a common probability distribution (such as normal distribution); if it presents a multi-peak or skewed shape, it means that the pore size distribution has multiple different modes or obvious bias, which may be related to the formation process of the porous medium or the special structure inside it.



2. Pore shape analysis

Similarly, based on the pore shape properties set for the nodes in the random geometry model, we can analyze the pore shape of porous media.

The pore shapes are classified and counted, and the proportion of pores of different shapes in the entire porous medium is calculated. For example, the proportion of spherical pores, cylindrical pores, irregular pores, etc. is determined.

Pores of different shapes have different effects on the performance of porous media[29]. Taking catalytic reactions as an example, in porous catalyst materials, spherical pores may provide a more uniform diffusion environment, which is conducive to the uniform diffusion of reactant molecules in the pores; while irregular pores, due to their complex internal structure, may complicate the diffusion path of reactant molecules in the pores, thereby increasing the contact opportunities between reactant molecules and catalyst active sites, which may improve the efficiency of catalytic reactions in some cases.

In addition to proportional statistics, we can also analyze the spatial distribution relationship between pores of different shapes. Observe whether there are pores of a certain shape that tend to cluster together, or whether there is a specific arrangement pattern between pores of different shapes. This spatial distribution relationship may affect the connectivity of porous media and the transmission path of substances in them. For example, if spherical pores are concentrated in a large number in a local area, while irregular pores are scattered in other areas, then in the area where spherical pores are concentrated, substances may be more easily transported along relatively regular paths, while in the area of irregular pores, material transmission may be more hindered by their complex shapes.

3. Analysis of pore distribution

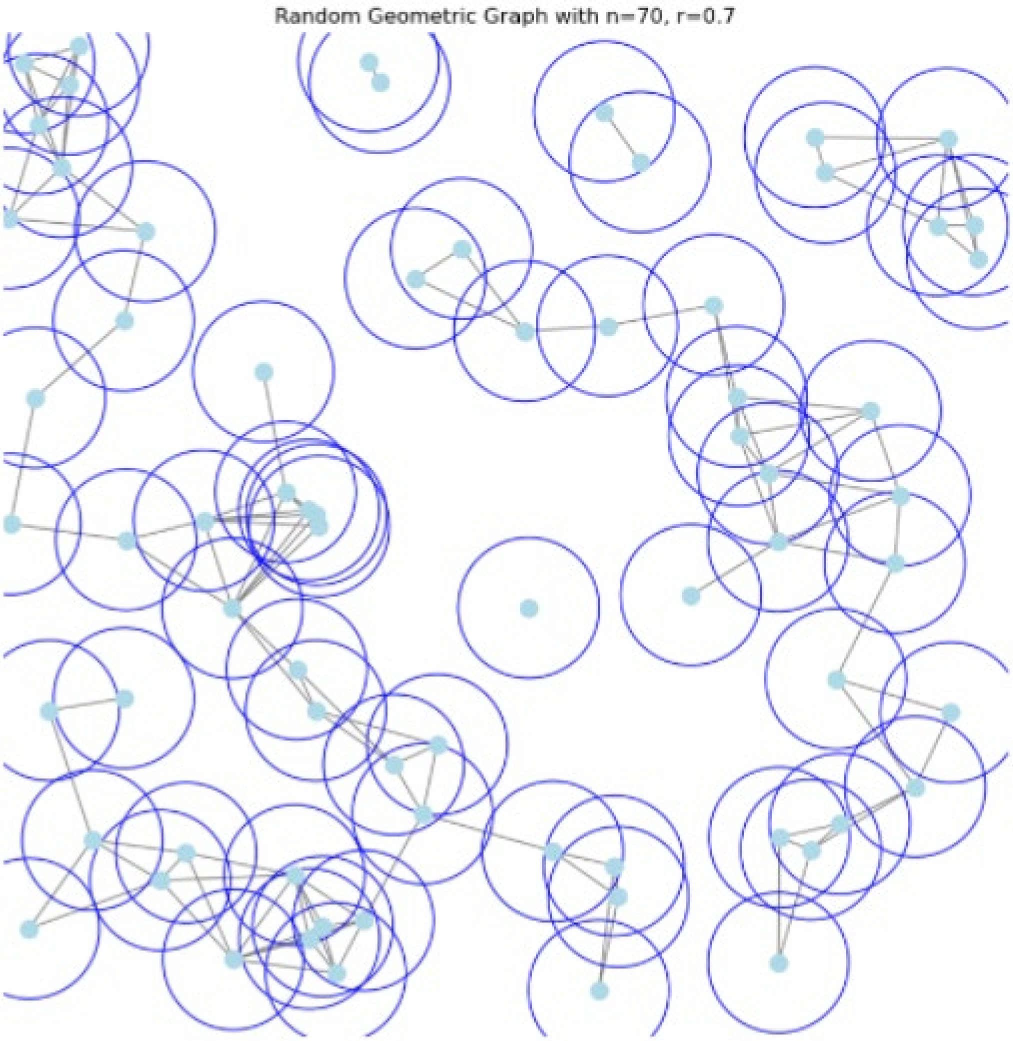
With the help of the random distribution of nodes in the random geometric graph model (corresponding to the distribution of pore centers in porous media), we can deeply study the distribution law of pores in space.

Calculate the spatial density distribution function of the pores, that is, determine the relative density of the pores at different spatial positions. By analyzing this function, we can find out whether there are areas with higher pore density (such as local pore-rich areas) or areas with lower pore density (such as relatively dense areas) in the porous medium. The existence of these areas will have an important impact on the transport of substances in porous media. For example, in the process of oil extraction, if there are areas with higher pore density in the reservoir rock, then the oil may flow and gather more easily in these areas, thus affecting the oil extraction efficiency and extraction path.

We can also further understand the pore distribution law by analyzing the statistical information of the distance between nodes. Calculate indicators such as the average inter-node distance and the standard deviation of the inter-node distance. The average inter-node distance can reflect the average spacing between pores in space. A larger average inter-node distance means that the pores are relatively dispersed in space, while a smaller average inter-node distance means that the pores are relatively dense. The standard deviation of the inter-node distance reveals the uniformity of the pore distribution, similar to the role of the pore size standard deviation.

In addition, by visualizing the spatial distribution of nodes (such as drawing a two-dimensional or three-dimensional scatter plot), we can more intuitively observe the distribution of pores in space, whether they are uniformly distributed, randomly clustered, or have other specific distribution patterns [30]. This kind of visual analysis helps us quickly grasp the overall characteristics of the pore structure of porous media and its relationship with related properties such as material transport.

By analyzing the pore size, pore shape and pore distribution in the random geometric graph model, we can fully and deeply understand the pore structure characteristics of porous media, which provides a solid foundation for further studying its connectivity, seepage characteristics and applications in various fields.



2. Connectivity analysis

In porous media, connectivity is a key property that determines the transport path of substances within the medium and the overall physical processes such as percolation. With the help of random geometric graph models, we can conduct a detailed and quantitative analysis of the connectivity of porous media, thereby gaining a deeper understanding of the relationship between its internal structure and the material transport mechanism.

1. Calculation of the number of connected branches and the maximum connected branch size

In the random geometric graph model, nodes represent pores in porous media, and edges represent the connectivity between pores. We first focus on the number of connected branches and the size of the largest connected branch.

A connected branch is a set of nodes in a graph that are connected to each other through edges, and are not connected to nodes in other connected branches. By traversing all nodes and edges in a random geometric graph model and applying appropriate graph algorithms (such as depth-first search or breadth-first search algorithms), all connected branches can be accurately identified and their number can be counted.

The significance of calculating the number of connected branches is that it can intuitively reflect the degree of fragmentation of the connectivity between pores in porous media [31]. If the number of connected branches is large, it means that there are more relatively independent pore connection areas in the porous medium, and the material may need to cross these different connection areas when it is transported, which will increase the complexity and uncertainty of the transmission [32]. For example, in the groundwater system, if the number of connected branches of porous media such as soil or rock is large, the flow path of groundwater will become more complicated, and local stagnation or diversion may occur, affecting the rational allocation and utilization of water resources.

At the same time, it is also crucial to determine the size of the largest connected branch (usually measured by the number of nodes it contains). The size of the maximum connected branch reflects the size of the largest connected pore area in the porous medium. In many practical application scenarios, this maximum connected branch often undertakes the main task of material transmission [33]. For example, in the process of oil extraction, the largest connected branch in the reservoir rock is like a major "oil transmission channel", and its size directly affects the efficiency of oil extraction from the reservoir to the wellhead. If the maximum connected branch is small, it may mean that it is difficult for oil to form an effective large-scale flow channel during the extraction process, resulting in increased difficulty in extraction and limited production.

2. Correlation between connectivity indices and seepage characteristics

In addition to calculating the two basic indicators of the number of connected branches and the maximum connected branch size, we also need to explore the intrinsic relationship between connectivity indicators and the seepage characteristics of porous media.

Seepage refers to the flow of fluid in porous media. It is an extremely important physical process in many fields related to porous media (such as petroleum engineering, groundwater hydrology, etc.). In the seepage model established on the basis of random geometric graph model, connectivity plays a key role in regulating seepage.

Generally speaking, the better the connectivity (i.e., the fewer connected branches and the larger the largest connected branch), the easier it is for fluid to seep through porous media and the faster the seepage speed [34]. This is because good connectivity provides a relatively continuous and unobstructed flow channel for the fluid, allowing the fluid to migrate smoothly between pores, reducing the resistance caused by unconnected or poorly connected pores [35]. For example, in reservoir rocks with good connectivity, oil can flow quickly to the wellhead along the connected pore channels, improving the efficiency of oil extraction.

On the contrary, when the connectivity is poor (the number of connected branches is large and the largest connected branch is small), the seepage of the fluid will be significantly hindered, the seepage speed will slow down, and even in some local areas, it may not be able to seep. This is because the numerous relatively independent connected branches make it difficult for the fluid to form a continuous flow path in the entire porous medium, and it needs to constantly cross different connected areas, which increases the difficulty of flow and energy loss. For example, in some dense and poorly connected soils, the infiltration rate of rainwater will be very slow, and it may even form water accumulation on the surface, because the connectivity between the soil pores is poor, which hinders the normal seepage of rainwater.

We can quantify this correlation through further theoretical analysis and numerical simulation. For example, by using the percolation theory and combining it with the connectivity index of the random geometric graph model, we can establish a mathematical relationship between the seepage threshold and connectivity [36].

The seepage threshold refers to a critical condition (such as porosity, pressure, etc. reaching a certain value) corresponding to when the fluid begins to seep significantly in a porous medium. Through research, it is found that the better the connectivity, the lower the seepage threshold is, that is, it is easier to reach the conditions for seepage; and the worse the connectivity, the higher the seepage threshold, and higher conditions (such as greater porosity or pressure) are required for the fluid to begin to seep.

Through the connectivity analysis of random geometric graph models, we can not only clearly understand the connectivity structure between pores inside porous media, but also deeply explore its close connection with seepage characteristics, which is of great guiding significance for accurately predicting the transport behavior of substances in porous media and optimizing related engineering practices (such as oil extraction plans, groundwater hydrological management, etc.).

3. Study on seepage characteristics

The seepage process plays an extremely important role in many fields related to porous media, such as oil extraction, groundwater hydrology, soil pollution diffusion, etc. With the help of random geometric graph models, we can deeply and systematically study the seepage characteristics of porous media, reveal its internal laws, and provide strong theoretical support for related practical applications.

1. Establish a seepage model based on a random geometric graph model

In order to accurately study the seepage characteristics of porous media, we first need to establish a suitable seepage model based on the random geometric graph model. In this seepage model, we regard the nodes in the random geometric graph (corresponding to the pores in the porous media) as the locations where the fluid can occupy or pass through, and the edges (representing the connectivity between pores) as the channels for the fluid to flow between pores.

Based on the percolation theory, we can describe and analyze the seepage process. Percolation theory is an important theoretical framework for studying the percolation phenomenon in disordered systems. It provides a method to understand how fluids in complex media (such as porous media) gradually develop from local flow to macroscopic seepage. In our percolation model, when certain conditions are met (such as the fluid pressure in the pores reaches a certain threshold, or the proportion of the pores occupied by the fluid reaches a certain degree, etc.), the fluid can flow between nodes through the edges [37]. This process is similar to the transition from microscopic local connectivity to macroscopic overall seepage described in the percolation theory.

Specifically, we can define a seepage probability parameter p , which represents the probability that any edge in the random geometric graph model can allow fluid to pass through. This probability can be associated with the physical parameters in the actual porous medium (such as pore permeability, fluid pressure, etc.). For example, when the pore permeability is higher or the fluid pressure is higher, the corresponding seepage probability p will also increase accordingly, which means that more edges will allow fluid to pass through, which is more conducive to the occurrence of seepage.

Through this setting, we constructed a percolation model based on the random geometric graph model and percolation theory, which laid the foundation for the subsequent in-depth study of the percolation characteristics.

2. Study on percolation threshold

The seepage threshold is a key concept in the study of seepage characteristics. It refers to a critical condition corresponding to the start of obvious seepage of fluid in porous media. In the seepage model based on the random geometric graph model, we can study the variation law of the seepage threshold by changing the seepage probability p and other related model parameters (such as node density, connection radius, etc.).

When the seepage probability p is small, the flow of fluid in the random geometric graph model is mainly limited to local nodes and edges, and no macroscopic seepage phenomenon has yet formed [38]. As the seepage probability p gradually increases, it will reach a critical value, at which time the fluid begins to form a relatively obvious seepage path in the entire model (i.e., the simulated porous medium). This critical value is what we call the seepage threshold pc .

The study found that the percolation threshold pc is closely related to many parameters of the random geometric graph model. For example, the node density has an impact on the percolation threshold. Generally speaking, a higher node density may lead to a lower percolation threshold. This is because a higher node density means that there are more pores (nodes), which increases the possibility of forming connecting channels between pores, making it easier for the fluid to reach the overall percolation condition at a lower percolation probability.

The connection radius also affects the percolation threshold. A larger connection radius makes it easier to form edges between nodes (i.e., pores are more easily connected), which may also reduce the percolation threshold. Because in this case, even if the percolation probability p is relatively low, the fluid can flow between nodes through more channels due to the easier formation of edges, making it easier to reach the state of macroscopic percolation.

In addition, the seepage threshold is also related to the physical properties of the porous medium itself (such as porosity, pore size distribution, etc.). By comparing the seepage threshold obtained based on the random geometric graph model with the experimental measurement value of the actual porous medium, the accuracy of the model can be verified and the intrinsic connection between the physical properties of the porous medium and the seepage threshold can be further understood.

3. Study on seepage velocity

In addition to the percolation threshold, the percolation velocity is also one of the important indicators of percolation characteristics. In the percolation model based on the random geometric graph model, we can calculate the percolation velocity by analyzing the flow of fluid between nodes and edges.

When fluid flows in a random geometric graph model, we can assume that the flow speed of the fluid on each edge is related to the properties of the edge (such as channel permeability, edge length, etc.). For example, for an edge, the higher the channel permeability and the shorter the edge length, the faster the fluid flows on this edge.

We can calculate the seepage velocity by establishing the corresponding mathematical formula. Assuming v_{ij} represents the flow velocity of the fluid on the edge connecting node i and node j , k_{ij} represents the channel permeability of this edge, and l_{ij} represents the length of this edge, then v_{ij} can be calculated according to Darcy's law or its modified formula , for example

$$v_{ij} = \frac{k_{ij}}{\mu l_{ij}}$$

where μ is the dynamic viscosity of the fluid.

By summing or weighted summing the flow velocities of all edges (determining the weights according to the specific situation), the seepage velocity V in the entire seepage model (i.e., the simulated porous medium) can be obtained.

The study found that the seepage velocity V is also affected by many factors. In addition to being directly related to the properties of the edge (channel permeability, edge length, etc.), it is also related to the overall connectivity of the random geometric graph model. As mentioned in the previous connectivity analysis, the better the connectivity (the fewer connected branches and the larger the maximum connected branch), the easier it is for the fluid to seep in the porous medium and the faster the seepage velocity [39,40]. Because good connectivity provides a relatively continuous and unobstructed flow channel for the fluid, the fluid can migrate smoothly between the pores, reducing the resistance caused by the disconnection or poor connectivity of the pores.

In addition, the percolation rate is also related to the percolation probability p . When the percolation probability p increases, more edges allow fluid to pass through, thereby increasing the flow channel of the fluid, which generally leads to an increase in the percolation rate.

Through the study of seepage velocity, we can better understand the flow efficiency of fluids in porous media, which is of great significance for practical applications involving the flow of fluids in porous media in fields such as oil extraction and groundwater hydrology.

4. Comparison and verification with experimental results and existing theories

In order to verify the effectiveness and accuracy of the seepage model based on the random geometric graph model, we need to compare and verify the seepage characteristics (such as seepage threshold, seepage velocity, etc.) obtained by the model with the experimental results of actual porous media and existing theories.

For experimental results, we can collect a large amount of experimental data related to porous media seepage, such as simulating the seepage process of oil in reservoir rocks in the laboratory, or measuring the seepage of groundwater in soil, etc. The prediction results such as seepage threshold and seepage velocity obtained based on the random geometric graph model are compared in detail with these experimental data.

If the model prediction results are consistent with the experimental data, it means that our model can simulate the seepage characteristics of porous media well and has high effectiveness and accuracy [41]. On the contrary, if there is a large deviation, it is necessary to further analyze the cause of the deviation, which may be due to oversimplification of model assumptions, unreasonable model parameter settings, etc., so as to improve and perfect the model.

At the same time, we also need to compare the seepage model based on the random geometric graph model with existing theories (such as the classic Darcy's law, percolation theory, etc.). Although we have applied the percolation theory when constructing the seepage model, we still need to check whether the performance of the model in specific applications conforms to the general conclusions of existing theories [42]. For example, Darcy's law is the basic law that describes the linear seepage of fluids in porous media. We need to verify whether our seepage model also conforms to the predictions of Darcy's law when certain conditions are met (such as the seepage is in the linear region).

By comparing and verifying with experimental results and existing theories, we can continuously improve the seepage model based on the random geometric graph model so that it can more accurately simulate the seepage characteristics of porous media and provide a more reliable theoretical basis for practical applications in related fields.

5. Model Verification and Evaluation

1. Comparison of experimental data

In order to comprehensively evaluate the effectiveness and accuracy of the random geometric graph model as a mathematical model of porous media, it is a crucial step to compare and analyze the prediction results based on the model with the actual collected experimental data related to porous media. This comparison process can intuitively reveal the degree of consistency between the model and the actual situation, and thus provide a basis for further improvement and optimization of the model.

1. Experimental Data Collection

First, a series of targeted experiments need to be carried out on different types of porous media and various physical processes related to them to obtain rich and reliable experimental data.

In terms of porosity measurement, a variety of experimental methods can be used, such as mercury intrusion method, gas adsorption method, etc. The mercury intrusion method is to press mercury into the pores of the porous medium and calculate the porosity based on the volume of mercury entering the pores under different pressures; the gas adsorption method is to use the adsorption phenomenon of gas molecules on the surface of porous media and combine it with relevant theoretical models to determine the porosity. Through these methods, the porosity values of different porous media samples can be accurately measured, and the corresponding sample characteristic information, such as sample source, medium type, processing conditions, etc., can be recorded.

For the collection of seepage experimental data, a special experimental device can be built to simulate the seepage process of fluids in porous media under different scenarios. For example, when studying the seepage of groundwater in soil, an experimental device similar to a soil column can be constructed, and fluid simulating groundwater can be injected at the top of the device, and sensors can be set at different locations to monitor the flow rate, flow rate, pressure and other parameters of the fluid over time [43]. Similarly, when simulating the seepage of oil in reservoir rocks, a special core displacement device can be used to inject fluid simulating oil to observe and record the seepage characteristics of the fluid in the core, including the seepage rate, seepage threshold (that is, the conditions when obvious seepage begins), etc.

In addition to data related to porosity and seepage characteristics, experimental data related to other characteristics of porous media can also be collected, such as pore size distribution data (which can be observed and analyzed through equipment such as scanning electron microscopes), pore connectivity data (which can be determined with the help of tracer experiments and other methods), etc., in order to fully verify the model from multiple angles.

2. Comparative analysis process

After obtaining sufficient experimental data, we began to conduct a detailed comparative analysis between the predicted results based on the random geometric graph model and these experimental data.

Taking porosity as an example, the porosity values predicted by the model are compared with the porosity values obtained by experimental measurement one by one. The difference between the two is calculated, and the size and distribution of the difference are analyzed. If the difference is small and within a reasonable error range, it means that the model has a certain accuracy in predicting porosity; on the contrary, if the difference is large, it indicates that the model may have problems in simulating pore structure or related parameter settings, and further in-depth analysis of the reasons is needed. For example, it may be that the correspondence between nodes and pores in the model assumptions is not accurate enough, or the parameters such as node density are set unreasonably, resulting in the node distribution generated by the model not being able to well reflect the actual pore distribution, thereby affecting the prediction accuracy of porosity.

The comparison of seepage characteristics is also crucial. The seepage threshold, seepage velocity and other seepage characteristic indicators predicted by the model are compared with the corresponding indicators obtained from experimental measurements.

In the comparison of seepage thresholds, observe whether the seepage threshold predicted by the model is close to the seepage threshold determined by the experiment [44]. If the difference between the two is not large, it means that the model can better capture the critical conditions for the fluid to start to seep significantly in the porous medium; if the difference is large, it is necessary to check whether the parameter settings related to the seepage probability in the model (such as the setting of the seepage probability parameter in the seepage model established based on the random geometric graph model) and the model's simulation of pore connectivity (because connectivity has an important influence on the seepage threshold) are consistent with the actual situation.

In terms of the comparison of seepage velocity, the seepage velocity predicted by the model is compared with the seepage velocity measured experimentally. The velocity difference between the two and the reasons for the difference are analyzed. Factors that may affect the accuracy of the prediction of seepage velocity include whether the attribute parameter settings of the edge in the model (such as channel permeability, edge length and other parameters) are reasonable, and whether the model accurately simulates the connectivity of the entire porous medium. Because as mentioned above, the seepage velocity is closely related to the attributes and connectivity of the edge. If the model simulation in these aspects deviates from the actual situation, it will lead to inaccurate prediction of the seepage velocity.

The comparison of other characteristics such as pore size distribution and pore connectivity is also carried out in a similar way. The pore size distribution predicted by the model is compared with the actual pore size distribution observed by equipment such as scanning electron microscopes to check whether the model can accurately reflect the distribution law of pore size in porous media. In the comparison of pore connectivity, the connectivity reflected by the model through the calculation of indicators such as the number of connected branches and the maximum connected branch size can be compared with the actual connectivity determined by methods such as tracer experiments to check whether the model accurately simulates the connectivity relationship between pores.

3. Analysis of causes of deviation

During the comparison process, if it is found that there is a deviation between the model prediction results and the experimental data, it is necessary to conduct an in-depth analysis of the cause of the deviation in order to make targeted improvements to the model.

In addition to the deviations mentioned above that may be caused by unreasonable model assumptions and parameter settings, there are other factors that may affect the comparison results.

For example, actual porous media often have complex microstructures and inhomogeneities, and the model may have made some idealized assumptions in the process of building it to simplify the analysis, such as assuming that the porous media has a certain degree of uniformity in a certain area (the uniformity assumption mentioned in the model assumptions section), or assuming that the generation of nodes (pores) and the formation of edges are independent processes (independence assumption). These assumptions may not be consistent with the actual situation in some cases, resulting in deviations between the model prediction results and the experimental data.

In addition, the experimental data itself may also have certain errors. Different experimental methods have their limitations. For example, when measuring porosity using the mercury intrusion method, certain errors may occur due to factors such as the interaction between mercury and the surface of the porous medium. In the seepage experiment, the accuracy of the sensor and the sealing of the experimental device may also affect the accuracy of the measurement results [45]. Therefore, when analyzing the causes of the deviation, it is also necessary to consider the error factors of the experimental data itself in order to more comprehensively and accurately evaluate the degree of fit between the model and the actual situation.

By conducting a comprehensive and in-depth comparative analysis of the prediction results based on the random geometric graph model and the actual experimental data, and carefully analyzing the causes of the deviations, we can more clearly understand the advantages and disadvantages of the model, and provide strong guidance for the subsequent improvement and optimization of the model, so that it can better simulate and predict the various characteristics of porous media.

2. Comparison with other models

In order to more comprehensively evaluate the performance and advantages of the random geometric graph model as a porous media mathematical model, it is necessary to compare it in detail with other traditional porous media mathematical models [46]. Through this comparison, we can clearly see the uniqueness of the random geometric graph model in describing the characteristics of porous media, as well as its possible shortcomings compared with other models, thus providing a reference for its further development and application.

1. Select the traditional porous media mathematical model

Among many traditional porous media mathematical models, we selected the continuum model and the network model as the objects for comparison with the random geometric graph model [47]. These two models are widely used in the field of porous media research, and each has its own representative characteristics and modeling ideas.

The continuum model is based on the continuum mechanics hypothesis, which regards the porous medium as a uniform continuum and describes the overall characteristics of the medium by defining some macroscopic parameters (such as porosity, permeability, etc.). It can provide a relatively effective analysis method when dealing with some macroscopic phenomena, but it has certain limitations when it comes to issues such as pore structure at the microscopic level, connectivity between pores, and local heterogeneity.

The network model abstracts the pores in the porous medium as nodes in the network and the connecting channels between the pores as edges in the network, thus constructing a network structure similar to a graph to describe the porous medium.

Although it takes into account the connectivity between pores to a certain extent, it often oversimplifies the actual shape and distribution of the pores and is difficult to accurately reflect the real microstructural characteristics of the porous medium.

2. Comparison of pore structure descriptions

There are significant differences between random geometric graph models, continuum media models, and network models in describing the pore structure of porous media.

For the continuum model, since it treats the porous medium as a uniform continuum, it almost ignores the pore structure information at the microscopic level. It cannot provide a detailed description of the size, shape, distribution of pores and the connectivity between them. It can only roughly characterize the overall characteristics of the medium such as the degree of porosity from a macro perspective through parameters such as porosity. For example, when studying porous media such as soil, the continuum model can only give the overall porosity of the soil, but cannot accurately represent details such as the size and shape of specific pores in the soil and how they are connected to each other.

Although the network model abstracts pores as nodes and represents connectivity through edges, its simplified treatment of pore shapes makes it inaccurate in describing pore structures. It usually assumes that pores have relatively simple shapes (such as spheres or cylinders), while the pore shapes in actual porous media are often diverse and irregular. In addition, network models often require a lot of assumptions and empirical data when determining the properties of nodes and edges, and the way these properties are determined may not be accurate enough, thus affecting the accurate description of the pore structure. For example, when simulating the pore structure of reservoir rocks, network models may not accurately reflect the complex pore shapes in the rocks and the actual connectivity between different pores.

In contrast, the random geometric graph model has obvious advantages in describing pore structure. It can more accurately reflect the distribution of pores in space by assigning nodes to the center of the pores and determining the edges (i.e., the connectivity between the pores) based on the distance between the nodes. At the same time, by assigning attributes such as pore size, shape, and channel permeability to the nodes and edges respectively, it can more comprehensively describe the various characteristics of the pores [48]. For example, when studying a new type of porous material, the random geometric graph model can set corresponding attributes for the nodes and edges based on the actual measured data, thereby accurately presenting details such as the size, shape, distribution of the pores in the material, and the connectivity between them, providing a more accurate basis for further research and application of the material.

3. Comparison of seepage characteristics description

These three models also have their own characteristics in describing the seepage characteristics of porous media.

The continuum model describes the seepage of fluids in porous media based on macroscopic theories such as Darcy's law. It regards porous media as a uniform continuum, so when dealing with seepage problems, it mainly focuses on parameters such as macroscopic seepage velocity and flow rate, while it is difficult to accurately grasp details such as the pore connectivity at the microscopic level during the seepage process and the specific flow path of the fluid between different pores. For example, when studying the seepage of groundwater in soil, the continuum model can predict the overall seepage velocity and flow rate of groundwater, but it cannot conduct in-depth analysis of how groundwater passes through specific pores in the soil and the impact of the connectivity between these pores on seepage.

Although the network model takes into account the connectivity between pores in describing the seepage characteristics, it has certain difficulties in accurately predicting the seepage characteristics due to its simplified treatment of the pore structure and the inaccuracy of the attribute determination. For example, when simulating the seepage of oil in reservoir rocks, the network model may not accurately reflect the real structure and connectivity of the pores in the rock, resulting in a large deviation between the predicted seepage characteristic indicators such as seepage velocity and seepage threshold and the actual situation.

The random geometric graph model is a seepage model based on the percolation theory and other theories, and it has shown good performance in describing seepage characteristics. It can more deeply analyze the specific flow path of the fluid between different pores during the seepage process, as well as seepage characteristic indicators such as the seepage threshold and seepage velocity, by accurately simulating the connectivity between pores and assigning attributes such as channel permeability to the edges. For example, when studying the seepage characteristics of a porous medium with a complex pore structure, the random geometric graph model can set the attributes of nodes and edges according to the actual measured data, thereby accurately predicting parameters such as the seepage threshold and seepage velocity, and by comparing and verifying with experimental results, its prediction results are often consistent with the actual situation.

4. Summary of model advantages and disadvantages

Through the above comparison of the descriptions of pore structure and seepage characteristics, we can summarize the advantages and disadvantages of the random geometric graph model compared with the continuous medium model and network model.

The advantages of random geometric graph models are:

It can more accurately describe the pore structure, including details such as the size, shape, distribution of pores, and the connectivity between them, and provide a more comprehensive description of pore characteristics by assigning corresponding attributes to nodes and edges.

In terms of describing seepage characteristics, the seepage model established based on the percolation theory can more deeply analyze the microscopic details of the seepage process, such as the specific flow path of the fluid between different pores and seepage characteristic indicators such as the seepage threshold and seepage velocity, and the predicted results are consistent with the actual situation.

However, random geometric graph models also have some shortcomings:

The model construction process is relatively complex and requires the setting of multiple parameters (such as node density, connection radius, node attribute parameters, edge attribute parameters, etc.), and the values of these parameters need to be flexibly adjusted according to factors such as the specific porous media type, research purpose, and available experimental data, which increases the difficulty and complexity of model application.

Although it can more accurately describe the characteristics of porous media in theory, in practical applications, due to the complexity and heterogeneity of actual porous media, the model may not be consistent with the actual situation due to some assumptions (such as uniformity assumption, independence assumption, etc.), resulting in a certain deviation between the predicted results and the actual situation.

In comparison, the advantage of the continuum model is that it is simple and easy to use, and can provide a more effective analysis method when dealing with macroscopic phenomena, but its disadvantage is that it cannot accurately describe the pore structure and seepage characteristics at the microscopic level.

The advantage of the network model is that it takes into account the connectivity between pores, but its disadvantage is that the simplified treatment of the pore structure makes it unable to accurately describe the true microstructural characteristics and seepage characteristics of the pores.

Through a comprehensive comparison with the continuum model and the network model, we have a clearer understanding of the performance of the random geometric graph model in describing the characteristics of porous media, which helps us choose the appropriate model according to specific needs in practical applications and provides a direction for the further improvement and optimization of the random geometric graph model.

3. Model evaluation indicators

In order to quantitatively evaluate the performance of the random geometric graph model as a mathematical model of porous media, we need to introduce a series of appropriate evaluation indicators. These indicators can objectively measure the degree of consistency between the model and the actual situation from different perspectives, as well as the accuracy and effectiveness of the model in describing various characteristics of porous media, thus providing a clear basis for further improvement and optimization of the model.

Table 1: Comparison of Traditional Mathematical Models and Random Geometric Graph Models

Evaluation Indicator	**Calculation Method**	**Significance**
Mean Squared Error (MSE) - Porosity	$\frac{1}{n}\sum_{i=0}^{n}(y_i-\dot{y}_i)^2$(y_i is the actual porosity value, $\dot{y}_i$ is the predicted porosity value by the model, n is the number of samples)	Measures the difference between the predicted porosity by the model and the actual measured value. The smaller the MSE, the more accurate the porosity prediction by the model.
Mean Squared Error (MSE) - Percolation Threshold	$\frac{1}{n}\sum_{i=1}^{n}(t_i-\dot{t}_i)^2$(t_i is the actual percolation threshold sequence, $\dot{t}_i$ is the predicted percolation threshold sequence by the model)	Evaluates the deviation between the predicted percolation threshold by the model and the actual measured value. The smaller the MSE, the more accurate the percolation threshold prediction.
Mean Squared Error (MSE) - Seepage Velocity	$\frac{1}{n}\sum_{i=1}^{n}(v_i-\tilde{v}_i)^2$(v_i is the actual seepage velocity sequence, $\tilde{v}_i$ is the predicted seepage velocity sequence by the model)	Reflects the error between the predicted seepage velocity by the model and the actual measured value. The smaller the MSE, the more accurate the seepage velocity prediction.
Correlation Coefficient (R) - Porosity	$\frac{\sum_{i=1}^{n}(y_i-\bar{y})(\hat{y}_i-\bar{\hat{y}})}{\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2\sum_{i=1}^{n}(\hat{y}_i-\bar{\hat{y}})^2}}$ ($\bar{y}$ is the average of the actual porosity values, $\bar{\hat{y}}$ is the average	Measures the degree of linear correlation between the predicted porosity value by the model and the actual measured value. The

	of the predicted porosity values by the model)	closer R is to 1, the higher the degree of linear correlation and the better the performance of the model in predicting porosity.
Correlation Coefficient (R) - Percolation Threshold	Similar to the calculation of the correlation coefficient for porosity (using the percolation threshold)	

1. Mean Squared Error (MSE)

The mean square error is a commonly used indicator to evaluate the difference between the model prediction value and the actual observation value. When applying the random geometric graph model to the prediction of porous media properties, we can calculate the mean square error for key properties such as porosity, seepage threshold, and seepage velocity.

Taking porosity as an example, suppose we obtain the actual porosity values of a set of porous media samples through experimental measurement, denoted as y_i ($i=1, 2, ..., n$, where n is the number of samples), and predict the corresponding porosity values through the random geometric graph model, denoted as $\dot{y}_i$. Then, the mean square error calculation formula for porosity prediction is:

$$MSE_{孔隙率} = \frac{1}{n}\sum_{i=0}^{n}(y_i - \dot{y}_i)^2$$

The smaller the value of the mean square error, the smaller the difference between the porosity predicted by the model and the actual measured value, which means that the model is more accurate in predicting porosity. Similarly, we can calculate the mean square error of the prediction of properties such as seepage threshold and seepage velocity in a similar way.

For the percolation threshold, if the actual measured percolation threshold sequence is t_i and the model predicted percolation threshold sequence is $\dot{t}_i$, then the mean square error of the percolation threshold prediction is:

$$MSE_{渗流阈值} = \frac{1}{n}\sum_{i=1}^{n}(t_i - ṫ_i)^2$$

For the seepage velocity, let the actually measured seepage velocity sequence be vi and the model-predicted seepage velocity sequence be $\tilde{v}_i$, and the mean square error calculation formula is:

$$MSE_{渗流速度} = \frac{1}{n}\sum_{i=1}^{n}(v_i - \tilde{v}_i)^2$$

By calculating the mean square error of the predictions of these key features, we can intuitively understand the degree of deviation of the model from the actual situation in various aspects, and thus have a preliminary quantitative assessment of the overall performance of the model.

2. Correlation coefficient (R)

The correlation coefficient is another important evaluation metric, which is used to measure the degree of linear correlation between the model predictions and the actual observed values. When evaluating random geometry models, the correlation coefficient can also be calculated for different porous media properties. Taking porosity as an example, the correlation coefficient Rporosity between the predicted porosity value and the actual measured value is calculated . The calculation formula of the correlation coefficient is based on covariance and standard deviation, as follows:

$$R_{孔隙率} = \frac{\sum_{i=1}^{n}(y_i - \bar{y})\left(\hat{y}_i - \bar{\hat{y}}\right)}{\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2 \sum_{i=1}^{n}\left(\hat{y}_i - \bar{\hat{y}}\right)^2}}$$

Among them, $\bar{y}$is the average of the actual porosity values and $\bar{\hat{y}}$is the average of the porosity values predicted by the model.

The value range of the correlation coefficient $R_{porosity}$ is between -1 and 1. When $R_{porosity}$ = 1 , it means that the model prediction value is completely positively correlated with the actual measured value, that is, the model prediction is very accurate; when $R_{porosity}$ = -1 , it means a complete negative correlation, which is usually unreasonable in actual situations; when $R_{porosity} = 0$, it means that there is no linear correlation between the two. Generally speaking, the closer the $R_{porosity}$ is, the higher the degree of linear correlation between the model prediction value and the actual measured value, and the better the performance of the model in predicting porosity.

Similarly, we can calculate the correlation coefficients $R_{Percolation\ Threshold}$ and $R_{PercolationRate}$ for the prediction of properties such as percolation threshold and percolation rate to evaluate the performance of the model in these aspects.

By calculating the correlation coefficient, we can not only understand the linear relationship between the model's predicted value and the actual value, but also evaluate the accuracy of the model from another perspective, complementing the mean square error indicator to more comprehensively characterize the performance of the model.

3. Coefficient of determination (R^2)

The coefficient of determination is the square of the correlation coefficient, that is, $R^2_{porosity} = (R_{porosity})^2$, $R^2_{seepage\ threshold} = (R_{seepage\ threshold})^2$, $R^2_{seepage\ velocity} = (R_{seepage\ velocity})^2$, etc.

The significance of the coefficient of determination is that it indicates the proportion of the variation of the actual observed values that the model predicted values can explain. For example, if $R^2_{porosity} = 0.8$, it means that the porosity values predicted by the model can explain 80% of the variation of the actual measured porosity values , which means that the model has performed well in predicting porosity, but there is still 20% of the variation that cannot be explained by the model, which may be due to the limitations of the model or the complexity of the actual situation.

The closer the value of the coefficient of determination is to 1 , the stronger the predictive ability of the model is and the better it can explain the variation of the actual observations. Together with the mean square error and the correlation coefficient, it constitutes a relatively complete evaluation system to evaluate the performance of the random geometric graph model from different angles.

4. Model evaluation and scope determination

By calculating the above evaluation indicators (mean square error, correlation coefficient, determination coefficient, etc.), we can perform a comprehensive quantitative evaluation of the random geometric graph model.

If the mean square error is small, the correlation coefficient is close to 1 , and the determination coefficient is also high, it means that the model performs well in describing the corresponding porous media properties (such as porosity, seepage threshold, seepage velocity, etc.) and has high accuracy and effectiveness. At this point, we can further determine the scope of application of the model based on the specific circumstances of the actual research.

For example, when the model predicts the seepage characteristics of a certain type of porous media (such as a specific rock type or soil type), all evaluation indicators show good results, then we can preliminarily judge that the model is applicable to the study of the seepage characteristics of this type of porous media. However, if in other cases, such as for some porous media with special microstructures or high heterogeneity, the evaluation indicators of the model are not ideal, then we need to recognize that the model may have limitations in these cases, and further improvements or the use of other more suitable models are needed.

By quantitatively evaluating the model and determining its scope of applicability, we can apply the stochastic geometric graph model more scientifically, so that it can play the best role in studying the characteristics of porous media, and also provide a clear direction for the improvement and optimization of the model.

6. Conclusion and Outlook

1. Research conclusions

This study conducted an in-depth study on the use of random geometric graphs as mathematical models for porous media. Through a series of theoretical construction, characteristic analysis, and verification and evaluation, the following important research conclusions were obtained:

1. Successfully constructed and verified the ability of random geometric graph models to describe the characteristics of porous media

The random geometric graph was successfully constructed as a mathematical model of porous media, and the model assumptions, parameters and generation algorithms were reasonably set according to the actual physical properties of porous media [49]. Through this model, the key characteristics of porous media such as pore structure, connectivity and seepage can be more comprehensively and meticulously characterized.

In terms of pore structure, the model uses the setting that nodes correspond to pore centers and edges represent pore connectivity, combined with the pore size, shape and other attributes assigned to nodes and edges, to achieve an effective description of pore size, shape, distribution, etc. By comparing with the actual porous media microstructure image, it is found that the pore structure characteristics presented by the model are consistent with the actual situation to a certain extent, verifying the effectiveness of the model in describing pore structure.

For the connectivity characteristics, by analyzing the number of connected branches and the maximum connected branch size in the random geometric graph model, the connectivity architecture between the pores inside the porous media is clearly revealed. In addition, the close relationship between connectivity and seepage characteristics is further explored, indicating that connectivity plays a vital role in the seepage process. Good connectivity is conducive to the occurrence and progress of seepage, while poor connectivity will hinder seepage, which is consistent with the physical phenomena in actual porous media.

In the study of seepage characteristics, the seepage model established based on the random geometric graph model, combined with the percolation theory, successfully revealed the law of changes in seepage characteristics such as seepage threshold and seepage velocity with model parameters (such as node density, connection radius, etc.). Through comparison and verification with experimental results and existing theories, it was found that the key indicators such as seepage threshold and seepage velocity predicted by the model were relatively close to the actual measured values, proving that the model has high accuracy in describing seepage characteristics.

2. Demonstrated performance advantages over traditional models

A comprehensive comparison with traditional porous media mathematical models (such as continuum models and network models) highlights the advantages of random geometric graph models in describing the characteristics of porous media.

Compared with the continuous medium model, the random geometric graph model no longer regards the porous medium as a simple uniform continuum, but can go deep into the microscopic level and describe the pore structure and the microscopic details of the seepage process in detail [50]. The continuous medium model has obvious limitations in dealing with microscopic characteristics, while the random geometric graph model can accurately present the size, shape, distribution and connectivity of the pores, and in the description of seepage characteristics, it can analyze the specific flow path of the fluid between different pores and indicators such as the seepage threshold and seepage velocity, which is beyond the reach of the continuous medium model.

Compared with the network model, although the network model takes into account the connectivity between pores, the random geometric graph model is more accurate in describing the pore structure. The network model often oversimplifies the actual shape and distribution of the pores, and has many deficiencies in determining the properties of nodes and edges, which affects its accurate description of the pore structure and seepage characteristics. The random geometric graph model can more realistically reflect the actual situation of porous media by reasonably setting the attribute parameters of nodes and edges, and also shows better accuracy in predicting seepage characteristics.

3. Provides new perspectives and tools for in-depth understanding of porous media properties

This study used random geometric graph models to analyze and understand the characteristics of porous media from a completely new perspective, providing researchers in related fields with a powerful tool that is different from traditional methods.

By converting the complex porous media structure into the form of random geometric graphs, the study of pore structure, connectivity, seepage and other characteristics is more intuitive and quantitative. This modeling method based on graph theory not only enriches the research methods of porous media, but also helps to deeply explore the intrinsic connection between the microstructure and macroscopic characteristics of porous media. For example, when studying the influence of pore structure on seepage characteristics, the random geometric graph model can clearly observe how different pore sizes, shapes and distribution patterns change the seepage characteristics such as seepage threshold and seepage velocity by affecting connectivity, thus providing a clearer idea for a deeper understanding of the physical nature of porous media.

In summary, this study has achieved remarkable results in describing the characteristics of porous media, demonstrating the advantages of the model, and providing new research perspectives by constructing random geometric graphs as mathematical models of porous media, laying a solid foundation for further in-depth research on porous media and applications in related fields.

2. Research Deficiencies and Improvement Directions

Although this study has achieved certain results in the construction and application of random geometric graphs as a mathematical model of porous media, it is also recognized during the research process that the model has some shortcomings that need to be improved and perfected in future research. The following is a detailed analysis of these shortcomings and corresponding suggestions for improvement:

1. Limitations of model assumptions

When constructing the random geometric graph model, in order to simplify the analysis process, we put forward some assumptions, such as the uniformity assumption (assuming that the physical properties such as the pore distribution density, pore size distribution, and pore shape in the porous media region under study are statistically uniform) and the independence assumption (assuming that the generation of each node in the random geometric graph and the formation of the edges between them are independent processes). However, the actual porous media are often highly complex and non-uniform, and these assumptions may deviate greatly from the actual situation in some cases.

For example, when dealing with some porous media with obvious local differences (such as rocks with different mineral compositions that lead to large differences in pore structure, or soils with different degrees of pollution that affect pore properties), the uniformity assumption is difficult to accurately reflect their true physical properties [51]. Similarly, in some pore systems with strong interactions (such as under certain special geological conditions, the pores have obvious mutual influence due to factors such as pressure changes or chemical reactions), the independence assumption is no longer applicable.

Improvement directions:

In view of the limitations of the uniformity assumption, future research can consider adopting a more refined zoning modeling method. According to the actual distribution of physical properties of porous media, the study area is divided into multiple sub-areas, and more realistic local parameters are set in each sub-area to more accurately simulate the pore structure and characteristics of different areas. For example, for rock samples, they can be divided according to the distribution areas of different mineral components, and the porosity, pore size distribution and other parameters of each area can be determined separately.

For the problem of independence assumption, it is necessary to further study the interaction mechanism between pores and incorporate it into the model. More complex mathematical relationships can be established to describe the mutual influence of node (pore) generation and edge formation. For example, considering the influence of deformation and connectivity changes between pores under specific conditions (such as temperature and pressure changes) on the entire porous media structure, a random geometric graph model that is closer to the actual situation can be constructed.

2. Uncertainty of parameter values and difficulty of adjustment

The random geometric graph model involves multiple key parameters, such as node density, connection radius, node attribute parameters (pore size, shape, etc.) and edge attribute parameters (channel permeability, edge length, etc.). The values of these parameters need to be flexibly adjusted according to factors such as the specific porous media type, research purpose, and available experimental data. However, in actual operation, there is often a certain degree of uncertainty in determining the accurate values of these parameters.

On the one hand, different porous media have different physical properties, and accurately obtaining their corresponding parameter values requires a lot of precise experimental measurements and data analysis, but the limitations of experimental conditions and measurement errors may lead to inaccurate parameter values. On the other hand, even if relatively accurate parameter values are obtained, due to the mutual influence between these parameters, adjusting one of the parameters may have a complex impact on the performance of the entire model, making the parameter adjustment process difficult and time-consuming.

Improvement directions:

In order to reduce the uncertainty of parameter values, it is necessary to further improve experimental techniques and methods in the future to improve the accuracy of measuring the physical properties of porous media. For example, more advanced porosity measurement instruments and pore size analysis equipment should be developed to obtain more accurate experimental data for determining parameter values.

In terms of parameter adjustment, advanced data analysis and optimization algorithms can be used. For example, by using parameter optimization algorithms in machine learning (such as genetic algorithms, particle swarm optimization algorithms, etc.), by setting appropriate objective functions (such as minimizing the mean square error between model prediction results and experimental data, etc.), the algorithm can automatically search for the optimal parameter combination, thereby improving the efficiency and accuracy of parameter adjustment and enabling the model to better adapt to different porous media and research needs.

3. The adaptability of the model to extremely complex media needs to be improved

Although the random geometric graph model has shown good performance in describing the characteristics of general porous media, its adaptability is still insufficient for some extremely complex porous media, such as special materials or geological structures with ultra-high porosity, extremely irregular pore shapes, and highly complex connectivity.

In these extreme cases, the model may not be able to accurately capture all the complex physical properties and relationships. For example, for some new materials with nanoscale pores and fractal pore shapes, the random geometric graph model based on the conventional node and edge settings may not be able to fully reflect its microstructure and permeation characteristics.

Improvement directions:

In order to improve the adaptability of the model to extremely complex media, the basic framework of the model needs to be expanded and innovated. New mathematical concepts and methods can be introduced, such as fractal geometry theory, to describe pore shapes and complex connectivity structures with fractal characteristics [52]. By combining fractal geometry with random geometric graphs, a more adaptable hybrid model can be constructed to better handle extremely complex media.

At the same time, we should strengthen the in-depth study of extremely complex media and understand their special physical properties and internal laws in detail so as to improve and optimize the model in a targeted manner according to these characteristics. For example, for materials with ultra-high porosity, we should study their pore formation mechanism and connectivity laws so as to set relevant parameters and simulate their characteristics more accurately in the model.

By analyzing the above research deficiencies and clarifying the corresponding improvement directions, it is expected that the performance of random geometric graphs as a mathematical model of porous media will be further improved in the future, so that it can more accurately and comprehensively describe the characteristics of various types of porous media, providing relevant Provide stronger support for research and application in the field.

(3) Application prospects

The random geometric graph constructed in this study serves as a mathematical model of porous media. Although there are still some areas to be improved, it has shown significant advantages in describing the characteristics of porous media. Based on these advantages, this model has broad application prospects in multiple fields related to porous media, and is expected to provide strong support for solving related practical problems and in-depth theoretical research. The following is a specific outlook on its application prospects:

1. Application prospects in the field of petroleum engineering

In petroleum engineering, accurate understanding of the pore structure and seepage characteristics of reservoir rocks is crucial for efficient oil extraction. Random geometric graph models can accurately describe the pore size, shape, distribution and connectivity of reservoir rocks, and thus accurately simulate the seepage process of oil in them.

Through this model, we can deeply analyze the differences in seepage characteristics of different reservoir areas and provide a basis for formulating more targeted oil production plans. For example, according to the parameters such as seepage velocity and seepage threshold predicted by the model, we can determine the optimal injection and production well layout, optimize the production process, and improve oil recovery. At the same time, in terms of reservoir numerical simulation, the random geometric graph model can be used as an effective supplementary tool, combined with traditional reservoir simulation methods, to more comprehensively describe the complex physical processes of the reservoir, thereby providing more accurate decision support for petroleum engineers, and is expected to play an important role in increasing oil production and reducing production costs.

2. Application prospects in the field of materials science

The field of materials science pays great attention to the optimization and design of porous materials, and these properties depend largely on the pore structure and connectivity of the materials. Stochastic geometric graph models can be used to deeply study the microstructural properties of various porous materials, such as porous ceramics and porous metals.

With the help of this model, it is possible to analyze the effects of pores of different shapes, sizes, and distributions on material properties (such as strength, thermal conductivity, permeability, etc.), thereby providing theoretical guidance for material designers to help them manufacture porous materials with better performance according to specific application requirements. For example, when designing porous ceramic materials for filtration, the model is used to analyze the effects of pore structure on filtration efficiency and fluid flow capacity, and the material preparation process parameters are adjusted to achieve optimal filtration performance. In addition, the model can also be used to study the development of new porous materials. By simulating their potential pore structure and seepage characteristics, the feasibility and application potential of the materials can be evaluated in advance, accelerating the transformation of new porous materials from the laboratory to practical applications.

3. Application prospects in environmental science

In the field of environmental science, porous media play an important role in groundwater hydrology, soil pollution diffusion, etc. Stochastic geometric graph models can more accurately analyze the flow path and seepage characteristics of groundwater in porous media such as soil and rock, as well as the migration and diffusion laws of pollutants in them.

For groundwater hydrology research, the model can be used to predict groundwater level changes, water flow velocity and distribution of water resources under different geological conditions, providing a scientific basis for the rational development and protection of groundwater resources. In terms of soil pollution diffusion, by simulating the migration process of pollutants in porous media, combined with actual soil pore structure and connectivity data, the diffusion range and diffusion rate of pollutants can be accurately predicted, thereby formulating more effective pollution prevention and control measures, which will help reduce the harm of soil pollution to the ecological environment.

4. Application prospects in modeling other similar complex systems

As an effective modeling tool for complex systems, the random geometric graph model has application potential not limited to the field of porous media. With the in-depth study of various complex systems, the model is expected to be extended to other systems with similar structural characteristics.

For example, in the study of the microstructure of biological tissues, the connection and material transfer between cells are similar to the connectivity and material permeation of pores in porous media. The random geometric graph model can be used to simulate the distribution, connection relationship and material exchange process of cells in biological tissues through appropriate adjustment and improvement, providing new modeling ideas and analysis methods for biomedical research. For another example, in the modeling of urban transportation networks, the relationship between traffic nodes (such as intersections, stations, etc.) and traffic routes (such as roads, tracks, etc.) can also be analogized to nodes and edges in random geometric graphs. By introducing relevant parameters such as traffic flow and congestion level, the model can be used to analyze urban traffic congestion and optimize traffic route planning.

In summary, as a mathematical model of porous media, the random geometric graph model has broad application prospects in many fields such as petroleum engineering, materials science, environmental science, and other similar complex system modeling. With the continuous improvement and improvement of the model, it is believed that it will make greater contributions to the development of related fields in the future and provide more powerful tools for solving practical problems and promoting theoretical research.

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