A Detailed Digital Model of the Human Arterial System

N.N.Kizilova
Department of Theoretical Mechanics , Kharkov National University, Ukraine

Abstract – **A detailed model of the human circulation is developed. The large systemic arteries are presented by the branching system of straight viscoelastic tubes which corresponds topology of the human circulation. Terminal elements at the outlets of the system are presented by tree-like systems with a given topology (with/without anastomoses) and certain geometrical relations between the lengths and diameters of the vessels of different branching orders and the relation between the maximal total length of the vasculature and diameter of the feeding artery. The relations have been obtained by analysis of the morphometric data. They allow correct calculations of the hydraulic resistance and wave impedance of the arterial beds of different organs. The proposed outflow boundary conditions are more preferable then the Windkessels and the regular tree-like systems because they describe both resonant properties of the intraorgan vasculatures and the distributed sources of the reflected waves. The model describes realistic pressure and flow waves and pressure-flow dependences either in the aorta or in the feeding arteries of the inner organs. The latter underlies possibility of the novel noninvasive diagnostics of the state (normal or pathological) of the intraorgan circulation by non-invasive measuring the wall oscillations and blood flow velocity in any cross-section of the feeding artery of the organ.**

Keywords – **circulatory system, arterial networks, hydraulic conductivity, input admittance, wave propagation.**

I. INTRODUCTION

Geometry of the arterial vasculatures is an important factor that defines pressure-flow relationships in different parts of the circulatory system and pulse wave propagation and reflection. The arterial systems of different individuals exhibit significant variations in lengths and diameters of the arterial segments and topology of the vasculatures. A multi-branched model of the human extraorgan arteries that includes large systemic arteries has been proposed in [1-2]. Wave reflection at the terminuses can be modeled by the three-element Windkessels [3] and by regular self-similar tree-like branching systems of compliant tubes [4]. The parameters of the terminal elements have to be selected to fit the in vivo pressure oscillations in the aorta [1-4]. Usually the intraorgan arterial beds are considered as bifurcating trees whereas real arterial vasculatures possess different topology that includes the tree-like systems (lung, kidneys, liver, heart), systems with a few anastomoses between the large intraorgan arteries (stomach, limbs) and systems with numerous anastomoses (large and small intestine). Morphometric analysis revealed some regularity in geometry of different intraorgan vasculatures [5-9]. Theoretical estimations of the input admittance Y of the intraorgan beds that is calculated as the ratio of the flow rate Q to pressure P in the inlet of the feeding artery of the organ have revealed some differences in Fourier spectra of the admittances of the vasculatures with different geometry that can be used in medical diagnostics [10–13].

On the basis of the latest morphometric data we propose a novel detailed digital model of the human arterial system that includes the realistic topology, branching regularities of the extraorgan and intraorgan arterial systems of the main inner organs of a human. The detailed analysis of the dynamics of arterial blood flow and pulse wave propagation taking into account the distributed nature of the pulse wave reflections at bifurcations and terminal arterial vasculatures can be carried out on the model.

II. REGULARITIES IN THE STRUCTURE OF THE INTRAORGAN ARTERIAL BEDS

Geometry and topology of the arterial beds of the inner organs and extraorgan large arteries have been invesigated on the plastic casts of the arterial system of the corpses of the healthy young human whose death was not connected with circulatory diseases [5]. It is well known that arterial beds of the inner organs possess different topology [14]. Vasculature of the liver, kidneys, spleen, lungs and heart resembles tree-like branching systems (see the model number 1 in Fig.1). The most part of the branches are dichotomous divisions of the parent vessel into two daughter branches. Nevertheless according to the morphometric observations [9] some part of the branches is always presented by trichotomous divisions. In the organs with tree-like arterial beds some anastomoses between the arteries of the $n \geq 4$ branching order can also be found. Some other arterial vasculatures are presented by systems with anastomoses between the arteries of the $n = 2 - 3$ branching order. In stomach the branches of the feeding artery form four arteries which are connected into two loops (number 3 in Fig.1). The downstream vasculatures begin in several points along the loops. The points can be considered as bifurcations though the flows in the bifurcations in some of the points converge. The most important extraorgan systems of this kind are vasculatures of the upper and lower limbs (number 2 in Fig.1). In the upper limbs the radial and ulnar arteries form two palmar arcs and the digital arteries which supply the fingers begin in several points along the arcs. Due to significant individual variations in topology of the arterial systems the arcs can be presented in different forms [14,15]. The last group of the inner organs possesses the vasculature with numerous loops formed by bifurcations of their feeding arteries. The first order branches of the large and small intestine form a complicated system of loops (the model number 4 in Fig.1). Anatomical variations of the vasculature include different number of loops, presence of some incomplete peripheral loops and different number of the terminal tree-like structures penetrated into the tissues of the organ [14,15].

Anastomoses are very important as additional pathways for blood supply to the organ. Moreover when the pressure and flow waves propagate through the vasculature with loops the additional pathways for the wave propagation and reflection produce an increase in the wave amplitudes due to superposition of the waves [16]. The waves have small phase shifts due to high wave velocity in the intraorgan arteries. That is a possible explanation of using some arteries of the limbs for the pulse palpation and diagnosis of the pathologies in the oriental medicine [12,16].

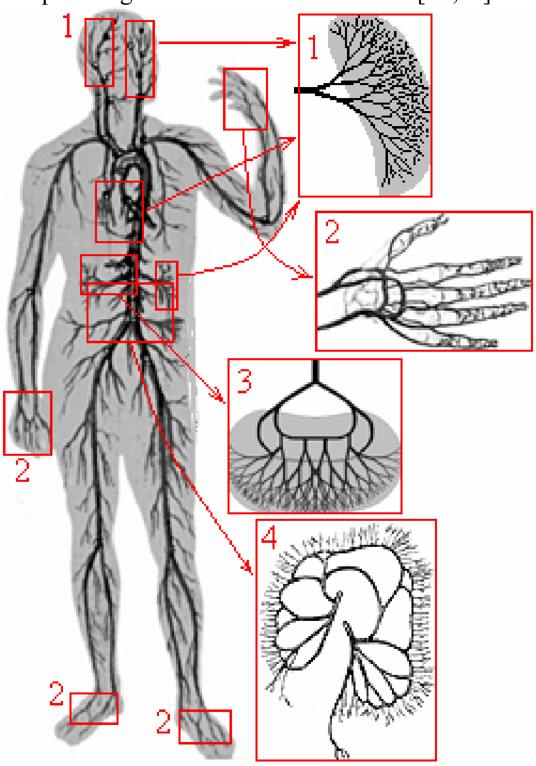

Fig.1. The model of the human arterial system including the tree-like (1) intraorgan beds, the beds with 1-2 (2,3) and numerous loops (4).

Basing on the detailed data on geometry of the arterial vasculatures of the inner organs the dependences between the lengths L_j, diameters d_j, branching angles φ_j of the vessels of the branching order j (according the Strahler ordering) have been investigated. Perfect agreement of the dependences $\varphi_j(d_j)$ and the relationships between the diameters $d_j^{1,2}$ and d_j^0 of the daughters and parent vessels in the bifurcations j to the optimal principle based on the minimization of the total energy expenses in the arterial system (Murray's law $(d_j^0)^\gamma = (d_j^1)^\gamma + (d_j^2)^\gamma$) [5,6,17,18] has been found for the tree-like arterial structures. The dependence between the average diameters of the daughter vessels $<d_j^{12}> = (d_j^1 + d_j^2)/2$ is presented in Fig.2. It is close to linear dependence ($<d_j^{12}> = a(d_j^0)^b$, $a \in [0.562, 0.878]$, $b \in [0.982, 1.086]$, $<R^2> \in [0.881, 0.945]$) for all the examined systems. In that way the diameters of the daughter vessels are determined by the inflow conditions produced by the blood flow in the parent vessel.

The dependence of the optimal parameter $\mu = ((d_j^1)^3 + (d_j^2)^3)/(d_j^3)^3$ of the bifurcation on the branching asymmetry coefficient $\nu = \min\{d_j^{1,2}\}/\max\{d_j^{1,2}\}$ is presented in Fig.3. The solid line corresponds to the asymmetrical bifurcations with $\max\{d_j^{1,2}\} = d_j^0$ when the bigger daughter vessel can be regarded as a prolongation of the parent vessel.

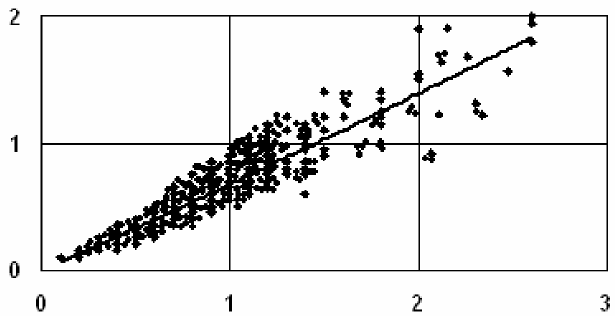

Fig.2. The average diameter of the daughter arteries $<d_j^{12}>$ (mm) as a function of the diameter d_j^0 (mm) of the parent vessel.

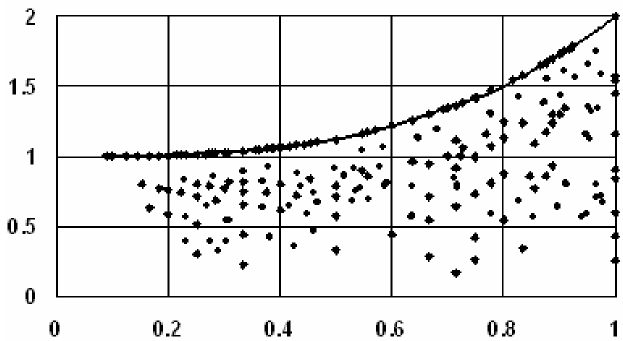

Fig.3. Non-dimensional Murray's parameter μ versus the branching asymmetry coefficient ν.

Dependences $L_{\Sigma}(d_1)$ between the length L_{Σ} of the longest arterial path in the vasculature from the feeding artery to the smallest arteries and the diameter d_1 of the feeding artery have also been calculated for all the intraorgan vasculatures. For the coronary arteries the dependence $L_{\Sigma}(d_1)$ has been obtained in [19] and explained by the suggestion about the uniform shear stress in the blood vessels [20]. Based on the detailed data [9] the approximations $L_{\Sigma}(d_1)$ have been found for the inner organs by the least-square method [21]. The obtained relations can be used for computer generation of the realistic arterial systems. The models possess the realistic structure of the corresponding arterial systems, the total hydraulic resistance Z_{Σ} and the input wave impedance $Z_w(\omega)$ of the vasculatures where ω is the wave frequency.

II. MODEL OF HUMAN CIRCULATION

The proposed model of the human arterial circulation includes the large extrtaorgan arteries with given lengths, diameters, wall thicknesses and Young's modules [1,2] and intraorgan vasculatures that are considered as branching systems with anastomoses and without them depending on the morphometric data. The corresponding relationships between the diameters of the consequent arterial segments as well as between the total length of any vasculature and the diameter of its feeding artery which are proper to different intraorgan arterial beds are embedded into the model. The smallest intraorgan arteries in the model ($d \le 0.1$ mm) terminate in three-element Windkessels which properties reflect the microcirculation state in the organ. The beds are combined into the total circulatory system (Fig.4).

Each artery in the model is considered as a thick-walled viscoelastic tube with Poiseuille hydraulic resistance $Z_j = 128\mu(d_j)L_j/(\pi(d_j)^4)$. The hydraulic resistances of separate intraorgan beds and the total circulatory system have been calculated by taking into consideration the type of connection of the tubes, where μ is the blood viscosity which is a function of the diameter for the small vessels due to the Fahreus-Lindquist effect. For the tree-like systems the parallel and series connections of the segments give the simple iterative procedure for calculation Z. For the systems with loops the pressure P and volumetric rate Q continuity conditions $P_j^0 = P_j^1 = P_j^2$, $Q_j^0 = Q_j^1 + Q_j^2$ and Poiseuille law for each tube $P_j^0 - P_{j+1}^0 = Q_j Z_j$ give the algebraic system of equations for pressures and flow rates in the tubes.

Possible normal and pathological variations in topology of the vasculatures (absence of some loops or additional loops at different orders of branching) as well as physiological variations in the lengths ($\pm$ 30%) and diameters ($\pm$ 10%) of the tubes have been considered. Different pathologies that are connected with narrowing/widening of separate tubes (stenosis/aneurisma), increasing the rigidity and thickness of the wall (atherosclerosis, hypertension or age-induced changes), increasing/decreasing the resistive and compliant properties of the terminal elements (different pathologies at a capillary level) can be modeled by corresponding variations of the geometrical and mechanical parameters of the tubes at a given topology of the system.

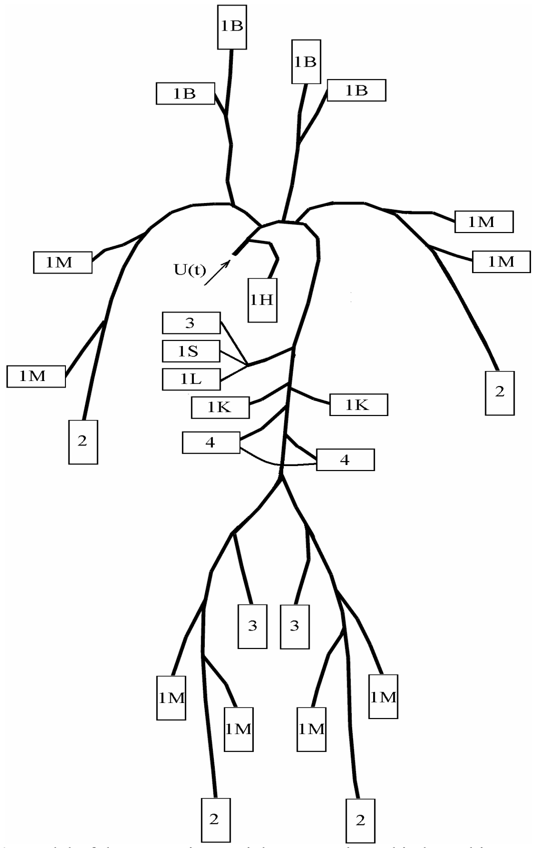

Fig.4. Model of the systemic arterial tree terminated in branching systems with different topologies: tree-like vasculatures of the brain (1B), muscles (1M), liver (1L), spleen (1S), kidneys (1K), heart (1H), arterial beds of the limbs where 1-2 arcs are presented (2), intraorgan systems with a few arcs (3) and numerous arcs (4).

Wave input impedance for the tree-like systems (the models number 1 in Fig.4) can be calculated in the same way with the characteristic impedances of the separate tubes $Z_{wj} = 4\rho c_j/(\pi(d_j)^2)$ instead of the Poiseuille resistances, where c_j is the wave velocity, ρ is density of the blood [22]. The characteristic admittance corresponds to the tube without wave reflections at the end of the tube (the open end). In the real vessels the observed wave shapes are always the composition of the forward and numerous backward waves. The input impedance taking into consideration the wave reflections can be calculated basing on the Womersley model of the wave motion of the viscous fluid through the thick-walled viscoelastic tube [23] namely:

$$P_i(t,x_i) = P_i^0 \left(e^{i\omega(t-x_i/c_i)} + \Gamma_i e^{i\omega(t+(x_i-2L_i)/c_i)}\right)$$

$$Q_i(t,x_i) = Y_i^0 P_i^0 \left(e^{i\omega(t-x_i/c_i)} - \Gamma_i e^{i\omega(t+(x_i-2L_i)/c_i)}\right)$$

where $c_i = \left(\dfrac{E_i h_i (1-F_{01})}{2\rho R_i (1-\sigma_i^2)} e^{j\theta_i}\right)^{1/2}$, $F_i = 2J_1(\beta_i)/(\beta_i J_0(\beta_i))$, $\beta_i = \alpha_i (-1)^{3/4}$, $P_i^0 = P_i\big|_{x_i=0}$, $\alpha_i = R_i\sqrt{\omega\rho/\mu}$ is Womersley number, $Y_i^0 = (Z_i^0)^{-1}$, Γ_i is the complex reflection coefficient $\Gamma_i = P_b / P_f$, where $P_{b,f}$ are amplitudes of the forward and backward going pressure waves.

For the tree-like systems the method of calculation the total input impedance is described in [22]. For the arterial systems with anastomoses (models number 2-4 in Fig.4) the pressure and volumetric rate boundary conditions give the system of non-linear equations for the unknown pressure amplitudes P_i^0 and the reflection coefficients Γ_i. The modified Newton's numerical method for calculating the sets $\left\{P_i^0, \Gamma_i\right\}_j$ is presented in [12].

III. Results and Discussions

An important feature of the proposed model is connected with so-called resonant properties of the intraogran vasculatures of different inner organs [10–12,24]. That implies that the input admittance of the intraorgan vasculature possesses a set of resonant harmonics. Any variation in the parameters of reflection conditions at the terminal arteries and in blood rheology cause noticeable alterations of the amplitudes of resonant harmonics and negligible alterations of the amplitudes of other harmonics. The set of resonant harmonics is independent of some variations in the tree geometry and correlate with the length of the feeding artery of the arterial system [11–13]. The sets are different for the tree-like arterial systems and the vasculatures with anastomoses [12]. In Fig.5 a,b the dependences of the dimensionless input wave admittances of the systems with/ without anastomoses versus the number of harmonics are presented. In the normal state the wave transmitted into the arterial system of the organ has its maxima and minima at certain harmonics. In contrary, the wave reflected at the bifurcation of the feeding artery of the organ is characterized by the minimal amplitudes of the resonant harmonics of the inner organ. In that way the pressure wave propagated along the aorta and reflected at its bifurcations carries some information about circulation in the inner organs. On the other hand when the pressure $P = P_f + P_b$ and flow $Q = Q_f - Q_b$ waveforms are measured in any cross-section of the feeding artery of the intraorgan vasculature the propagated waves P_f, Q_f carry information from the inlet of the arterial system whereas the reflected waves P_b, Q_b carry information on the state of the intraorgan circulation that can be used for diagnostics.

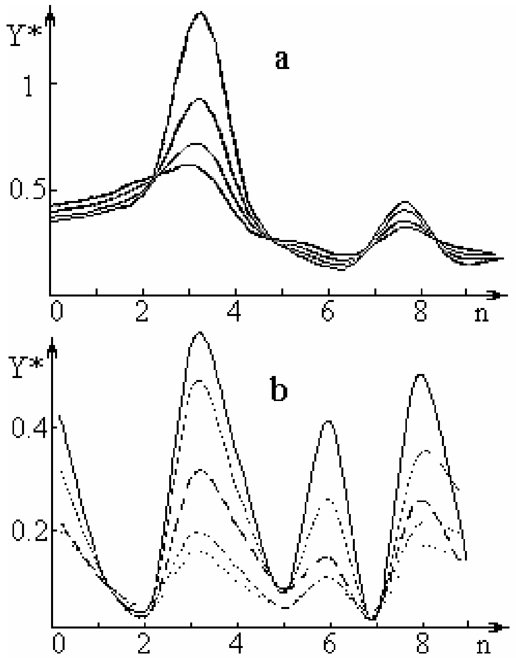

Fig.5. Dependences of the non-dimensional admittance Y* on the harmonics number n for a tree-like vasculature (a) and a system with anastomoses between the vessels of the second branching order (b).

The exact data on geometry of any intraorgan arterial system contains a few thousand values which are characterized by significant individual variations. When the intraorgan system is considered as a three element Windkessel the single wave reflection with a complex reflection coefficient $\Gamma = \Gamma_1 + i\Gamma_2$ is taken into account. Optimal tree-like systems can be generated by the diameter of the initial (feeding) artery d_1 and the coefficients in the dependence $L_j = L_j(d_j)$ that is used in the model, the asymmetry coefficient ν and the Murray power γ which are usually kept constant for the tree. The sets of the parameters $\left\{d_1, L_j(d_j), \nu, \gamma\right\}$ are different for different inner organs. When the model of the systemic arteries contains 20-25 terminal elements [1,2,4] the wide variation range of the parameters of the terminal trees allows obtaining the realistic wave patterns in different sections of the aorta [4]. The sets of the parameters of the intraorgan vasculatures are taken from the statistical analysis of the morphometric data and can not be varied in an arbitrary way to obtain the best fit to the experimental pressure and flow curves. The only unknown parameters are reflection conditions at the terminal elements which reflect the microcirculation and can vary both in the normal state and in pathology.

Basing on the detailed model of the human circulation the calculations of the pressure and flow waveforms and pressure-flow relationships have been calculated and compared to the experimental data on the blood flow and wave propagation in different parts of the aorta, in the carotid, subclavian, brachial, radial and kidney arteries [25–27]. Individual variations in the lengths ($\pm 30\%$) and diameters ($\pm 10\%$) of all the arteries in the model have been set by the random function. Variations in the reflection coefficients at the terminal elements $\Gamma_j = \Gamma_j^1 + i\Gamma_j^2$, $\Gamma_j^{1,2} \in [-1,1]$ correspond to all possible microcirculation state including open and closed end and negative reflections [28]. As a result a lot of different curves have been calculated and some of them are presented in Fig.6. Due to different reflection conditions in the peripheral vessels the radial and carotid waveforms undergo significant variations in the number and relative amplitudes of the peaks, position of the dicrotic wave(s) and the incisura and only one of them are presented in Fig.6 b,c. The variations of the terminal conditions in the model lead to some variations in the shape of the waves whereas the number of the dicrotic waves and position of the incisura on the descending part of the wave remain quite stable. That can be explained by the fixed topology of the arterial systems of the upper limbs (2 arcs) while significant variations on the wave patterns can be connected with variation in topology of the palmar arcs (unclosed arcs and others [14,15]) and different elastic properties of the arterial walls. Variation of the mechanical parameters of the wall within the physiological limits produce a great variety of the wave shapes which correspond to age-induced and pathological conditions. In that way different pathological variations of the pressure-flow relationships can be in-depth investigated on the basis of the developed model. On the other hand the model provide the input conditions for the intraorgan systems and the intraorgan blood circulation can be investigated using the realistic input pressure conditions in the inlet of the feeding artery of the inner organ. Our preliminary investigations confirmed existence of the resonant properties of the intraorgan arterial beds as it was shown in the models with a simple sinusoidal input [12,13] and in experimental measurements in the systems of latex tubes and in the acute experiments with rats [10,11,24].

IV. CONCLUSIONS

The model gives realistic values for the total hydraulic resistance of the circulatory system and separate inner organs that can be compared to physiological data. Parameters of the pulse wave in the aorta and radial arteries correspond to the experimental measurements. In contrast to the tree-like systems the vasculatures with numerous loops exhibit insignificant changes in the total input impedance Z

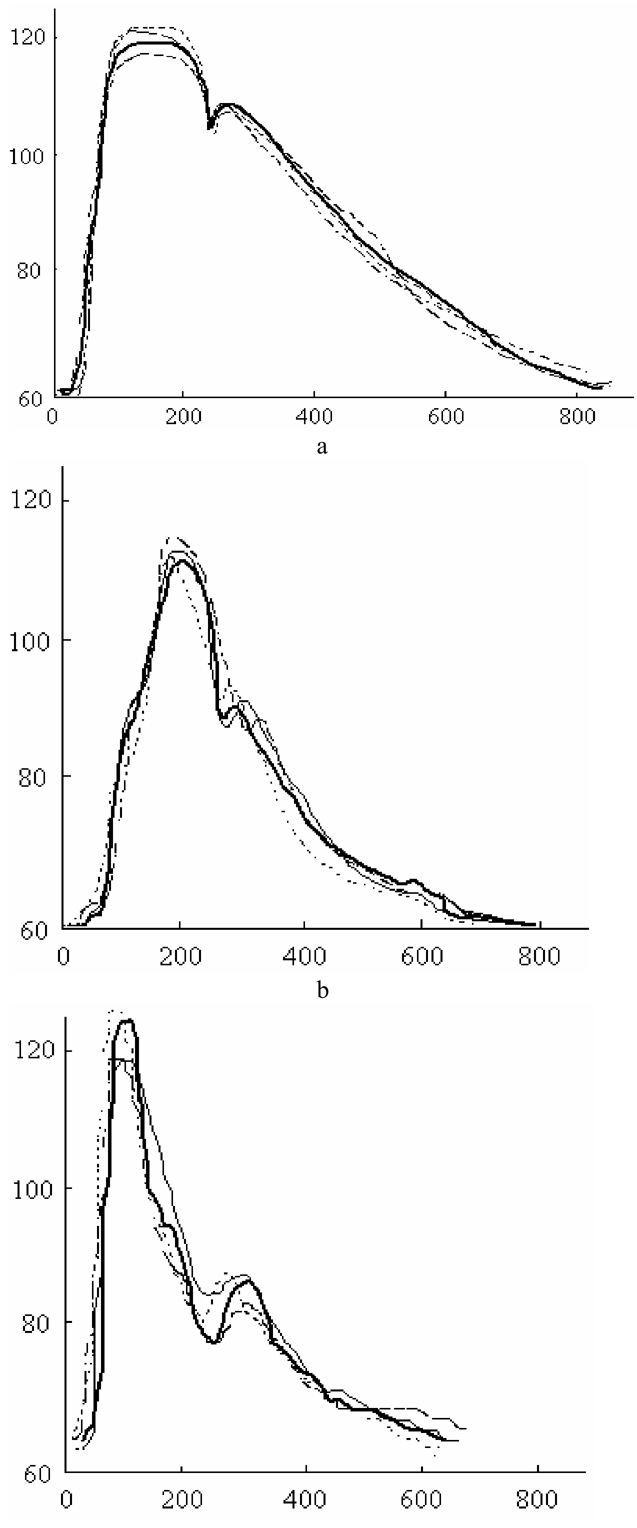

Fig.6. Dependences of the pressure P (mm Hg) on time t(ms) in the thoracic aorta (a), carotid artery (b) and radial artery(c) at some variations of the reflection conditions at the terminal elements (thin, dashed and dotted lines) in comparison with the experimental curves (solid line).

some large arteries are narrowed due to stenosis or other of the vasculature and blood flow in the smallest tubes when pathology. The computer modeling has revealed existence of the resonant harmonics in vasculatures with different topology. Amplitudes of the resonant harmonics significantly vary when the reflection conditions at the terminal elements are changed due to the pathology at a microcirculatory level whereas the variations of the amplitudes of the other harmonics are insignificant. The tree-like vasculatures and the systems with loops possess different unique sets of the resonant harmonics (Fig.5) that makes possible the diagnostics of different circulatory pathologies. The results and the proposed detailed model can be used for improvement the methods of pulse wave analysis in medical diagnostics. Additional information about the state of the intraorgan circulation can be obtained by estimation of parameters of the pressure-flow loops and wave intensity analysis of the pressure and flow curves in the feeding arteries of the inner organs and superficial arteries of the upper and low extremities which are accessible for the noninvasive measurements.

REFERENCES

[1] N. Westerhof, F. Bosman, C.J. de Vries, and A.Noordegraaf, "Analog studies of the human systemic arterial tree," *J. Biomech.*, vol. 2, no.1, pp.121–143, 1969.

[2] A. P. Avolio, "Multi-branched model of the human arterial system," *Med.&BiolEng&Comput.*, vol.18, no.6, pp.709–718, 1980.

[3] P. Segers, N. Stergiopulos, P. Verdonk, and R.Verhoeven, "Assessment of distributed arterial network models," *Med.&Biol.Eng.&Comput*, vol. 35, no.6, pp. 729–736, 1997.

[4] M.S. Olufsen, „Structured tree outflow condition for blood flow in larger systemic arteries," *Amer.J.Physiol.*, vol. 276, pp. H257–H268, 1999.

[5] T.F. Sherman, "On connecting large vessels to small: the meaning of Murrays's law,"J.Gen.Physiol., vol. 78, pp. 431–453, 1981.

[6] M. Zamir, and D.C. Bigelow, "Cost of departure from optimality in arterial branching," *J.Theor.Biol.*, vol. 109, pp. 401–409, 1984.

[7] M. Zamir, and H. Chee, "Branching characteristics of human coronary arteries," *Canad.J. Physiol. Pharmacol.*, vol.64, pp. 661–668, 1986.

[8] Y.Zhou, G.S. Kassab, and S.Molloi, "On the design of the coronary arterial tree: a generalization of Murray's law," *Phys.Med. Biol*, vol. 44, no.12, p. 2929–2945, 1999.

[9] O. K.Zenin, V. K.Gusak and G. S. Kirjakulov, *Arterial system of a human in figures and formulas*, Donetzk, 2002.

[10] Y. Y. Wang, S. L. Chang, Y. E. Wu,. T. L. Hsu, and W. K. Wang, "Resonance. The missing phenomenon in hemodynamics," *Circulat. Res.*, vol. 69, no.1, pp. 246–249, 1991.

[11] Y. Y. Wang, W. C. Lia, H. Hsiu, M. Y. Jan and W. K. Wang, "Effect of length on the fundamental resonance frequency of arterial models having radial dilatation," *IEEE Transact.BME*, vol. 47, no. 3, pp. 313–318, 2000.

[12] N. N. Kizilova, " ulse wave reflections in branching arterial networks and pulse diagnosis methods," *J. Chinese Inst. Eng.*, vol. 26, no. 6, pp. 869–880, 2003.

[13] N. N. Kizilova, "Reflection of Pulse Waves and Resonance Characteristics of Arterial Beds," *Fluid Dynamics*, vol. 38, no.5, pp.772–781, 2003.

[14] D. Luzsa, *X-Ray Anatomy of the Vascular System*, Budapest: Acad. Kiado, 1973.

[15] H.Lipert, R.Pabst, *Arterial variations in man: classification and frequency*, Munich: J.F.Bergman, 1985.

[16] N.N. Kizilova , "Biomechanical model of pulse wave diagnostics method based on palpation of a radial artery," in First Karazin Scientific readings. Mathematical symposium, 2004.

[17] C.D. Murray, "The physiological principle of minimum work. I. The vascular system and the cost of blood volume," *Proc. Nat. Acad. Sci. USA.*, vol.12, pp.207–214, 1926.

[18] C.D. Murray, "The physiological principle of minimum work applied to the angle of branching of arteries," *J.Gen. Physiol*, vol.9, pp. 835–841, 1926.

[19] Y. Zhou, G.S. Kassab, S. Molloi, "On the disign of the coronary tree: a generalization of Murray's law, " *Phys.Med.Biol.*, vol.4, pp.2929–2945, 1999.

[20] G. Kassab, Y. Fung, "The pattern of coronary arteriolar bifurcations and the uniform shear hypothesis," *Ann.Biomed.Eng.*, vol..23, pp.13–20, 1995.

[21] N.N. Kizilova, O.K. Zenin, I. Karabash, A. Chvala, O.Nikitin "Computer generated models of intraorgan arterial beds: hydraulic conductivity and input admittance," in ICSSA 2005 conference special issue, Singapore, in press.

[22] M. G. Taylor, "The input impedance of an assembly of randomly branching elastic tubes," *Biophys. J.*, vol. 6, no. 1, pp. 29–51, 1966.

[23] J.R.Womersley, "An elastic tube theory of pulse transmission and oscillatory flow in mammalian arteries," *Tech.Report TR-56–614,* 1957.

[24] G.L. Yu, Y.L. Wang, W.K. Wang, "Resonance in the kidney system of rats," *Amer.J.Physiol* vol.267, no.4, pp.H1544–H1548, 1994.

[25] R.D. Latham, N. Westerhof, P. Sipkema, B.J. Rubal, P. Reuderink, and J.P. Murgo, "Regional wave travel and reflections along the human aorta: a study with six simultaneous micromanometric pressures," *Circulation*, vol.72, no.6, pp.1257–1269, 1985.

[26] K. Kohara, Y.Jiang, M. Igase, and K. Hiwada, "Effect of Reflection of Arterial Pressure on Carotid Circulation in Essential Hypertension," *Amer.J.Hypertension*, vol.12, no 10, pp.1015–1020, 1999.

[27] C.H. Chen, E. Nevo, B. Fetics, P.H. Pak, F.C. Yin, W.L. Maughan, D.A. Kass, "Estimation of Central Aortic Pressure Waveform by Mathematical Transformation of Radial Tonometry Pressure," *Circulation*, vol.95, pp.1827–1836, 1997.

[28] E.H. Hollander, J.J. Wang, G.M. Dobson, K.H. Parker, J.V. Tyberg, "Negative wave reflections in pulmonary arteries," *Amer.J.Physiol.*, vol.281, no.2, pp.H895-902, 2001.