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ON NEW REPRESENTATIONS OF WELL-KNOWN PHYSICAL PHENOMENA

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New theoretical description of well-known physical phenomena are announced. 1. The description of wave interaction in plasma in terms of spontaneous and stimulated processes is discussed. Such a description is not only attractive from the methodological point of view, but also offers new possibilities for the understanding of physical processes related to the interaction of radiation with matter. Moreover, there exists an intimate relationship between spontaneous and stimulated processes that can simplify the calculation of nonlinear terms for systems with multi-wave interactions. 2. It is shown by the example of two-level system that generation of coherent radiation is realized in excess of the threshold, determined by the equality of the squared population inversion to the half of the total number of all possible states. 3. The spectrum of waves emitted by oscillator, trapped in an external potential well is analyzed. It is assumed that the eigenfrequency of the oscillator is much greater than the frequency of oscillations in the potential well. The effect of the recoil on the absorption and emission of the oscillator is discussed. Since the energy of the slow oscillations in well is equal to the recoil energy, the intensity of the absorption and emission lines at the eigenfrequency exceeds the intensity of other spectral lines. 4. The formation of gravity surface waves with abnormally high amplitude, that occurs only in initial stage of nonlinear regime of modulation instability in the ocean, is considered. 5. The intensive long-wave Langmuir oscillation in plasma has been generated by a high-current charged-particle beam and a maser radiation is unstable. It being known the field energy density often exceeds the thermal energy density of plasma. In this case the modulation instability of intensive oscillation results one plasma density cavity over a wavelength of the intensive oscillation. It is shown, that kinetic limitation mechanism of cavity deepening is the local capture of ions. It should be noted, the potential of cavity is quite low for capture of great part of electrons with considerable kinetic energy.
KEY WORDS: spontaneous and stimulated processes, multi-wave interactions, threshold of coherent radiation, emission by oscillator, trapped in an potential well, modulation instability.

ПРО НОВІ ОПИСИ ДОБРЕ ВІДОМИХ ФІЗИЧНИХ ЯВИЩ

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Анонсовано нові описи відомих фізичних явищ. 1. Обговорюється опис хвилевих взаємодій в плазмі у термінах спонтанних та вимушених процесів. Такий опис привертає увагу не тільки з методологічної точки зору, а також відкриває нові можливості для розуміння фізичних процесів, що відносяться до взаємодії випромінювання з речовиною. Крім того існує значний зв'язок між спонтанними та вимушеними процесами, що дозволяє спростити обчислення нелінійних доданків в виразах для багатохвильових процесів. 2. На прикладі дворівневої системи показано, що виникнення когерентного випромінювання можливо при перевищенні порогу, що визначається рівністю квадрата інверсії половині повного числа станів системи. 3. Проаналізовано спектр хвиль, що генерує осцилятор, який захвачено в зовнішню потенційну яму. Власна частота осцилятора набагато більша частоти його коливань в потенційної ямі. Враховано ефект віддачі осцилятора при випромінюванні та поглинанні. В тому випадку, коли енергія віддачі дорівнює енергії повільних коливань осцилятора в потенційної ямі, інтенсивність ліній випромінювання та поглинання на його власної частоті значно перевищує інтенсивність інших ліній спектру. 4. Показано, що формування гравітаційних поверхневих хвиль аномально великої амплітуди в океані можна побачити тільки на начальної стадії нелінійного режиму модуляційної нестійкості хвиль в океані. 5. Інтенсивні довгохвилеві ленгмюрівські коливання, що генеруються пучками заряджених часток та мазерним випромінюванням, виявляються модуляційне нестійкими. Досить часто густина енергії поля значно перевищує густину теплої енергії плазми. В цьому випадку результатом модуляційної нестійкості є виникнення однієї каверни густини плазми на масштабі довжини інтенсивної хвилі. Показано, що кінетичним механізмом обмеження процесу поглиблення каверни є локальний захват іонів. Відмічається, що потенціал каверни замалий для захвату значної частини електронів, які мають значну кінетичну енергію.
КЛЮЧОВІ СЛОВА: спонтанні та вимушені процеси, багатохвильові взаємодії, поріг когерентного випромінювання, випромінювання осцилятора, що рухається в зовнішньої потенційної ямі, модуляційна нестійкість.

О НОВЫХ ОПИСАНИЯХ ХОРОШО ИЗВЕСТНЫХ ФИЗИЧЕСКИХ ЯВЛЕНИЙ

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Анонсированы новые описания известных физических явлений. 1. Обсуждается описание волновых взаимодействий в плазме в терминах спонтанных и индуцированных процессов. Такое описание привлекательно не только с методологической точки зрения, но и открывает новые возможности для понимания физических процессов, относящихся к взаимодействию излучения с веществом. Кроме того, существует глубокая связь между спонтанными и индуцированными процессами, которая позволяет упростить вычисления нелинейных слагаемых в выражениях для многоволновых взаимодействий. 2. На примере двухуровневой системы показано, что возникновение когерентного излучения возможно при

превышении порога, определяемого равенством квадрата инверсии населенностей половине полного числа состояний системы. 3. Изучен спектр волн, генерируемых осциллятором, захваченным во внешнюю потенциальную яму. Собственная частота осциллятора при этом много больше частоты его колебаний в потенциальной яме. Учитывается эффект отдачи осциллятора при излучении и поглощении. В том случае, если энергия отдачи равна энергии медленных колебаний осциллятора в потенциальной яме, интенсивность линий излучения и поглощения на его собственной частоте превышает интенсивности других спектральных линий. 4. Показано, что формирование гравитационных поверхностных волн аномально большой амплитуды, происходит только на начальной стадии нелинейного режима модуляционной неустойчивости океанского волнения. 5.Генерируемые пучками заряженных частиц или мазерным излучением интенсивные длинноволновые ленмюровские колебания плазмы оказываются модуляционно неустойчивы. Причем часто плотность энергии поля достигает и порой превосходит плотность тепловой энергии плазмы. В этом случае результатом модуляционной неустойчивости является возникновение каверны плотности плазмы на масштабах длины интенсивной волны. Показано, что кинетическим механизмом стабилизации дальнейшего углубления каверны плотности является локальный захват ионов. Отмечено, что потенциал каверны оказывается недостаточным для захвата большей части электронов, обладающих значительной кинетической энергией.
КЛЮЧЕВЫЕ СЛОВА: спонтанные и индуцированные процессы, многоволновые взаимодействия, порог когерентной генерации, излучение осциллятора, захваченного в потенциальную яму; модуляционная неустойчивость.

1. INTRODUCTION

That paper announces new theoretical description of following well-known physical phenomena and new results of its analyses. At first we apply this concept for a description of some processes in plasma in terms of spontaneous and stimulated emission. By spontaneous emission, one means the process by which an emission source such as a charge or current emits an electromagnetic wave and this emission does not depend on the external electromagnetic field at the same frequency. By stimulated emission (or absorption), we mean the process, caused by the interaction between an emission source (absorption channel) and an external electromagnetic field at the corresponding frequency. It is not difficult to generalize the method suggested by Einstein [1] and derive [2,3] the equation for the radiation energy density $W_k = \hbar\omega \cdot N_k$:

$$dW_k / dt = S_2 + \frac{\partial S_2}{\partial(\hbar\omega)} W_k \,, \tag{1}$$

here N_k - number of emitted quanta, S_n the change in energy density on level *n* caused by spontaneous processes. The term $(S_2 - S_1)N_k = W_k \cdot \partial S_2 / \partial(\hbar\omega)$ corresponds to stimulated processes. Interaction between waves and currents has an allocated character and depends on the integral phase relations, which causes some difficulties in the interpretation of these processes as radiative. But one can formulate a simple criterion for the existence of spontaneous radiation. If the work done by the field on the current, which generates this field, has a non-zero real part, then spontaneous radiation takes place. This criterion becomes very important when the current and the related field occupy the whole interaction space, e.g. the current is not localized and the field in the far-field zone cannot be analyzed. In the 2 part one can see that oscillations, generated by nonlinear currents at combined frequencies, demonstrate all the characteristics of spontaneous and stimulated emission.

The major problem is the interpretation of an induced radiation in quantum description as coherent radiation. In contrast to the classical description here is impossible to say something about field phases of atoms and molecules radiating. But though Charles Townes considered that "…energy is radiated from molecular systems has the same field distribution and the same frequency as external emission, and consequently permanent (probably zero-order) phase difference" [4]. Stimulative problem for thinking are degree of coherence in maser; inversion level, which is the necessary value for coherent radiation and the role of noncoherent radiation in depletion of inversion. The other unsolved problem is the physical role of noise emission into the process dynamics of generation for traditional and plasma electronics devices. That problems [5] are proposed for discussion in the 3 part.

The scattering of high-energy photons by free electrons results in a decrease in energy of photons due to the recoil effect (the Compton effect). This fact together with the phenomenon of the photoelectric effect confirmed the basic principles of quantum theory of radiation [6-8]. The processes underlying the interaction of radiation and matter are characterized by an impressive variety and form the basis for many physical research directions [9]. One of the problems, that arises when considering the processes of absorption and emission by a substance, is the problem of interaction with the external radiation field of the oscillating particle trapped in the potential well formed by the spatial structure of the medium. This problem requires the use of the quantum model for describing the behavior of the excited oscillator in a potential well, take into account the effect of the recoil. The purpose of the 4 part is discussion [10] of the quantum-mechanical model of the interaction between a charged particle-oscillator trapped in the potential well and an external electric field.

The modulation instability of regular spatial patterns form the spectra of perturbations, which growth rates have local maxima both in the bands of small and large scales [11-12]. The modulation instability occurs, for example, in systems described by the Lighthill equation (or NSE) [11]. It also underlies the instability of Langmuir waves in plasma described by Zakharov equations [13] or Silin equations [14]. In recent years, the attention of researchers was attracted to the phenomenon of large-amplitude short-lived waves observed in various nonlinear dispersive wave media. These waves were named as freaks or rogue waves. Of particular interest are the experimental observations of extreme ocean waves. A complete review on the various phenomena yielding to rogue waves in ocean can be found in the book [15]. Later, it has been found experimentally that freak waves can be generated in optical systems [16-17] and in space plasma [18]. Zakharov and co-authors (see detailed review [19] and book [20]) have formulated the theory of the nonlinear stage of modulation instability based on excites the spectrum satisfying the conditions of space-time synchronism of the form $2\omega_0 = \omega(k) + \omega(-k)$ and interaction by pairs of waves symmetric with respect to the pump $\omega(k) + \omega(-k) = \omega(k') + \omega(-k')$. This model, as shown in the 5 part, allows to analyze some specific futures of the instability, in particular, the symmetry breaking of the excited spectrum during the progress of the modulation instability in a medium with strong dispersion and the formation of gravity waves with abnormally high amplitude [21].

The intensive long-wave Langmuir oscillation in plasma has been generated by a high-current charged-particle beam and a maser radiation is unstable. It being known the field energy density often exceeds the thermal energy density of plasma [14]. In this case the modulation instability of intensive oscillation results one plasma density cavity over a wavelength of the intensive oscillation [13,14]. In the 6 part one may see the potential well bottom is capable of ions capture, but the potential of cavity is quite low for a capture of great part of electrons. The energy of captured ion considerably exceeds initial energy of electron. But capture region is too small and the total energy of ions is substantially smaller the initial total energy of system. That local capture of ions is the kinetic (but non hydrodynamic) limitation mechanism of cavity deepening.

2. WAVE INTERACTIONS IN PLASMA IN TERMS OF SPONTANEOUS AND STIMULATED PROCESSES

Let us consider the interaction of three ion-acoustic waves in non-isothermal plasma. Let two ion-acoustic waves with frequencies ω_2 and ω_3 propagate in a nonlinear medium and excite a nonlinear current, $\tilde{j}_{23}$, capable of radiating proper waves of the medium at a frequency ω_1 under conditions of spatial–temporal synchronism $\omega_1 \approx \omega_2 + \omega_3$ and $\vec{k}_1 = \vec{k}_2 + \vec{k}_3$. The radiation of quanta ω_1 by the nonlinear current $\tilde{j}_{23}$ can be considered as a spontaneous process if one takes into account the action of waves of frequencies ω_2, ω_3 on $\tilde{j}_{23}$. The Fourier transform of the current at combination frequency $\omega_2 + \omega_3$ can be represented as follows:

$$j_{23}(\omega, k) = (k_2 + k_3)\frac{n_0 e^3 \{E_2 E_3\}_\omega}{m_i^2 \omega_1 \omega_2 \omega_3}\delta(k_1 - k_2 - k_3)\,, \tag{2}$$

where for E_2, E_3 and $\{E_2 E_3\}_\omega$ we use the expressions

$$\{E_2 E_3\}_\omega = \{E_2 E_3\}_0 \frac{1}{\Delta_{\Omega 23}\sqrt{\pi}} \exp\left\{-(\omega - \omega_2 - \omega_3)^2 / \Delta^2{}_{\Omega 23}\right\}, \tag{3}$$

By applying the Borel theorem at small detunings $-(\omega_1 - \omega_2 - \omega_3) = \Delta_{-1,2,3} = \Delta$, ($\Delta_{\Omega 23}$ - is dispersion spreading) the rate of emission at the frequency ω_1 due to the interaction of the field with the nonlinear current at combination frequency $\omega_2 + \omega_3$ is determined by the following expression:

$$-(E_{23}^{(2)*} j_{23}^{(2)} + E_{23}^{(2)} j_{23}^{(2)*})/2 = \alpha \left(\frac{e}{m_i v_s}\right)^2 W_2 W_3 \frac{8}{3}\frac{\omega_1}{\omega_2 \omega_3} \tag{4}$$

where $j_{23}^{(2)}$ is the nonlinear current, which is generated by two ion-acoustic waves with frequencies ω_2 and ω_3, and $E_{23}^{(2)}$ is the field at a frequency ω_1 which is generated by that current in turn. Note that the sign of the rhs of (4) is positive and does not depend on phases of interacting waves, which correspond to the emission process. This sign-definiteness is characteristic of spontaneous processes. In addition, the generation at the frequency ω_1 is provided by an external source in relation to the wave at the frequency ω_1 (here by the waves at frequencies ω_2 and ω_3) that also common to spontaneous processes. We have used the following notation: $W_1 = \frac{1}{8\pi}\omega_1 \frac{\partial \varepsilon}{\partial \omega_1} |E_1|^2 = 2\frac{\Omega_i^2}{8\pi\omega_1^2}|E_1|^2$ - is the intensity of oscillations at the frequency ω_1 and $\Delta_{-1,2,3} = (-\omega_1 + \omega_2 + \omega_3) = 3\omega_1\omega_2\omega_3 / 2\Omega_i^2$ is a frequency detuning via dispersion,

$$\alpha = \pi \cdot \frac{3\omega_1\omega_2\omega_3}{2\Omega_i^2}\frac{1}{\Delta_{\Omega 23}\sqrt{\pi}}\exp\left[-(\omega_1 - \omega_2 - \omega_3)^2 / \Delta^2{}_{\Omega 23}\right] \tag{5}$$

A similar equations can be written for the number of quanta $N_i = W_i / \hbar\omega_i$:

$$\frac{\partial N_1}{\partial t}=\alpha\frac{8\hbar}{3}\left(\frac{e}{m_i v_s}\right)^2 N_2N_3-\mathrm{Re}\frac{2\Omega_i^2 eE_2E_3E_1^*}{\pi m_i\hbar v_s\omega_2\omega_3\omega_1}+$$
$$+\frac{8\hbar}{3}\left(\frac{e}{m_i v_s}\right)^2\left[\alpha' N_1N_1+\alpha N_1(N_2+N_3)\right]. \tag{6}$$

and for the slow phase of oscillations at frequency ω_1:

$$N_1\frac{\partial\phi_1}{\partial t}=\frac{8\hbar}{6}\left(\frac{e}{m_i v_s}\right)^2 N_2N_3-\mathrm{Im}\frac{\Omega_i^2 eE_2E_3E_1^*}{\pi m_i\hbar v_s\omega_2\omega_3\omega_1}-$$
$$-\frac{8\hbar}{6}\left(\frac{e}{m_i v_s}\right)^2\left[N_1N_1-N_1(N_2+N_3)\right]. \tag{7}$$

The first term in the rhs of (6) and (7) describes the spontaneous effects, the second term governs the interaction of all three waves and the third term is responsible for stimulated self-action ($\propto N_1^2$) and cross-modulation effects, which can be obtained by direct calculation. Equation (6) can be rewritten without considering the self-action term $\propto N_1^2$ as follows (compare with (1)):

$$dW_1/dt=S+\left\{j_{23}E_1^*+j_{23}^*E_1\right\}+\frac{\partial S}{\partial(\hbar\omega)}W_1 \tag{8}$$

where $S=\alpha\frac{8}{3}\left(\frac{e}{m_i v_s}\right)^2\omega_1\frac{W_2}{\omega_2}\frac{W_3}{\omega_3}$.

Unexpectedly, ex facte, the equation for the slow phase (7) can be rewritten by the same sort. The term $\propto N_1^2$ is responsible for stimulated self-action can be obtained by such procedure [22]. Note that the first terms in (6) and (7) are of the same order as the last terms of these equations. This gives reason to believe sometimes that the physical mechanisms for which they are responsible are of the same type, which is not true. Note that spontaneous emission of nonlinear currents gives rise to terms of fourth order on the field and has no appreciable effect on the dynamics of wave processes, where the third-order terms play a dominating role. Moreover, if one knows the term of emission at the frequency ω_1 due to the interaction of the field with the nonlinear current at combination frequency $\omega_2+\omega_3$, one will find all other terms of fourth order on the field. Such fourth order terms can be decisive in determining the steady-state emission spectra of plasma systems.

3. ON THRESHOLD OF COHERENT GENERATION

According to conception, formulated by A. Einstein, the description of two level system in presence of radiation with transition frequency $\varepsilon_2-\varepsilon_1=\hbar\omega_{12}$ the following:

$$\partial n_2/\partial t=-\left(u_{21}+w_{21}\cdot N_k\right)\cdot n_2+w_{12}\cdot N_k\cdot n_1,$$
$$\partial n_1/\partial t=-w_{12}\cdot N_k\cdot n_1+\left(u_{21}+w_{21}\cdot N_k\right)\cdot n_2, \tag{9}$$

where general number of particles of system is invariable $n_1+n_2=Const$, $u_{21}\cdot n_2$ - the rate of change of quantum number on second excited level at the expense of spontaneous emission. The expressions $w_{21}\cdot N_k\cdot n_2$ and $w_{12}\cdot N_k\cdot n_1$ determine the rate of change of quantum number at the expense of induced emission and absorption, respectively. Here N_k - quantum number on transition frequency. The equation, which describes the behavior of N_k is

$$\frac{\partial N_k}{\partial t}=\left(u_{21}+w_{21}\cdot N_k\right)\cdot n_2-\left(w_{12}\cdot N_k\right)\cdot n_1\cdot \tag{10}$$

It will be recalled that the oscillator radiates with the same frequency and phase as the outer emission. Note, the outer radiation and induced emission of oscillator are appeared coherent [6,23,24] (also see [25-29]). The more intensive coherent component of outer radiation, the more energy in unit time the oscillator loses. The spontaneous emission does not depend on the outer emission and is noncoherent. Assume on qualitative level the terms in right side of equation (9), (10), which are proportional to N_k, to be responsible for coherent processes [5]. It is reasonable to submit $\mu=n_2-n_1$, $N_k=N_k^{(incoh)}+N_k^{(coh)}$ and represent the equations (9)-(10) in a form

$$2\partial n_2/\partial\tau=\partial\mu/\partial\tau=-2n_2-2\mu\cdot N_k^{(coh)};\qquad \partial N_k^{(incoh)}/\partial\tau=n_2;\qquad \partial N_k^{(coh)}/\partial\tau=\mu\cdot N_k^{(coh)}, \tag{11}$$

where $u_{21}=w_{21}=w_{12}$, $\tau=u_{21}\cdot t$. From the solution (11) one can see the threshold of coherent generation [5]

$$\mu=\mu_{TH2}=\sqrt{2N}, \tag{12}$$

where $N=n_1+n_2$ - general number of states. Let illustrate the dynamics of generation process by the instrumentality of numerical solution of equations (12) in handy shape

$$\partial \mathrm{N}_{inc}/\partial T = N_0/2; \qquad \partial \mathrm{N}_c/\partial T = \mathrm{M}\cdot \mathrm{N}_c; \qquad \partial \mathrm{M}/\partial T = -N_0 - 2M\cdot \mathrm{N}_c \tag{13}$$

where $\mathrm{N}_{inc} = N_k^{(incoh)}/\mu_0$, $\mathrm{N}_c = N_k^{(coh)}/\mu_0$, $\mathrm{M} = \mu/\mu_0$, $T = w_{21}\cdot\mu_0\cdot t = \mu_0\cdot\tau$, the single free parameter is $N_0 = N/\mu_0^2$, initial conditions: $\mathrm{M}(T=0)=1$, $\mathrm{N}_{inc}(T=0) = 0{,}001$, $\mathrm{N}_c(T=0)=0{,}001$. The results of the calculations are represented on Fig.1. The value of normalized quantities N_{inc} (solid curve 1), M (solid curve 2), N_c (solid curve 3), and the stationary solution of Eqs. (9)-(10) N_k/μ_0 (upper dotted line) and $\mathrm{M} = \mu/\mu_0$ (lower dotted line) are demonstrated in the line of Y-direction.

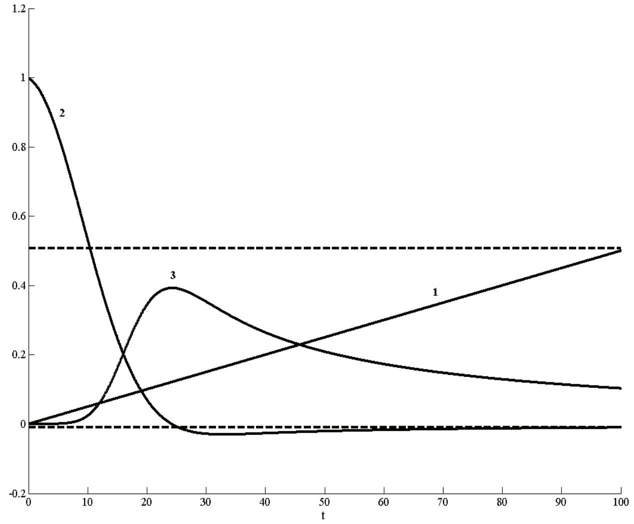

Fig.1. The dynamic behavior of normalized noncoherent quanta number N_{inc} (1), relational inversion M (2) and normalized coherent quanta number N_c (3); $N_0 = N/\mu_0^2 = 0.01$. The dashed lines - the stationary solution of Eqs. (9)-(10): N_k/μ_0 (upper dotted line) and $\mathrm{M} = \mu/\mu_0$ (lower dotted line).

In case of sufficiently large initial inversion $\mu_0 = 10\sqrt{N}$, exponentially fast ($T \propto 20$) current value of inversion tent to zero, coherent quanta number $N_k^{(coh)}$ reaches $\mu_0/2$, it is typical of super-radiation. Noncoherent component is smallish under such conditions. After that in what follows normalized coherent quanta number N_c decreases, but noncoherent quanta number N_{inc} on the contrary is increasing (Fig.1). In case of small initial inversion $\mu_0 = \sqrt{N/30}$, growing noncoherent component rapidly depletes current value of inversion and coherent field is depressed by stimulated absorption. The growth (even not exponential) of coherent component N_c below threshold (12) is impossible.

4. ON THE EMISSION AND ABSORPTION SPECTRA OF OSCILLATOR, TRAPPED IN THE POTENTIAL WELL

The important problems that arises when considering the processes of absorption and emission by a substance is the problem of interaction with the external radiation field of the oscillating particle trapped in the potential well. Suppose that a charge Q is at rest near the bottom of the outer potential well. We also assume that the recoil energy is not enough to ensure that the particle has left the potential well in which it is located. Upon emitting the high-frequency quantum $E_\nu = \hbar(\omega_0 + \Omega)$, the charged particle obtains the recoil momentum $M_Q V_Q$, becomes the excited oscillator with the proper frequency ω_0, which executes a slow periodic motion in the well. The particle motion can be represented as $x = b\sin\Omega t$. The conservation laws at this take the form:

$$\hbar(\omega_0 + \Omega)/c = M_Q V_Q, \qquad \hbar\Omega = M_Q V_Q^2/2. \tag{14}$$

Note that the energy of oscillations in the potential well is equal to $\hbar\Omega$. That is why the energy of the absorbed photon must exceed the energy of the oscillator excitation $\hbar\omega_0$ namely on this value. The frequency of oscillations in the potential well can be found from Eqs. (14)

$$\Omega \approx \hbar\omega_0^2/2M_Q c^2. \tag{15}$$

Here we assume the condition $\hbar\omega_0 << 2M_Q c^2$ is fulfilled. On the other hand, Eq. (15) implies the circle frequency of oscillations in the potential well is equal $\Omega = V_Q/b$. Since the energy of the quantum $\hbar\Omega$ is equal to the recoil energy, it follows [30]

$$\omega_0 b/c = kb \approx 2. \tag{16}$$

Note that the potential well should be significantly wider than b and the wavelength $\lambda = 2\pi/k = \pi b$ and deeper than the recoil energy (15). The highest intensity of the absorption and emission lines is observed on the eigenfrequency of the oscillator just when the recoil energy is equal to energy of the quantum of low-frequency oscillations in the potential well [10]. The Hamiltonian's correction of interaction between the oscillator and the field can be written in the form ($Q = e, M_Q = m$)

$$\mathrm{H}' = -e\cdot v_x A_x/c. \tag{17}$$

The interaction energy (17) is proportional to the product of the velocity of the oscillator on the field at the point where it is located. Note, $v_x = v_{x0}\cdot cos\omega_0 t$ - velocity of the oscillator , $A_x = \sqrt{2}\cdot q_0 Cos((\omega \pm \Omega)t)\cdot Cos\{kb\sin\Omega t + \delta\}$ - component of the vector potential. Minus sign corresponds to absorption, when the energy of the absorbed quantum must exceed the value of the excitation energy on the value of the recoil energy (14). Plus sign corresponds to emission when the energy of the emitted quantum must be less than the value of the excitation energy on the value of the recoil energy. The interaction energy under condition is nonzero at $\omega = \omega_0 + (m \mp 1)\Omega$:

$$\mathrm{H}' = -\frac{e \cdot v_{x0}}{c} q_0 \sqrt{2} \cdot \sum_m J_m(kb) \cdot \cos\delta. \tag{18}$$

Authors of [10] consider the cases of absorption ($m = -1$) and emission ($m = +1$) on the proper frequency (eigenfrequency) of the oscillator ω_0 and when the oscillator is not at rest on the bottom of the potential well. That is the frequencies of the absorbed and emitted field are the same and equal to ω_0. The probability of transition with the emission (upper sign) and with absorption (lower sign) on the proper frequency of the motionless oscillator ω_0

$$P_{if} = \frac{8\pi e^2}{hc^3} \omega_0^2 (|x_{ab}|^2 + |y_{ab}|^2) \cdot J_1^2(kb) \cdot Cos^2\delta \cdot \begin{Bmatrix} n+1 \\ n \end{Bmatrix}. \tag{19}$$

Note that under condition (16), the intensity of the absorption and emission lines at the frequency ω_0 in $J^2{}_1(kb) / J^2{}_0(kb)$ times exceeds the intensity of spectral lines $\omega = \omega_0 \pm \Omega$.

5. MODULATION INSTABILITY OF GRAVITY WAVES IN OCEAN WITH DAMPING AND FORCING

Consider the modulation instability of externally driven wave in a medium with sufficiently strong dispersion and weak dissipation, which can be observed in plasma wave-guides, as well as on the water surface and other physical situations. We use below the following dispersion relation that characteristic, as an example, of gravity waves on deep water [31]:

$$\omega = \sqrt{gk}\left\{1 + A^2k^2/2 + ...\right\}, \tag{20}$$

where g is some dimensional coefficient (for the gravity waves on deep water it is the acceleration of gravity).

Experimental data for ocean waves give us the following characteristics [32]: the maximal steepness of stable waves = $H/\lambda = 0.13 - 0.14$, where $H \propto 2A$ and λ are the vertical distance between the wave crest and the deepest trough preceding or following the crest and wave-length correspondingly. It follows from here that $Ak < 1$. Denote frequency ω_0, the average wave amplitude as $|A_0|$, the average wave height as $\bar{H} = 2|A_0|$. The gravity waves with $H = (2 \div 3) \cdot 2|A_0|$ are considered as extremely high. It follows from here that $(4 \div 6)|A_0| k_0 / 2\pi \propto 0.13$ and it is easy to see that the width of spatial spectrum of the instability in this case is not so small. The simplified representation [19-21], which describes only in initial stage of nonlinear regime of modulation instability, one takes into consideration to the following diagrams $2\omega_0 = \omega(k) + \omega(-k)$ and interaction by pairs of waves symmetric with respect to the pump $\omega(k) + \omega(-k) = \omega(k') + \omega(-k')$. Introducing real amplitudes and phases $A_K = |u_K| \exp(i\varphi_K)$, we have obtained the system of equations, which describes the modulation instability in a medium with strong dispersion [21]:

$$\frac{\partial u_K}{\partial \tau} = -\delta u_K + (1+K)^{2.5}\left[u_{-K}u_0^2 \sin\Phi_K + u_{-K}\sum_{\mathrm{K} \neq K,0} u_{\mathrm{K}} u_{-\mathrm{K}} \sin(\Phi_{\mathrm{K}} - \Phi_K)\right], \tag{21}$$

where $\Phi_K = 2\varphi_0 - \varphi_K - \varphi_{-K}$ is the total phase (or the phase of the instability channel). A distinction needs to be drawn between modes with wave numbers K and K and phases Φ_K and $\Phi_{\mathrm{K}} = \varphi_K + \varphi_{-K} - \varphi_{\mathrm{K}} - \varphi_{-\mathrm{K}}$.

$$\frac{\partial \varphi_K}{\partial \tau} = -\frac{2}{\alpha}\left(\sqrt{(1+K)} - 1\right) - (1+K)^{2.5}\left[2u_0^2 + u_K^2 + 2\sum_{K' \neq K,0} u_{K'}^2 + \frac{u_{-K}}{u_K}u_0^2 \cos\Phi_K + \frac{u_{-K}}{u_K}\sum_{\mathrm{K} \neq K,0} u_{\mathrm{K}} u_{-\mathrm{K}} \cos(\Phi_{\mathrm{K}} - \Phi_K)\right], \tag{22}$$

where the mode frequencies are $\omega(K) - \omega_0 = \omega[k_0(1+K)] - \omega(k_0) = \sqrt{gk_0(1+K)} - \sqrt{gk_0}$. We use the following notations $\omega_0 t/2 = \tau/\alpha$, $\alpha = k_0^2 |A_0|^2$, $K = (k - k_0)/k_0$, $A_K / A_0(\tau = 0) = a_K = u_K \exp\{i\varphi_K\}$, and also $\Delta_K = 2\left(\sqrt{(1+K)} - 1] + \sqrt{(1-K)} - 1\right)/\alpha$, $\mathrm{P}_K = 2(1+K)^{2.5} + 2(1-K)^{2.5} - 2.$

In order to analyze the wave height distribution (e.g. the distribution of vertical distances between the wave crest and the deepest trough preceding or following the crest), we take a third of highest waves. Then we find the average height of all waves U_{CP}, average height of a highest third U_{SWH} and the maximum wave height U_{MAX} in consideration domain ($\zeta \subset L = 2\pi/(\Delta K/k_0) = \pi N/K_m = \pi N/\sqrt{2\alpha}$, where $\Delta K = 2K_{\max}/N$, $\zeta = k_0 x$, x - spatial value). Calculations were performed for 600 modes in the spectrum. The ratio of dissipation level δ to the maximum growth rate was chosen as 0.1 (e.g. $\delta = 0.1$). In order to provide the unit amplitude of the fundamental wave at the initial stage of the instability we have also chosen the value of the external drive force as $G = \delta = 0.1$. The results of that calculations are represented on Fig.2.

Note that the criterion which defines the extremely high waves with amplitude U_{AG} is $U_{AG} > 2U_{SWH}$ or something like this, should be used with caution because this criterion as usual is applied to statistics obtained on sufficiently large observation periods, but the highest amplitudes are observed at the initial stage of the instability.

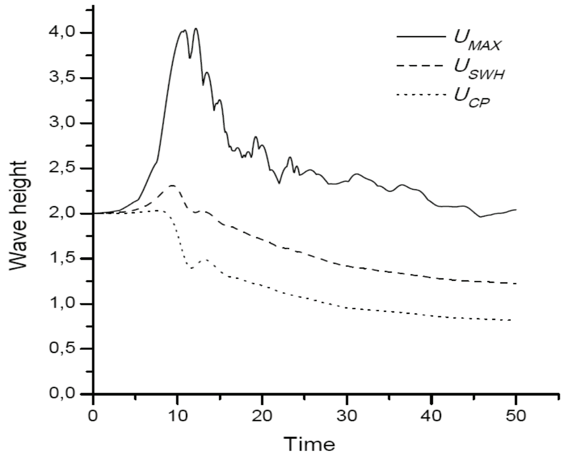

Fig. 2. Time evolution of wave height parameters: U_{CP} – the average height of all waves in the observation domain, U_{SWH} – the average height of a highest third, U_{MAX} – the maximum wave height [22].

The considered model shows that the appearance of waves with abnormally large amplitudes is characteristic of the initial stage of the modulation instability. The average and maximum wave heights noticeably decrease with the development of the instability. However, even in the later case ($\tau \propto 30$) abnormally high waves can be detected according to the criterion $U_{AG} > 2U_{SWH}$, although their amplitude is already one and a half - two times less than in the most interesting case ($\tau \propto 10$). There are few waves fits at the modulation length in the initial stage of the process and one of which may be abnormally large in some realizations (that result [21] comes to agreement with observed data [15,33]). At the stage of developed instability, the number of waves for the modulation length increases to three or four times. The probability of abnormally high wave events in our numerical simulations is qualitatively consistent with the known observations of ocean waves. It should be noted that the total energy of wave motion is approximately conserved with reasonable accuracy in the domain under consideration (both in time and space) and the condition that the amplitudes of individual side-band modes remain much smaller of the fundamental wave amplitude is satisfied during the entire simulation time [21]. The latter allows us to assume such a description of the modulation instability sufficiently correct.

6. THE LIMITATION MECHANISM OF THE DENSITY CAVITY DEEPING ON MODULATION INSTABILITY OF INTENSIVE OSCILLATION IN PLASMA

Let us consider the behavior of electrons ($\alpha = e$) and ions ($\alpha = i$) of cold plasma in the electric field of intensive Langmuir oscillation $E = E_0 \cdot Sin(\omega_0 t - k_0 x + \varphi)$ with the frequency ω_0 near Langmuir frequency $\omega_{pe} = \sqrt{4\pi e^2 n_0 / m_e}$ where e – charge, m_α - mass of a particles, E_0 , φ - slowly varying amplitude and phase of the field. The generalized 1D Silin's equations, which are describing the modulation instability [34] (see also [35]) of intensive oscillations with excitation of wave spectra (the wavelength of which is $2\pi / nk_0$, where $n \le 100$) are

$$\partial N_n / \partial \tau - i\Delta N_n - iM_n \cdot J_1(na)/2n = 0.5i\sum_m M_m \cdot [N_{n-m} \cdot J_0(ma) - N^*_{n-m} \cdot J_2(ma) \cdot \exp(2i\phi)], \tag{23}$$

$$\partial[a \cdot \exp(i\varphi)] / \partial \tau = 2i\sum_s M_s \cdot [J_0(sa)N_s + J_2(sa) \cdot N^*_s \cdot \exp(2i\varphi)], \tag{24}$$

$$\begin{aligned} \frac{d^2\xi}{d\tau^2} = \frac{\delta}{2\pi}\sum_n \{2J_1(na) \cdot \mathrm{Re}[N_n \exp\{-i\varphi\}] + \\ +n \cdot \sum_m J_0(na_n) N^*_m N_{n-m} - J_2(na)\mathrm{Re}(N^*_m N^*_{n-m} \exp\{-i\varphi\})\} \cdot \sin(2\pi n\xi) \end{aligned}, \tag{25}$$

where $\delta \cdot t = \tau$; $\delta = (m_e / m_i)^{1/3}$; $a_n = a \cdot n = n \cdot ek_0 E_0 / m_e \omega_0^{\ 2}$; $N_n = u_n / enn_0$ - relative disturbance of electron and $M_n = \delta^{-1} \cdot \int_{-0.5}^{0.5} d\xi_0 \cdot \cos(2\pi n\xi)$ - ion densities, $J_m(x)$ - Bessel function, $\Delta_n = \Delta_0 + \beta n^2 = -\delta^{-1}(1 - \omega_{pe}^2/\omega^2) + 3k_0^2 T_e / 4\delta m_e \omega^2$ - detuning; T_e - electron temperature in units of energy. Let us assume the condition $E_0^2 / 4\pi n_0 T_e = W / n_0 T_e >> 1$ is satisfied. The result of the solution of equations (23)-(25) is represented in [34] on conditions that $a(\tau = 0) = k_0 e E_0(\tau = 0) / m_e \omega_0^{\ 2} = v_0 / c = 0.06$; $n \le 100$, $\delta = 0.1$, which answer the requirements the high-current electron-beam excitation.

The energy of intensive long-wave ($\lambda_0 = 2\pi / k_0 = 2\pi c / \omega_{pe}$) oscillation in plasma reaches several tens of percent of total beam energy. The maximum value of modulation instability increment $\sim \delta \cdot \omega_0 \cdot J_1(na)$ is achieved when $an \sim 2$. It follows from this that the number of spectra modes $n \ge 30$. The electric field amplitude of mode is $E_n \sim N_n \omega_0^{\ 2} m_e / k_0 e$. The numerical experiment [34] shows $N_n \sim 10^{-2}$. The electrons on a scale of intensive oscillation length ($\lambda_0 = 2\pi / k_0 = 2\pi c / \omega_{pe}$) practically are not captured by the plasma density cavity. Therefore the spectra growth transmits on the modes with numbers $n_{\max} \sim 60$. HF energy flux becomes formed just because increment maximum moves in short-wave region $n \sim 2/a$ on account of intensive long-wave oscillations amplitude is decreasing.

The half of initial energy approximately is transferred in the short-wave HF spectra energy. The ratio of ion oscillation frequency at the bottom of potential well of plasma density cavity to linear increment of modulation instability is

$$\sqrt{ek_0\sum_s sE_s / m_i\omega_0^2\delta^2} = 10^{-1} n_{\max}\delta^{1/2} / 2 \sim 1 \cdot \qquad (26)$$

There is the fulfillment of conditions of ions capture. This fact explains the choice of ion kinetic description (25). The ions capture occurs on a small scale $\pi / n_{\max} k_0$, and the ion oscillation frequency at the bottom of cavity on the order of linear increment of modulation instability. Therefore at the bottom of cavity ion velocity is $\sim \pi\delta\omega_0 / n_{\max} k_0 \sim 0.05 \cdot c\delta$ and its energy considerably exceeds initial energy of electron. The major portion of ions on periphery of the cavity and a great distance away from it is moving in opposite direction, "fill up" the cavity, at the same time disintegrate previous interaction between the Langmuir spectrum and the ion oscillations. However, summarized energy of ions is remained substantially least of the initial total energy of system [34-37].

CONCLUSION

The purpose of that paper is to announce new theoretical description of following well-known physical phenomena.

The nonlinear current (2), which generated by the fields with frequencies ω_2, ω_3, radiates the wave with frequency ω_1 and this emission does not depend on the external electromagnetic field at the same frequency. That features is typical ones for emitter of spontaneous radiation. If such emitter characteristics are known, it is not difficult to determine all components of the induced radiation with frequency ω_1, which is proportional to the intensity of the spectrum line. Such procedures are applied to the phase expression.

If the induced radiation is believed an coherent radiation, and spontaneous radiation is random process, very low threshold and pulse shaping of coherent (maser) radiation can be find out. The pulse has sharp rise-up portion and very lengthy descending part.

The high-frequency oscillator in the external potential well is capable of linear spectrum emission. Let emission and absorption of HF quantum accompany the recoil effect. When the recoil energy is equal to energy of the quantum of low-frequency oscillations in the potential well, the highest intensity of the absorption and emission lines is observed on the eigenfrequency of the HF oscillator. That assertion is consequence of the quantum description, which is discussed in [10] and in that paper.

The theory of the nonlinear stage of modulation instability based on excites the spectrum satisfying the conditions of space-time synchronism of the form $2\omega_0 = \omega(k) + \omega(-k)$ and interaction by pairs of waves symmetric with respect to the pump $\omega(k) + \omega(-k) = \omega(k') + \omega(-k')$ is reviewed. That description is applied to analysis of sea waves instability and detects gravity surface waves with abnormally high amplitude, that occurs only in initial stage of nonlinear regime of that process. The high coincidence of theoretical results and observed data is succeed.

The plasma density cavity is formed as a result of the modulation instability of intensive long-wave Langmuir oscillations. The instability scales down of density cavity. There is no hydrodynamic mechanisms to stabilize the dimension of density cavity, except the dispersion and the damping of short-wave Langmuir spectrum. But often cavity dimension in hydrodynamic description comes not nearer to that critical scale [34-36, 38]. However there is the kinetic limitation mechanism of cavity deepening. The potential well bottom of cavity is capable of ions capture and the cavity will be not become deeper. The condition of ions capture (26) is determined the minimum dimension of cavity.

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