Numerical methods in blood flow modelling in the complex systems of distensible tubes with arbitrary geometry

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Abstract

A 1000-tube virtual model of human systemic arterial tree that includes the database on geometry of the arterial vasculatures, the software for computation and visualization of the blood flow and pulse wave propagation, diagnostic analysis of the pulse wave parameters and planning the cardiovascular surgery is elaborated. Blood flow in each tube is modeled as a sum of the Windkessel and pulsatile components. Pulse wave propagation and reflection at the bifurcations are considered as 2d waves in the fluid filled thick wall viscoelastic tubes. The results of the numerical computations of the pressure and flow wave evolution along aorta, simulations of the blood vessel occlusion, stenosis, aneurism and addition some collateral pathways as a model of arterial and venous grafts are presented.

Keywords: computational mechanics, blood flow, pulse wave, wave-intensity analysis, medical diagnostics

1. Introduction

Blood flow and pressure distribution, wave propagation and reflection in the blood vessel systems are important determinants of human hemodynamic. Important diagnostic information on blood circulation in the systemic arteries and on microcirculation in the inner organs can be obtained from the pressure P(t) and flow U(t) curves measured non-invasively by ultrasound technique or MRT. Data mining methods and biomechanical interpretation of the parameters are important for early diagnostics of different pathology.

The blood flow along the vasculature can be described by 1D [6-8] or 2D models [9-11] of the fluid flows in distensible tubes. The flow and wall equations are considered in each tube and the flow and pressure continuity equations are applied in each bifurcation to close the system [12]. The 1D model is presented by a hyperbolic equation and can be solved by the method of characteristics [13], which has been developed for the complex branching system of distensible tubes [14] and the corresponding numerical code has been elaborated and tested in the 55-tube model of human systemic circulation and some other simpler systems [15-17]. Basing on the realistic model of the vasculature different circulatory pathologies like hypertension, atherosclerosis, thrombosis, hyperlipidemia, stenosis, aneurism, microcirculatory disorders etc, can be modelled as high/low wall rigidity, occlusion/dilation of separate tubes, variations in the blood viscosity, reflection conditions at the terminuses and other model parameters [18].

2. Materials and Methods

Individual geometry of the inraorgan vasculatures has been studied on plastic casts of the inner organs and muscles. The lengths, diameters and branching angles of the arterial segments have been measured on five sets taken from corpses of young healthy people dead in accidents [1]. The parameters of the large extraorgan (systemic) arteries have been measured by post-mortem examination of the five corpses. Besides, the detailed ultrasound measurements on ten healthy individuals have been carried out. Geometry of the vasculatures has been described as a graph following topology of the vasculatures. As a result a dataset including 920-1150 arterial segments of the systemic arteries depending on individual geometry have been obtained. A comparative study of the datasets revealed that 880 segments were common for the five corpses and others are different. The “minimal” model containing 880 segments with lengths and diameters averaged over the five datasets and a “maximal” model containing all the segments presented in at least one corpse have been built. The terminal tubes in the models have been terminated by the models of the intraorgan and intramuscle vasculatures. Each of the terminal models included some 900-9000 tubes. The obtained virtual physiological human model allows numerical computations and modelling of different normal sates and pathologies.

The very first model of human systemic arterial tree based on the parameters of 55 large arteries terminated by Windkessel (lumped parameter) models has been obtained on corpses [2]. Later the dataset has been completed by some more arteries. The model presented here is the most complete and detailed at present.

3. Regularities in design of the arterial systems

Statistical analysis of the measurement data revealed some obvious regularity in construction of the blood vessel systems. In the bifurcations the parent (j=0) and daughter (j=1,2) vessels have been considered. The following parameters have been computed:

- branching asymmetry $\xi_j = \min\left(d_j^1, d_j^2\right) / \max\left(d_j^1, d_j^2\right)$;
- branching coefficient $K_j = \left((d_j^1)^2 + (d_j^2)^2\right) / (d_j^0)^2$;
- Murray’s optimality coefficient $\mu_j = \left((d_j^1)^3 + (d_j^2)^3\right) / (d_j^0)^3$;
- hydraulic conductivity $Y_j^h = 128\eta(d_j)L_j / \left(\pi d_j^4\right)$;
- wave input admittance $Y_j^{in} = \pi d_j^2 / \left(4\rho_f c_j\right)$,

where η and ρ_f are viscosity and density of blood, c_j is the pulse wave velocity, L_j and d_j are lengths and diameters of the vessels.

The dependences $d_{\min}(d_0)$ and $d_{\max}(d_0)$ corresponding to the Murray law [3] have been found in the systemic arterial tree as well as in the intraorgan vasculatures. The dependences $d_{\max} = \alpha d_0^{\beta}$ have been found for all the datasets, where $\alpha = 0.883; \beta = 0.99; R^2 = 0.915$ for the database measured on the corpses and $\alpha = 0.756$, $\beta = 1.05$, $R^2 = 0.902$ - for the database measured on the alive individuals. The difference is connected with maximal dilated state of the cadaveric blood vessels. Different families of vessel bifurcations (with K>1, K~1, K<1) as well as bifurcation asymmetries ($\xi < 1, \xi \le 1, \xi = 1$) have been found. The large extraorgan arteries are closer to optimal ones ($\mu = 3$) providing the blood delivery at total minimal energy costs (Murray law) and zero wave reflection (optimal waveguide) than the small arteries. The relationships $L = \alpha d^{\beta}$ are different for the inner organs and muscles. Using the obtained regularities a mathematical algorithm allowing reconstruction of an individual vasculature basing on the parameters of the feeding artery of an inner organ/muscle has been elaborated [4].

4. Mathematical Model

Blood flow in each tube is modeled as a sum of the Windkessel and wave components [5]. The Windkessel pressure $p_W(t)$ and flow rate $Q_W(t)$ are determined by the lumped parameter model

$$k\frac{d}{dt}p_W - Y_W p_W = Q_{in}(t), \quad Q_W = Y_W p_W \tag{1}$$

where k is the wall compliance, Y_W is the tube conductivity, $Q_{in}(t)$ is the inflow rate. For the network of tubes the equations (1) are considered for each tube taking into account the pressure and flow rate continuity conditions at the tube bifurcations.

Axisymmetric pulse wave propagation in the fluid-filled tubes is studied on the incompressible Navier-Stokes equations for the fluid and incompressible viscoelastic solid for the wall

$$div(\vec{v}) = 0, \ \rho_f \frac{\partial \vec{v}}{\partial t} = -\nabla p + \eta \Delta v \tag{2}$$

$$\rho_s \frac{\partial^2 \vec{u}}{\partial t^2} = div\hat{\Sigma}, \ \hat{\Sigma} = \hat{\sigma} - p_s \hat{I}, \ \tau_1 \frac{\delta\hat{\sigma}}{\delta t} + \hat{\sigma} = 2G\left(\hat{e} + \tau_2 \frac{\delta\hat{e}}{\delta t}\right)$$

where $v = (v_r, 0, v_x)$, p are fluid velocity and pressure, $u = (u_r, 0, u_x)$, ρ_s, p_s, $\hat{e}$, G, $\tau_{1,2}$ are solid displacement, density, pressure, strain rate tensor, shear modulus and relaxation parameters. The boundary conditions include velocity and stress continuity conditions at the fluid0solid interface, zero displacement or zero stress at the outer surface of the tube, flow and pressure continuity conditions at the tube bifurcations [6].

Solution of the linearized problem (2) has been found as a superposition of the incident and reflected waves

$$p_i(t, x_i) = p_i^+ + p_i^- = p_i^0 (e^{i\omega(t - x_i/c_i)} + \Gamma_i e^{i\omega(t + (x_i - 2L_i)/c_i)})$$

$$v_{xi}(t, x_i) = v_i^+ + v_i^- = Y_i^0 p_i^0 (e^{i\omega(t - x_i/c_i)} - \Gamma_i e^{i\omega(t + (x_i - 2L_i)/c_i)})$$

where $\Gamma_i = P_i^- / P_i^+$ is the wave reflection coefficient, P_i^+ and P_i^- are amplitudes of the incident and reflected waves.

5. Numerical results and discussions

Computations on the five measured cadaveric models as well as on the minimal 880-tube and maximal 1250-tube models have been carried out using (1) and (2). The solutions $P_i(t, x_i) = p_W(t) + p_i(t, x_i)$ and $Q_i(t, x_i) = Q_W(t) + \pi(0.5d_i + u_{xi}(t, x_i))^2 v_{xi}(t, x_i)$ have been computed for each tube. The wave reflection coefficients Γ_i have been computed using the data measured on the intraorgan and intramuscle vasculatures. Depending on geometry and terminal conditions (microcirculation) both positive $\Gamma > 0$ and negative $\Gamma < 0$ reflection conditions have been found and used for computations.

For all the studied systemic trees a realistic pressure and flow rate waves travelling along aorta [7] have been obtained. The flow waves computed in the middle cross-sections of different large arteries have been compared to the flow curves measured with Doppler ultrasound device on ten healthy volunteers and a very good correspondence have been found.

Modelling of stenosis/aneurism by decreasing/increasing of diameters of some arteries; modelling of the bypass surgery by addition of tubes in the system; modelling of the intraorgan pathology by variation the resistivity and compliance of the vasculature resulting in changes in Y_j^h and Y_j^{in} have been carried out. It was shown any mismatch in the hydraulic and wave conductivity produces additional reflected waves. Decomposition of the waves into the incident and a series of reflected waves allows estimation of the time delay between the wave fronts. Multiplying the time delay by the wave speed one can compute the distance to the reflection site and determine the pathology. Analysis of the pressure-flow curves $P(Q)$, phase curves $P_t^/(P)$, $Q_t^/(Q)$ and intensities $dI^{\pm} = dP^{\pm}dQ^{\pm}$ of the incident and reflected waves give very important information for medical diagnostics of cardiovascular and microcirculatory pathology. Basing on the computational results some novel integral parameters for differential clinical diagnostics are proposed. The model can be used for testing the treatment and planning the surgery and rehabilitation.

References

[1] Zenin, O.K., Kizilova, N.N. and Philippova E.N., Studies on the Structure of Human Coronary Vasculature, *Biophysics*, 52, pp.499–503, 2007.

[2] Westerhof, N., Bosman, F., de Vries, C.J. and Noordegraaf, A., Analog studies of the human systemic arterial tree, *J. Biomech.*, 2, pp.121-143, 1969.

[3] Rosen, R., *Optimality Principles in Biology*, Plenum Press, New York, 1967.

[4] Kizilova, N.N., Computational approach to optimal transport network construction in biomechanics, *Lecture Notes in Computer Sci*., 3044, pp.476-485, 2004.

[5] Tyberg, J.V., Davies, J.E., Wang, Zh., et al, Wave intensity analysis and the development of the reservoir–wave approach, *Med. Biol. Eng. Comput*., 47, pp.221-232, 2009.

[6] Kizilova, N.N., Pressure wave propagation in liquid-filled tubes of viscoelastic material, *Fluid Dyn*., 41, pp.434-446, 2006.

[7] Milnor, W.R., *Hemodynamics*, Williams and Wilkins, 1989.