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# ELECTROMAGNETIC WAVE DIFFRACTION BY METAL CYLINDER COATED WITH INHOMOGENEOUS MAGNETOACTIVE PLASMA SHEATH

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In our paper the task of diffraction of low frequency electromagnetic waves incident from a homogeneous magnetoactive plasma by a metal circular cylinder surrounded by radially inhomogeneous plasma sheath have been studied. The constant external magnetic field is parallel to a cylinder axis; the wavevector of incident plane wave is perpendicular to this axis. Plasma is cold and perturbations in it are governed by two-liquid hydrodynamic equations and Maxwell equations. On the basic of the exact solutions for fields in plasma sheath with linear profile of inhomogeneity the cross-section are obtained. The angular distribution of cross-sections is derived for various values of incident wavelength. It is shown that it is possible to control effectively this angular distributing by the change of wave frequency and strength and direction of the constant external magnetic field.
**KEY WORDS:** electromagnetic wave, diffraction, magnetoactive plasma, non-uniform plasma, cross-section, angular distributing.

### РОЗСІЯННЯ ЕЛЕКТРОМАГНІТНИХ ХВИЛЬ НА МЕТАЛЕВОМУ ЦИЛІНДРІ, ОТОЧЕНОМУ ШАРОМ НЕОДНОРІДНОЇ МАГНІТОАКТИВНОЇ ПЛАЗМИ

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У нашій роботі розв’язано задачу дифракції низькочастотної електромагнітної хвилі, що падає з однорідної магнітоактивної плазми на металевий циліндр, оточений радіально неоднорідним шаром плазми. Стале зовнішнє магнітне поле є паралельним до вісі циліндра, хвильовий вектор падаючої плоскої хвилі - перпендикулярний до цієї вісі. Плазма холодна, і збурення в ній описуються за допомогою дво-рідинних гідродинамічних рівнянь та рівнянь Максвела. На основі отриманих точних рішень для полів у межах плазмового шару з лінійним профілем неоднорідності отримано вираз для перерізу розсіяння. Для різних величин хвильового вектора падаючої хвилі отримано кутовий розподіл перерізу розсіяння. Показано, що зміною частоти хвилі та величиною та напрямом сталого зовнішнього магнітного поля можна ефективно керувати цим кутовим розподілом.
**КЛЮЧОВІ СЛОВА:** електромагнітна хвиля, дифракція, магнітоактивна плазма, неоднорідна плазма, переріз розсіяння, кутовий розподіл.

### РАССЕЯНИЕ ЭЛЕКТРОМАГНИТНЫХ ВОЛН НА МЕТАЛЛИЧЕСКОМ ЦИЛИНДРЕ, ОКРУЖЕННОМ СЛОЕМ НЕОДНОРОДНОЙ МАГНИТОАКТИВНОЙ ПЛАЗМЫ

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В нашей работе решена задача дифракции низкочастотной электромагнитной волны, которая падает из однородной магнитоактивной плазмы на металлический цилиндр, окруженный радиально неоднородным слоем плазмы. Постоянное внешнее магнитное поле является параллельным к оси цилиндра, волновой вектор падающей плоской волны - перпендикулярен к этой оси. Плазма холодная, и возмущения в ней описываются с помощью двужидкостных гидродинамических уравнений и уравнений Максвелла. На основе полученных точных решений для полей внутри плазменного слоя с линейным профилем неоднородности получено выражение для сечения рассеяния. Для разных величин волнового вектора падающей волны получено угловое распределение сечения рассеяния. Показано, что изменением частоты волны и величиной и направлением постоянного внешнего магнитного поля можно эффективно управлять этим угловым распределением.
**КЛЮЧЕВЫЕ СЛОВА:** электромагнитная волна, дифракция, магнитоактивная плазма, неоднородная плазма, сечение рассеяния.

Diffraction of electromagnetic wave by a metal cylinder embedded into magnetoactive plasma is of interest for solving a number of problems concerning with a metal object influence on the electromagnetic wave propagation in various plasma-like media. Models of such media are used very often in study from space to nano technologies.

Electromagnetic scattering by a metal cylinder immersed into homogeneous plasma or/and coated with homogeneous sheaths has been studied in many works [1,2]. Very often the different competitive processes give rise to inhomogeneous sheaths.

Aim of our paper is to study of an angular dependence of electromagnetic fields scattered by metal cylinder coated with inhomogeneous sheath being in homogeneous magnetoactive plasma.

## STATEMENT OF THE PROBLEM

We continue [3,4] to solve the problem of diffraction of low frequency $(\omega < \omega_i)$ magnetohydrodynamic (MHD) waves propagating in a homogeneous magnetoactive plasma ($r > \rho$) on a metal circular cylinder of radius $R_c$, surrounded by radially inhomogeneous plasma sheath $(1 < r < \rho)$; the radius $r$ is normalized by $R_c$. The external magnetic field $\vec{H}_0$ is parallel with a cylinder axis; the wavevector of incident plane wave is perpendicular to this axis. It is assumed that plasma is cold and disturbances in plasma are governed by Maxwell's and two-liquid hydrodynamic equations.

Let choose the linear profile of plasma density $N(r)$ inside the sheath in the form (Fig.1)

$$N(r) = A + B \cdot r \ . \tag{1}$$

where $N(r) = n(r)/n_0$; $n(r)$ - the unperturbed plasma densities in the inhomogeneous regions, $n_0$ - undisturbed density of the homogeneous plasma.

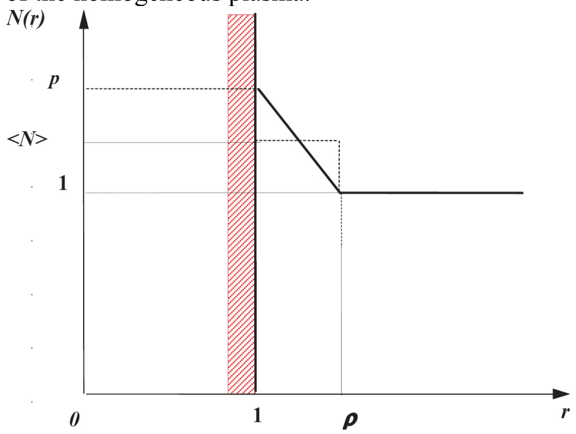


Fig.1. The normalized plasma density versus the normalized radius.

The incident plane low frequency $(\omega < \omega_i)$ wave has such components (TM -polarization)

$$H_z^{(i)} = E_0 \exp(-ik_T r) \ ;$$
$$E_x^{(i)} = -(\omega_i/\omega)(k/k_T) H_z^{(i)} \tag{2}$$
$$E_y^{(i)} = -(k/k_T) H_z^{(i)} \ ,$$

where $E_0$ is amplitude, $\omega_i = eH_0/m_i c$ is the ion gyro-frequency, $e$, $m_i$ - are ion charge and mass, respectively, $H_0$ - value of external magnetic field, $c$ - speed of light in vacuum. The factor $\exp(-i\omega t)$ was omitted because incident wave is assumed monochromatic.

Varying $A$ and $B$ it is possible to describe both layers with overdenced and rarefied adjacent plasma sheath near-by a metallic cylinder. Here $\langle N \rangle = \frac{1}{\rho - 1} \int_1^{\rho} N(r) dr$ .

The linear dispersion relation at low frequencies $\omega < \omega_i$ in a dense $\left(\Omega_{i0}^2 >> \omega_i^2\right)$ plasma for waves propagated in a homogeneous magnetoactive plasma perpendicular the external magnetic field $\vec{H}_0$ has a form

$$k_T = (\omega/c)(\Omega_{i0}/\omega_i) \ , \tag{3}$$

where $k_T$ - wavevector of incident plane wave, $\Omega_{i0} = \left(4\pi e^2 n_0/m_i\right)^{1/2}$ - ion Langmuir frequency.

Standard representation of a continuous plane MHD wave propagating in a homogeneous region $(r > \rho)$ via Bessel functions is :

$$H_z^{(i)} = E_0 \exp(-ik_T r) = E_0 \cdot \sum_{m=-\infty}^{\infty} (-i)^m J_m(k_T r) e^{im\varphi} \ ,$$
$$E_y^{(i)} = -\frac{(\omega/c)}{k_T} \cdot \sum_{m=-\infty}^{\infty} (-i)^m J_m(k_T r) e^{im\varphi} \ ; E_x^{(i)} = -(\omega_i/\omega)(k/k_T) E_0 \sum_{m=-\infty}^{\infty} (-i)^m J_m(k_T r) e^{im\varphi} \ . \tag{4}$$

We will assume that normally incident plane wave (2) generates a divergent cylindrical wave in result of diffraction on the given structure. It is possible to consider this wave as the locally plane wave of *TM*- polarization far from a cylinder. The diffraction field components in the homogeneous plasma region may be chosen in the following form :

$$H_z^{(s)} = E_0 \sum_{m=-\infty}^{\infty} (-i)^m a_m H_m^{(1)}(k_T r) e^{im\varphi} \ ,$$

$$E_r^{(t)} = \frac{-E_0}{k_T \sqrt{N(r)}} \sum_{m=-\infty}^{\infty} (-i)^m e^{im\varphi} a_m \left[ \frac{m}{r} H_m^{(1)}(k_T r) + k_T \frac{\omega}{\omega_i} \frac{dH_z^{(s)}(k_T r)}{dr} \right], \tag{5}$$

$$E_\varphi^{(t)} = \frac{-iE_0}{k_T \sqrt{N(r)}} \sum_{m=-\infty}^{\infty} (-i)^m e^{im\varphi} a_m \left[ \frac{m}{r} \frac{\omega}{\omega_i} H_m^{(1)}(k_T r) + k_T \frac{dH_z^{(s)}(k_T r)}{dr} \right],$$

where $H_m^{(1)}(k_T r)$ - are the Hankel functions of first kind.

Let us to express the fields inside the inhomogeneous sheath $(1 < r < \rho)$ via two linearly-independent solutions of equation (6):

$$\frac{d^2F_m}{dr^2}+\left[\frac{1}{r}-\frac{d}{dr}\ln N(r)\right]\frac{dF_m}{dr}+\left[k_T^2N(r)-\left(\frac{m}{r}\right)^2-\frac{m}{r}\frac{\omega}{\omega_i}\frac{d}{dr}\ln N(r)\right]\cdot F_m=0\,, \tag{6}$$

$$H_z^{(t)}=E_0\cdot\sum_{m=-\infty}^{\infty}(-i)^m\cdot e^{im\varphi}\cdot\left[b_m\cdot F_{1m}(r)+c_m\cdot F_{2m}(r)\right],$$

$$E_r^{(t)}=\frac{-E_0}{k_T\sqrt{N(r)}}\sum_{m=-\infty}^{\infty}(-i)^m e^{im\varphi}\left\{b_m\left[\frac{m}{r}F_{1m}(r)+\frac{\omega}{\omega_i}F'_{1m}(r)\right]+c_m\left[\frac{m}{r}F_{2m}(r)+\frac{\omega}{\omega_i}F'_{2m}(r)\right]\right\},$$

$$E_\varphi^{(t)}=\frac{-iE_0}{k_T\sqrt{N(r)}}\sum_{m=-\infty}^{\infty}(-i)^m e^{im\varphi}\left\{b_m\left[\frac{m}{r}\frac{\omega}{\omega_i}F_{1m}(r)+F'_{1m}(r)\right]+c_m\left[\frac{m}{r}\frac{\omega}{\omega_i}F_{2m}(r)+F'_{2m}(r)\right]\right\}.$$

## RESULTS

Let's find the solutions of equation (6) with help of Frobenius method [5]:

$$F_{1m}(r)=\sum_{l=0}^{\infty}\alpha_l r^l \quad,\quad F_{2m}(r)=a\cdot\ln r\cdot F_{1m}(r)+\sum_{k=0}^{\infty}\beta_k r^{-m+k} \tag{7}$$

For all $i<0$ $\alpha_i=\beta_i=0$ ; $\alpha_0=\beta_0=\beta_{2m}=1$, and all others coefficients will find from recurrent relations

$$\alpha_l\left[A\cdot l\cdot(l+2m)\right]+\alpha_{l-1}\cdot B\left[(m+l-1)\left[(m+l-2)-m^2-\mu\right]\right]+$$
$$+\alpha_{l-2}\cdot A^2\beta+\alpha_{l-3}\cdot 2AB\beta+\alpha_{l-4}\cdot B^2\beta=0 \quad;\quad \beta\equiv k_T^{\,2}\,. \tag{8}$$

For $k<2m$

$$\beta_k\cdot A\cdot\left[(-m+k)^2-m^2\right]+\beta_{k-1}\cdot B\left[(-m+k-1)\left[(-m+k-2)-m^2-\mu\right]\right]+\beta_{k-2}\cdot A^2\beta+\beta_{k-3}\cdot 2AB\beta+\beta_{k-4}\cdot B^2\beta=0\,, \tag{9}$$

For $k>2m$ : $k\equiv 2m+i$ ; $\beta_{2m+i}=\beta_i$

$$\beta_i\cdot A\cdot i\cdot(2m+i)+\beta_{i-1}\cdot B\left[(m+i-1)\left[(m+i-2)-m^2-\mu\right]\right]+\beta_{i-2}\cdot A^2\beta+$$
$$+\beta_{i-3}\cdot 2AB\beta+\beta_{i-4}\cdot B^2\beta+a\left\{2A(-m+i)\alpha_i+B\left[2(-m+i)-i\right]\alpha_{i-1}\right\}=0\,, \tag{10}$$

Then $a$ may be obtained by solving the equation:

$$a\cdot 2mA=\beta_{2m-1}\cdot B\left[(m-1)\left[(m-2)-m^2-\mu\right]\right]+\beta_{2m-2}\cdot A^2\beta+\beta_{2m-3}2AB\beta+\beta_{2m-4}\cdot B^2\beta \tag{11}$$

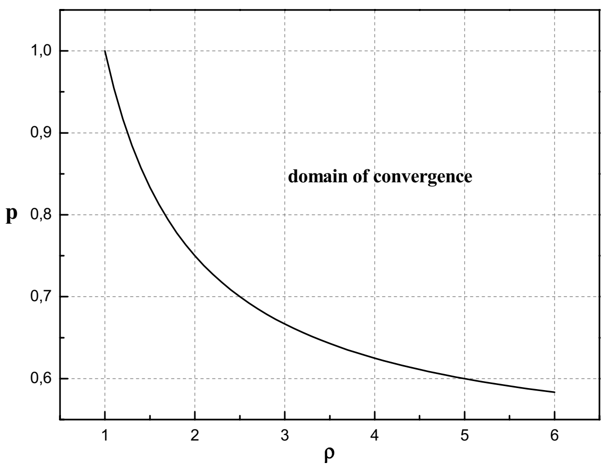


Fig.2. Domain of convergence for power series (7) for inhomogeneous plasma region with $N(r)=A+B\cdot r$

The power series (7) must converge so long as radius of convergence $R_{conv}\geq\rho$. In Fig.2 shows the region of the inhomogeneity parameters, in which there is convergence of the series (7). It is evident that for inhomogeneity of the plasma with increased density $p>1$ convergence is unconditional because $\rho>1$ by the problem formulation. In case of the inhomogeneity of the plasma density with lower than background ($p<1$) the convergence takes place only if the condition $p>(1+\rho)/2\rho$ is satisfied.

From boundary relations were determined the coefficients $a_m, b_m, c_m$. And finally we have

$$a_m=D_{ma}\cdot D_m^{-1},$$

$$D_m=-H_m^{(1)}(x_T)\cdot\Delta_{1m}+G_m\cdot\Delta_{2m};D_{ma}=J_m(x_T)\cdot\Delta_{1m}-Q_m\cdot\Delta_{2m};\quad \Delta_{1m}=P_{2m}(1)\cdot P_{1m}(\rho)-P_{1m}(1)\cdot P_{2m}(\rho);$$
$$P_{jm}(r)=\mu F_{jm}(r)+r\cdot F'_{jm}(r),j=1,2;\quad P_{jm}(r)=\mu F_{jm}(r)+r\cdot F'_{jm}(r),j=1,2;G_m=q_m\cdot H_m^{(1)}(x_T);Q_m=q_m\cdot J_m(x_T); \tag{12}$$
$$x_T=\rho k_T R_C\,;\ \mu=m(\omega/\omega_i)\,.$$

The differential scattering cross-section per unit length $\sigma(\varphi)$ may be introduced for study of scattering. It is equal to the ratio of the power diffracted at an angle $\varphi$ to the magnitude of the Poynting vector

$$\sigma(\varphi)=\lim_{r\to\infty}\frac{\pi r}{2\rho}\left(\left|H_z^{(s)}\right|^2\Big/\left|E_0\right|^2\right). \tag{13}$$

At far field zone the differential scattering cross-section can be written as

$$\sigma(\varphi) = \frac{4}{k_T R_c}\left|\sum (-1)^m a_m e^{im\varphi}\right|^2 . \tag{14}$$

Consider the angular distribution of the scattered fields for different values of ratio metal cylinder radius to length of incident low frequency wave.

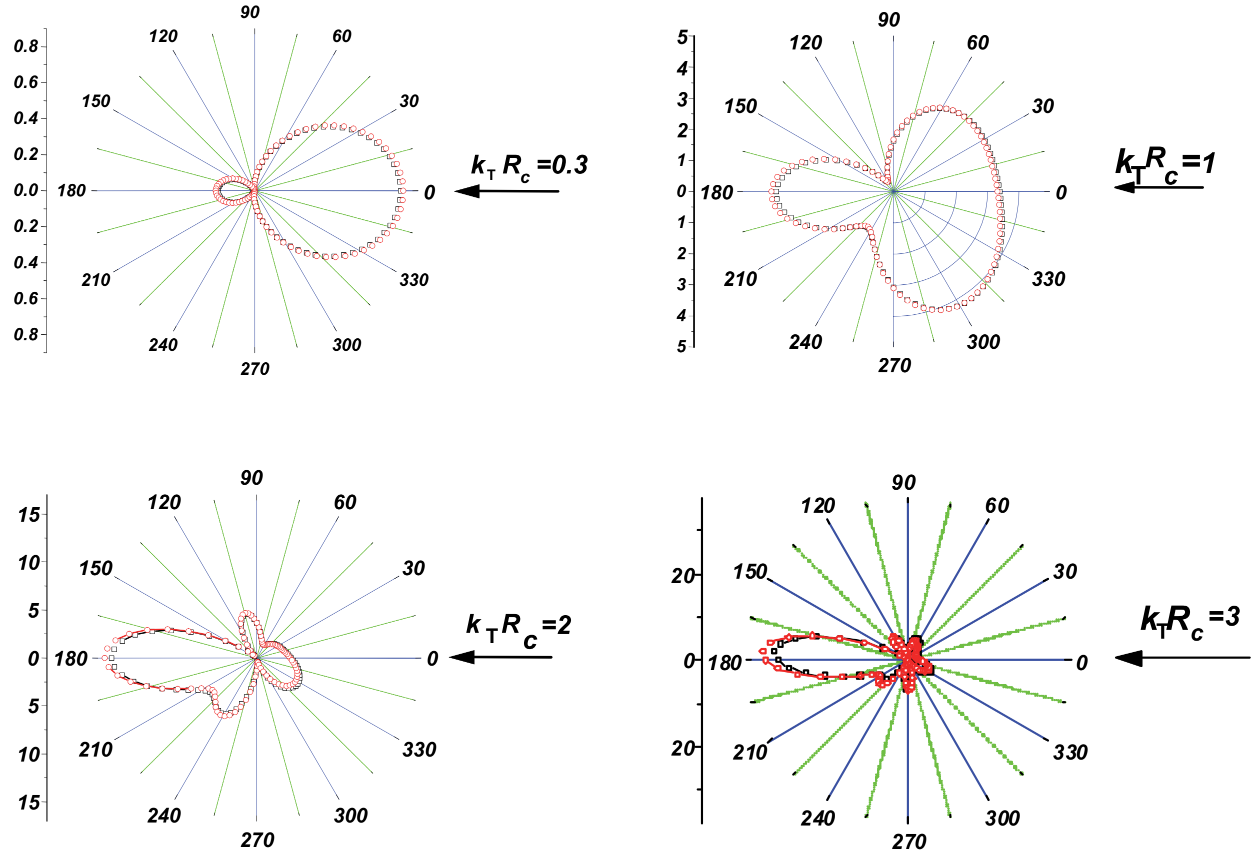


Fig. 3. Changes of scattering indicatrix for linear inhomogeneity $N(r) = A + B \cdot r$ (values of parameters $p = 1.9 : \rho = 2.2$ ).

## DISCUSSIONS AND CONCLUSION

From Fig. 3 evidently, that for small values $k_T R_C = 0.3$ practically all wave energy is back-scattered. It corresponds with the theory of diffraction of waves on a metallic cylinder, being in a vacuum [6]. If to increase ratio of radius of cylinder to a wave-length ( $k_T R_C = 1$ ), symmetry in scattering is violated, back-scattering diminishes, a maximum of scattering turns, and the part of energy, which forward diffracted increases as a result. In the case of small lengths of waves $k_T R_C = 2$ , $k_T R_C = 3$ practically all energy scatters in the wavevector direction of incident wave, and the values of cross-sections for back scattering approaches to the geometric optic limit, that partly confirms the rightness of the got results. In addition the scattering indicatrix becomes symmetric again with increasing of $k_T R_C$ .

Asymmetry of scattering is related to nonreciprocality of waves propagated in the external magnetic field $\vec{H}_0$ . The incident wave excites the electromagnetic waves at the boundary “metal- inhomogeneous plasma”. These waves can propagate around cylinder in two opposite directions, for which wave propagation are nonreciprocal in relation of direct of external magnetic field $\vec{H}_0$ . At substituting of direction of the field by opposite $\vec{H}_0 \to -\vec{H}_0$ , the changes of indicatrix of dispersion depending on a value $k_T R_C$ take place so that curves $\sigma(\varphi, -H_0)$ are the mirror symmetric of curves $\sigma(\varphi, H_0)$ in relation to direction of wavevector of incident wave. The change of strength of the external magnetic field gives changing of the ratio of length of incident wave to the cylinder radius.

Consequently, possibility of management of indicatrix orientation is opened if to change the wave frequency or/and the strength and orientation of the constant external magnetic field.

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