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Current-controlled filter on superconducting films with a tilted washboard pinning potential

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Abstract

The influence of an ac current of arbitrary amplitude and frequency on the mixed-state dc-voltage-ac-drive tilting-ratchet response of a superconducting film with uniaxial cosine pinning potential at finite temperature is theoretically investigated. The results are obtained in the single-vortex approximation, within the frame of an exact solution of the Langevin equation for non-interacting vortices. Both experimentally achievable, the dc ratchet response and absorbed ac power are predicted to demonstrate a pronounced filter-like behavior at microwave frequencies. Based on our findings, we propose a cut-off filter and discuss its operating curves as functions of the driving parameters, i.e, ac amplitude, frequency, and dc bias. The predicted results can be examined, e.g, on superconducting films with a washboard pinning potential landscape.

Keywords: Vortex ratchets, washboard pinning potential, absorbed power, superconducting devices
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1. Introduction

One of the unsolved problems in the field of the vortex dynamics in superconductors with a dc-biased pinning potential, also known as a tilting ratchet, consists in the calculation of the frequency dependence of the resistive responses, especially at microwave and gigahertz frequencies. From the viewpoint of applications, most important are the dc-voltage-ac-drive (dc ratchet) response and the power absorption in ac response. At sufficiently high frequencies, namely above $\simeq$ 1 MHz [1], both responses become strongly dependent on the frequency of the ac current input. A considerable amount of theoretical work about the general properties of different types of ratchet systems exists [2-4]; see Ref. [5] for a review. However, the question of the frequency-dependent ratchet effect has been theoretically addressed only recently [6], and only in the single-vortex approximation. Considering the vortex motion in a washboard pinning potential (WPP), a number of new effects have been predicted [6].

With regard to the related experiments, three recent investigations should be mentioned especially. The appearance of a rectified voltage in superconducting Al films patterned with symmetric antidots and subjected to a dc-biased ac driving current has been experimentally observed by de Souza Silva *et al.* [7]. They

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have shown that the rectification effect is due to the dc tilt inducing an asymmetry of the initially symmetric periodic pinning potential. The vortex rectification has been found to be strongly dependent on temperature and magnetic field. In that work, the data have been measured at one excitation frequency of 1 kHz. Later on, D. Perez de Lara *et al.* [8] have investigated ratchet effects in thin Nb films grown on top of arrays of Ni nanotriangles subjected to an ac current with a frequency up to 10 kHz. They have reported an adiabatic, i.e., frequency-independent behavior of the investigated responses [8].

Recently, B. B. Jin *et al.* [1] have measured the frequency dependence of the dc rectified voltage at large amplitudes of the ac driving force in a frequency range between 0.5 MHz and 2 GHz. In particular, they have found that (i) the transition from the non-adiabatic to the adiabatic case occurs at about 1 MHz, above which the ratchet window shifts upwards with the applied frequency due to the fact that the time for a vortex to escape from the pinning potential is comparable to the period of the applied rf driving current and (ii) a weakening of the ratchet effect at several GHz, attributed by the authors to the possibility of stronger inertia effects in the vortex motion at such high frequencies. The results of B. B. Jin *et al.* [1] quantitatively agree with our theoretical predictions [6]. In the present work, we put emphasis on some new aspect of this problem relevant especially for applications. Namely, we analyze in detail the frequency dependences of the ratchet voltage E^d and the real part of the nonlinear impedance ρ_1 and propose to experimentally examine these dependences by the development of a current-controlled cut-off filter.

2. Results and discussion

Our theoretical treatment of the system relies upon the Langevin equation for a vortex moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B} = \mathbf{n}B$ ($B \equiv |\mathbf{B}|$, $\mathbf{n} = n\mathbf{z}$, $\mathbf{z}$ is the unit vector in the z direction and $n = \pm 1$) which, neglecting the Hall effect, has the form [9]

$$\eta\mathbf{v} = \mathbf{F}_L + \mathbf{F}_p + \mathbf{F}_{th}, \tag{1}$$

where $\mathbf{F}_L = n(\Phi_0/c)\mathbf{j}\times\mathbf{z}$ is the Lorentz force (Φ_0 is the magnetic flux quantum, and c is the speed of light). $\mathbf{j} = \mathbf{j}(t) = \mathbf{j}^{dc} + \mathbf{j}^{ac}\cos\omega t$, where $\mathbf{j}^{dc}$ and $\mathbf{j}^{ac}$ are the dc and ac current density amplitudes and ω is the angular frequency. $\mathbf{F}_p = -\nabla U_p(x)$ is the anisotropic pinning force, $U_p(x) = (U_p/2)(1 - \cos kx)$ is the periodic washboard pinning potential with $k = 2\pi/a$ [9], where U_p is its depth and a is the period. $\mathbf{F}_{th}$ is the thermal fluctuation force represented by a Gaussian white noise and η is the vortex viscosity.

The Langevin equation (1) has been solved in Ref. [9] in terms of a matrix continued fraction allowing one to exactly calculate the main quantities of physical interest in the problem, namely (i) the time-independent (but frequency-dependent) dc electrical field response and (ii) the stationary ac response at a given frequency ω, independent of the initial conditions. Both these are determined by the appropriate components of the average electric field induced by the moving vortex system [6] (see Eqs. (2-5) therein).

We now turn to the analysis of the frequency dependences of the dc electric field $\mathbf{E}^d$ and the real part of the nonlinear impedance $\rho_1 = \mathrm{Re}Z_1$, which accounts for the absorbed power, scaled by the flux-flow resistivity $\rho_f \equiv B\Phi_0/\eta c^2$, as functions of the dimensionless external driving parameters, i.e., dc bias $\xi^d = |\mathbf{j}^{dc}|/j_c$, amplitude $\xi^a = |\mathbf{j}^{ac}|/j_c$, and frequency of the ac input $\Omega = \omega\hat{\tau}$ with $\hat{\tau} \equiv 2\eta/U_pk^2$ being the relaxation time and $j_c \equiv cU_pk/2\Phi_0$ [9]. For simplicity, below we put emphasis on the case when both the dc and ac currents flow along the WPP channels provoking the vortex movement perpendicular to them, though the model [6, 9] allows one to exactly calculate both responses for any intermediate current flow angles with regard to the guiding direction of the WPP. We furthermore limit the discussion to the responses' components along the WPP channels, so as to provide the most intuitive figure data results which can be obtained from our analysis.

With regard to the development of superconducting filters, one of the main questions in the study of the operation curves $E^d(\xi^a|\xi^d, \Omega)$ consists in the determination of the frequency and dc bias dependences of the ac amplitude threshold value, $\xi^a_c(\Omega, \xi^d)$, which can be considered as an ac critical current magnitude for the dc ratchet response E^d such that $E^d = 0$ for $\xi^a < \xi^a_c$. To accomplish this, we start with the analysis of the frequency dependences of the ratchet response E^d taken for a series of fixed ξ^d values at $\xi^a = 1$, as representative for intermediate ac amplitudes (see Fig. 1a). All the figure data are calculated for the dimensionless inverse temperature $g \equiv U_p/2T = 100$ [9] representing a reasonable value, experimentally achievable, e.g., for thin Nb films [10-13].

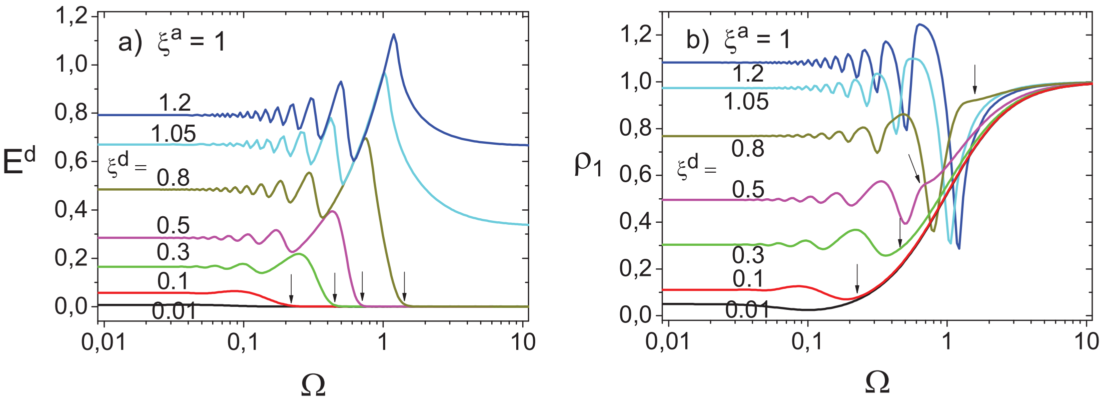

Fig. 1. The ratchet voltage E^d (a) and the real part of the nonlinear impedance ρ_1 (b) versus Ω at the intermediate ac current drive $\xi^a = 1$ for a set of biases $\xi^d = 0.01; 0.1; 0.3; 0.5; 0.8; 1.05; 1.2$, as indicated. At a fixed ac drive, the cut-off frequencies Ω_c are defined by the bias values and for $\xi^a = 1$ are shown by the arrows.

At intermediate ac drives $\xi^a = 1$ and different tilt values the behavior of the curves varies strongly. If the tilt values are small, the vortices are localized at the bottoms of the WPP wells which results in a non-dissipative state, thus the response vanishes regardless of the frequency. With gradual increase of the dc bias, specifically from $\xi^d = 0.01$ to $\xi^d = 0.8$ still corresponding to a subcritical tilt, the voltage drop gets substantially higher at low frequencies, whereas a zero-voltage tail spreads over the high-frequency range. The former is a consequence of the *running* vortex state, whereas the latter is a clear signature of a *localized* vortex state, see labels in Fig. 2a. For the filter development it is important that these regions are separated by a well-defined threshold frequency $\Omega_c \equiv \Omega(\xi^d, \xi^a_c)$, as shown by the arrows in Fig. 1a. The tilt, ξ^d, determines the asymmetry of the WPP and the time needed for a vortex to get from one to the next WPP well, whereas ξ^a represents the ac driving force for a vortex which also competes with the height of the initially symmetric WPP. This physically means that, if the ac driving frequency Ω is much less than the depinning frequency $\omega_p \sim \hat{\tau}^{-1}$, the running state of the vortex is established and it can visit several potential wells during the ac period. For a fixed ac amplitude ξ^a and frequency Ω, the number of visited wells increases strongly with the increase of the tilt, thus resulting in the upward shift of the threshold frequency Ω_c. Another interesting feature in the operating curves $E^d(\Omega)$ appears as a maximum at $\Omega \simeq 1$. Its magnitude increases with increase of the dc bias. For overcritical tilts, $\xi^d > 1$, the efficiency of the filter is getting lower with the increase of the bias value. Summarizing, the calculated ratchet response $E^d(\xi^a|\xi^d, \Omega)$ demonstrates a cut-off filter behavior at intermediate ac drives ξ^a and subcritical tilts ξ^d.

We now turn to the analysis of the frequency dependence of the real part of the nonlinear impedance $\rho_1 = \mathrm{Re}Z_1$ calculated for a series of dc densities ξ^d and at intermediate ac density $\xi^a = 1$. In Fig. 1b the curves demonstrate either a step-like behavior for $\xi^d \ll 1$ or a pronounced non-monotonic behavior for $\xi^d \gtrsim 0.5$. The monotonic curves at $\xi^d = 0$ agree with the results of Coffey and Clem [14] who calculated in linear approximation in ξ^a the temperature dependence of the depinning frequency in a nontilted cosine pinning potential. In contrast to this monotonic behavior, the nonmonotonic curves ($\xi^d \gtrsim 0.5$) demonstrate two characteristic features. First, a pronounced power absorption ensues at low frequencies. Second, a deep minimum in the power absorption occurs at $\Omega \simeq 1$. Finally leveling towards 1 takes place at high frequencies. With gradual increase of ξ^a, the value of ρ_1 at low frequencies remains the same, whereas the minimum shifts towards higher frequencies (not shown). In addition, at frequencies $\Omega \simeq 0.5$ peculiarities in the curves become more pronounced. These are smeared out when considering at higher temperatures ($g \ll 100$).

Summarizing the obtained findings, in Fig. 2b we propose a (functional) diagram illustrating the (ξ^d, ξ^a)-operation window of such a current-controlled filter. The operating window of the filter is restricted by dc (vertical axis) and ac (horizontal axis) current values corresponding to one of the limiting cases described below at which the filter operation is quenched. In the limiting case of a zero bias this quench occurs as

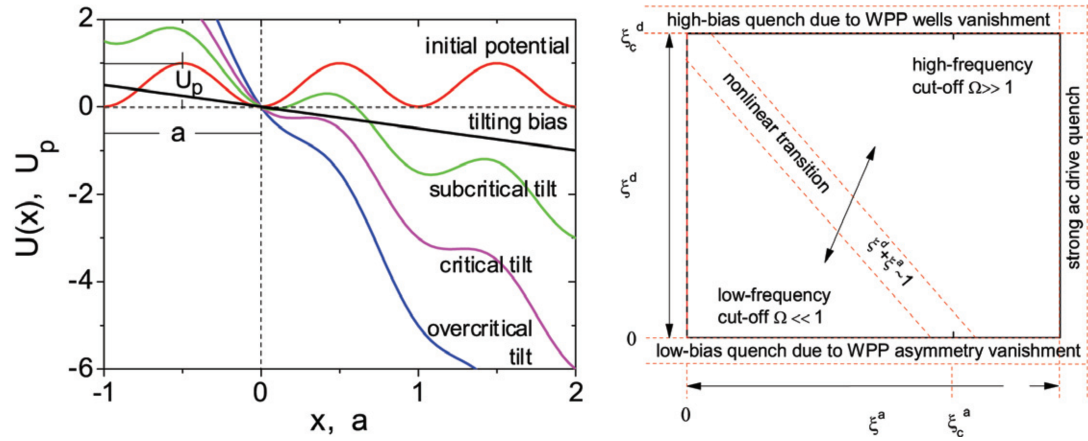

Fig. 2. a) Modification of the effective pinning potential $U(x) \equiv U_p(x) - Fx$ with gradual increase of the Lorentz force component in the x-direction F, where $U_p(x) = (U_p/2)(1 - \cos kx)$ is the WPP with its depth U_p and period $a = 2\pi/k$. As the initial WPP is symmetric, i.e., $U_p(-x) = U_p(x)$, it can establish ratchet properties only in the presence of an external dc bias F invoking its tilt. Depending on the bias value, in the absence of an ac current and assuming $T = 0$ K for simplicity, two qualitatively different modes in the vortex motion appear. (i) If $F < F_p$, thought the initial potential well is tilted, it maintains the average vortex position, i.e., the vortex is in the *localized* state. At the critical tilt value, i.e., when $F = F_p$, the right-side potential barrier disappears. (ii) At last, when $F > F_p$, the vortex motion direction coincides with the direction of the moving force F, i.e., the vortex is in the *running* state. b) The operating window of the proposed current-controlled filter (see text for details).

the asymmetry of the WPP vanishes, thus the system can no longer establish its ratchet properties. In the opposite case of overcritical biases a dc quench occurs as the potential wells vanish due to an excessively strong tilt (see Fig. 2a). This physically means that the effective pinning potential for the vortex movement disappears, thus no frequency dependence in the response is expected. The left-side vertical line corresponds to the absence of the ac input current which represents a trivial limiting case, as no output ratchet signal results in this case. The right-side quench is caused by excessively strong ac drives so that the WPP influence on the ratchet response becomes ineffective. The diagonal lines roughly define the area where $\xi^d + \xi^a \approx 1$. In order to widen the tunable range of the cut-off frequencies, the operating point of the filter should be chosen within this area, since a small deviation of one or both of the input parameters results in a drastic shift of the cut-off frequency of the filter. On the contrary, if a fine tuning of the cut-off frequency is needed, the operating point of the filter should be chosen out of the above-mentioned area taking into account a preferable frequency range of the filter operation.

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