New branches of spin waves spectra in non-ferromagnetic conductors and two-dimensional electron gas were predicted. The resonance (local, quasilocal and magnetoimpurity) electron states on impurity atoms were taken into account. Near frequencies of electron resonance transitions between spin split resonance levels and Landau levels there exist weakly damped spin magnetization oscillations called magnetoimpurity spin waves. A physical cause for existence of these waves is localization of electrons on impurity atoms stimulated by a magnetic field. The localization attenuates dissipation processes and leads to a possibility of propagation of new spin waves which are absent in pure samples. A spectrum and damping decrement of these waves were determined. New resonance features in an energy spectrum of inelastic neutron magnetic scattering on spin waves in conductors and two-dimensional electron gas with resonance states of electrons in a magnetic field were discovered.

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978-3-8484-9015-8

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Impressum/Imprint (nur für Deutschland/only for Germany)
Bibliografische Information der Deutschen Nationalbibliothek: Die Deutsche Nationalbibliothek verzeichnet diese Publikation in der Deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über http://dnb.d-nb.de abrufbar.

Coverbild: www.ingimage.com

Verlag: LAP LAMBERT Academic Publishing GmbH & Co. KG
Heinrich-Böcking-Str. 6-8, 66121 Saarbrücken, Deutschland
Telefon +49 681 3720-310, Telefax +49 681 3720-3109
Email: info@lap-publishing.com

Herstellung in Deutschland (siehe letzte Seite)
ISBN: 978-3-8484-9015-8

Imprint (only for USA, GB)
Bibliographic information published by the Deutsche Nationalbibliothek: The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at http://dnb.d-nb.de.

Cover image: www.ingimage.com

Publisher: LAP LAMBERT Academic Publishing GmbH & Co. KG
Heinrich-Böcking-Str. 6-8, 66121 Saarbrücken, Germany
Phone +49 681 3720-310, Fax +49 681 3720-3109
Email: info@lap-publishing.com

Printed in the U.S.A.
Printed in the U.K. by (see last page)
ISBN: 978-3-8484-9015-8

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LANDAU-SILIN SPIN WAVES IN CONDUCTORS WITH IMPURITY STATES

Theoretical investigation results of spin waves properties in nonmagnetic conductors and heterostructures with 2D electron gas in presence of impurity atoms be able to localize electrons are listed in monograph. The effect of quasi-local and magnetoimpurity states of electrons on dynamic spin susceptibility of electron gas, on spectrum and damping of spin waves are considered. New branches due to electron impurity states in spin wave spectrum are predicted. Methods of diagnostics of these waves in experiments with slow neutrons are proposed.

C O N T E N T S

I N T R O D U C T I O N

Impurity atoms in a conductor exert a complicated influence on quasi-particle energy spectrum. They cause shift and spread of energy zones as well as state density rearrangement accompanied by appearance of both local and quasi-local levels.

A local state concept in non-ideal crystalline lattice oscillation spectrum has been suggested by I.M.Lifshits [1] in 1940s. These investigations were continued in the Kagan and Iosilevskii [2, 3] and Brout and Visscher [4] articles in which impurity atom quasi-local oscillations in lattice was predicted. A theory of other quasi-particles – electrons and magnons – was been elaborated at the same time.

However, despite electron local states in semiconductors [5] were known long time ago, a quasi-local state concept in impurity conductors was given an intensive development during last decades (see articles [6-9] as well in [10-14]).

These works demonstrate that electron impurity levels ε_r in an insulated inclusion field can occur in a continuous spectrum region of assembled carriers. Being in resonance with zone states each of these levels splits into two sublevels which in their turn split as well and so on. This process is accompanied by appearance of a quasi-local level finite width Γ. One can consider that an electron is captured by an impurity for some time $\tau = \hbar / \Gamma$ ($\hbar$ - quantum constant) and then it is emitted. On a collision theory view point [15-17] such states correspond to complex poles $\varepsilon_r - i\Gamma$ of scattering amplitude. These poles are placed on a non-physical sheet of a Riemannian surface. Such states result into a sharp change of de Broglie wave phase on π (those waves are scattering by an impurity center), and scattering cross-section increase in λ / r_0 times (λ - electron wave length, r_0 - scattering center radius) when electron energy passes through the resonance value ε_r [15].

Though such resonances were known in quantum physics long ago [15] their role in solid state physics has become clear quite recently.

The situation changes in a magnetic field. Special electron bound states appear which are caused by influence of an attraction impurity and magnetic field on particle. The idea of magnetic localization of carriers at insulated impurity atoms goes back to works of V.G.Skobov [18] and Ju.A.Bychkov [19]. It is stated there that Landau quantization leads to bound state “multiplying”. A state under the condition mentioned above resembles a case of proper quasi-local states. Magnetoimpurity state physics was developed in the works [20-22].

A theory of spin waves in ferromagnetics and antiferromagnetics has been thoroughly elaborated [23]. Less attention was paid to spin waves in nonferromagnetic metals in

monographic literature. Here we deal with resonance electron state influence in a field of impurity atoms on properties of these waves.

Resonance states of electrons influence considerably on properties of spin waves in nonferromagnetic metals in a magnetic field. L.D.Landau [34] pointed out the possibility of such wave propagation without magnetic field and V.P.Silin [24] pointed out the possibility of such wave propagation in a magnetic field. They were recorded experimentally [25, 26].

A cause for spin wave appearance in nonferromagnetic metals in a magnetic field is an exchange interaction of assembled carriers [27-31]. The waves described in the works [28, 29] exist due to spin resonance of conductivity electrons [32] which form degenerated Fermi-liquid [28-31, 33-36]. Other resonances lead to new branches of collective excitation spectrum of metal spin system. For example, near frequencies of electron resonance transitions between spin split magnetoimpurity levels and Landau levels there exist slightly damped spin magnetization oscillations called magnetoimpurity spin waves [37]. They are analogous to magnetoimpurity electromagnetic waves [38-41]. A physical cause for existence of these waves is localization of electrons on insulated impurity atoms stimulated by a magnetic field. The localization attenuates dissipation processes and leads to a possibility of propagation of new electromagnetic and spin waves which are absent in pure samples.

The waves of this type exist also in the case when electron localization is due to just impurity atoms and it isn't connected with the magnetic field. That's why a theoretical prediction of such spin waves, studying their properties and characteristics as well as consideration of the experimental investigation methods are an important problem of solid state physics.

For this reason it's necessary to find out how quasi-local states of electrons on impurity atoms in nonferromagnetic metals influence characteristics of spin waves in a magnetic field. To gane this objective it's important to solve the following problems: with the help of the method of Green temperature functions it's necessary to calculate new resonance contributions in components of a tensor of dynamic spin susceptibility of electrons with a square isotropic and anisotropic dispersion law, to predict new branches of spin wave spectrum in nonferromagnetic metals with quasi-local states of electrons in a magnetic field, to consider neutron magnetic scattering on these waves as a method of experimental detection of new types of spin waves.

The components of a tensor of dynamic spin susceptibility of electrons of conductivity in nonferromagnetic metals with quasi-local states of carriers on impurity atoms in a magnetic field were calculated in the course of this work. Those components contain new resonance

contributions induced by electron transitions between quasi-local levels and Landau levels, the latter caused by variable magnetic field. New resonance contributions to components of a tensor of dynamic spin susceptibility of normal metals with quasi-local states of electrons whose Fermi-surface resembles a revolution ellipsoid were determined. These contributions depend on an angle between a direction of a tense vector of magnetic field in which the metal is placed and an axis of an ellipsoid revolution. New branches in a transversal spin wave spectrum were predicted while examining the processes of wave propagation in nonferromagnetic metals with quasi-local states of electrons on impurity atoms in a magnetic field. Transparency bands of these waves lie close to frequencies of resonance electron transitions between quasi-local levels and Landau levels accompanied by a spin flip. New type wave characteristics such as polarization, spectrum, damping decrement were calculated for the first time. An influence of quasi-local electron states on a spectrum and damping of quantum spin waves in nonferromagnetic metals in a magnetic field was investigated for the first time. It was shown that in this case a spin wave spectrum reorganization takes place in a region of intersection of spin wave dispersion curve with the frequency of resonance transitions of electrons between quasi-local levels and Landau levels accompanied by a spin flip. New branches of quantum spin waves appear in transparency windows.

A spectrum and damping decrement of these waves were determined. New resonance features were discovered in an energy spectrum of inelastic neutron magnetic scattering on spin waves in normal metals with quasi-local states of electrons in a magnetic field. A differential cross-section of magnetic scattering of neutrons with an excitation of spin waves of a new type was computed for the first time.

The results obtained broaden the notion about collective spin excitations in nonferromagnetic metals with quasi-local states of electrons on impurity atoms in a magnetic field and contribute into a development of a theory of spin waves in solids. A practical value of this work is assessed by a possibility of use of statements which were developed in it about new spectrum branches of spin waves which propagate in nonferromagnetic metals with elaboration of principles of creating new functional materials with definite properties for solid state microelectronics.

We express our gratitude to N.V.Gleizer, G.I. Rashba, A.D. Rudnev and A.D. Serdjuk for a discussion of the results used in this monograph, to V.V.Ulyanov for stimuli in work, to reviewers A.S.Kovalev and V.A.Yampol'skii for constructive remarks and also to Lyudmila Khristenko for fruitful collaboration.

CHAPTER I. SPIN WAVES IN A FERMI-LIQUID OF NONFERROMAGNETIC METALS AND IMPURITY STATES OF ELECTRONS

1.1. Spin waves in a Fermi-liquid of nonferromagnetic metals

In a nonferromagnetic metal spin waves can propagate due to exchange interaction of conductivity electrons. These waves were predicted by V.P.Silin [24]. Studying the spin waves is carried out on a basis of a kinetic equation for a vector spin density of electrons [29, 31]

$$\frac{\partial \delta\vec{\sigma}}{\partial t} + (\vec{v}\frac{\partial}{\partial \vec{r}} + \frac{e}{c}[\vec{v}\times\vec{H}]\frac{\partial}{\partial \vec{p}})(\delta\vec{\sigma} - \frac{\partial f_0}{\partial \varepsilon}\delta\vec{\varepsilon}_2) +$$

$$+ \frac{2\gamma}{\hbar}(\vec{H}\times\delta\vec{\sigma} - \frac{\partial f_0}{\partial \varepsilon}\delta\vec{\varepsilon}_2) = \left(\frac{\partial \delta\vec{\sigma}}{\partial t}\right)_{\text{col}}. \qquad (1.1)$$

Here $\vec{H}$ - magnetic field directed along z axis, $\delta\vec{\sigma}$ - a little non-equilibrium addition to a spin density ($\delta\vec{\sigma} \perp \vec{H}$), $\vec{p}$ and $\vec{v}$ - momentum and velocity of electron, $f_0(\varepsilon)$ - Fermi-function of distribution, γ - effective magnetic moment of a quasi-particle in a degenerated electron liquid,

$$\delta\vec{\varepsilon}_2 = -\mu\delta\vec{H} + \int \frac{d\vec{p}'}{(2\pi\hbar)^3}\psi(\vec{p},\vec{p}')\delta\vec{\sigma}(\vec{p}',\vec{r},t), \qquad (1.2)$$

μ - magnetic moment of electron, $\delta\vec{H}$ - variable magnetic induction, ψ - correlative function depending on an angle between $\vec{p}$ and $\vec{p}$ vectors lying on the Fermi sphere, $\vec{r}$ and t - radius-vector and time, e - the charge of electron, c - velocity of light, $\hbar$ - the quantum constant. Collision integral is marked by the “col” index.

If one neglects the collision integral in the right part of (1.1) than for excitations of $\exp[i(\vec{q}\vec{r} - \omega t)]$ type this expression gets equal

$$-i\omega\vec{g} + (i\vec{q}\vec{v} - \Omega\frac{\partial}{\partial \varphi})(\vec{g} + \delta\vec{\varepsilon}_2) + \frac{2\gamma}{\hbar}(\vec{H}\times\vec{g} + \delta\vec{\varepsilon}_2) = 0. \qquad (1.3)$$

Here $\vec{q}$ and ω - a wave vector and frequency of excitation, φ - an angle in (x, y) plane, Ω - cyclotron frequency and $\vec{g}$ function is defined by an equation

$$\delta\vec{\sigma} = -\frac{\partial f_0}{\partial \varepsilon}\vec{g}(\theta,\varphi), \qquad (1.4)$$

where $g(\theta,\varphi)=\sum_{n,m}a_{n,m}Y_{n,m}(\theta,\varphi)$ - g function expansion in series over spherical harmonics $Y_{n,m}$. Neglecting space dispersion ($\vec{q}=0$) and a wave magnetic field ($\delta\vec{H}=0$) in the kinetic equation (1.1) let's express it in a form of equations for g_z and $g_\pm=g_x\pm ig_y$ components:

$$-i\omega g_z-\Omega\frac{\partial}{\partial\varphi}(g_z+\delta\varepsilon_{2z})=0, \tag{1.5}$$

$$-i\omega g_\pm-\Omega\frac{\partial}{\partial\varphi}(g_\pm+\delta\varepsilon_{2\pm})\pm i\tilde{\omega}(g_\pm+\delta\varepsilon_{2\pm})=0, \tag{1.6}$$

where $\tilde{\omega}=\frac{2\gamma H}{\hbar}$.

For the g_z and $g_\pm$ functions let's use an expansion

$$g=\sum_{n,m}b_{n,m}Y_{n,m}(\theta,\varphi).$$

Thus for $\delta\varepsilon_{2z}$ and $\delta\varepsilon_{2\pm}$ values with taking into account the formula (1.2) we obtain $\delta\varepsilon_2=\sum_{n,m}b_{n,m}B_nY_{n,m}(\theta,\varphi)$. Here B_n - coefficients of an expansion of spin part of correlation Landau function in series of Legendre polynomials [29-31]. Substituting these expressions in the kinetic equations (1.5) and (1.6) we determine natural frequencies of longitudinal and transversal oscillations of magnetization of the system:

$$\begin{aligned}\omega&=\omega_{nm}^{z}=-m\Omega(1+B_n),\\ \omega&=\omega_{nm}^{\pm}=-(m\Omega\pm\tilde{\omega})(1+B_n).\end{aligned} \tag{1.7}$$

With taking into account an expression for the effective magnetic moment of quasi-particles $\gamma=\mu/(1+B_0)$ if $n=0$, $m=0$ formula (1.7) gives

$$\omega_{00}^{\pm}=\mp\tilde{\omega}(1+B_0)=\mp 2\mu H/\hbar=\mp\Omega_0, \tag{1.8}$$

where Ω_0 - frequency of a spin resonance of conductivity electrons [32].

Let us consider a case of longitudinal propagation of spin waves ($\vec{q}\|\vec{H}$) with a polarization corresponding to g_+ function. As this takes place, g_+ function doesn't depend on the angle φ and, therefore, an expression $\delta\varepsilon_{2+}=B_0g_0$ is valid for the value of $\delta\varepsilon_{2+}$, where g_0

is obtained by integration of g_+ value over all the directions: $g_0 = \frac{1}{4\pi}\int dOg_+$. The kinetic equation with taking into account spatial dispersion takes a form

$$g_+ = g_0 B_0 (qv_F \cos\theta + \tilde{\omega})(\omega - qv_F \cos\theta - \tilde{\omega})^{-1}. \quad (1.9)$$

Here v_F - Fermi velocity of an electron. Integrating (1.9) relation over the directions we obtain the dispersion equation in a form

$$\frac{1}{B_0} = \frac{s}{2}\ln\frac{s - s_0 + 1}{s - s_0 - 1} - 1, \quad (1.10)$$

where $s = \frac{\omega}{qv_F}$, $s_0 = \frac{\tilde{\omega}}{qv_F}$.

The dispersion equation (1.10) is real under those values of ω and q which lie beyond the region bounded by straight lines $\omega = \tilde{\omega} + qv_F$, $\omega = \tilde{\omega} - qv_F$ which define boundaries of a region of one-particle excitations which are connected with a spin flip of an electron. In other words, the dispersion curve of spin waves must lie outside the Stoner sector where the waves experience the Landau damping.

At low values of q the logarithm in the equation (1.10) can be expanded in the Taylor series. It will give the following dispersion law of the spin waves:

$$\omega = \tilde{\omega}(1 + B_0)(1 + \frac{v_F^2 q^2}{3B_0\tilde{\omega}^2} + \ldots). \quad (1.11)$$

B_0 value in metals is negative [28, 31]. The dispersion curve (1.11) is shown in Fig. 1. Notice that the formula (1.8) leads to a spin wave limit frequency equal to the frequency of the spin resonance.

The damping of the waves is defined by the collision integral in the right part of the equation (1.1). In a case of low temperatures, which are considered by the author of the [29], the only collisions of electrons with impurities are sufficient. Therewith one makes a distinction between collisions with a spin flip which change the magnetization and collisions with a momentum change not changing the magnetization. The collision integral is correspondingly written as [29]

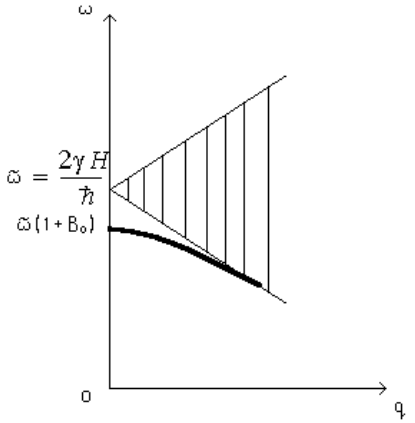


Fig. 1. The dispersion curve (1.11) for the spin waves of Silin ($B_0 < 0$) [30].

$$\left(\frac{\partial\delta\vec{\sigma}}{\partial t}\right)_{\text{col}} = -\left(\frac{1}{\tau_0}+\frac{1}{\tau}\right)\left(\delta\vec{\sigma} - \frac{\partial f_0}{\partial\varepsilon}\delta\vec{\varepsilon}_2\right) + \frac{\partial f_0}{\partial\varepsilon}\left(\int d\vec{p}'\frac{\partial f_0}{\partial\varepsilon'}\right)^{-1}\times$$

$$\times\int\frac{d\vec{p}'}{\tau(\theta)}\left[\delta\vec{\sigma}(\vec{p}') - \frac{\partial f_0}{\partial\varepsilon'}\delta\vec{\varepsilon}_2(\vec{p}')\right].$$

Here τ_0 and $\tau(\theta)$ characterize a momentum relaxation of electrons while τ - spin flip time. Let us use the expansion [29]

$$\frac{1}{\tau(\theta)} = \sum_{l=0}^{\infty}\frac{1}{\tau_l}(2l+1)P_l(\cos\theta),$$

where

$$P_l(z) = \frac{1}{2^l l!}\frac{d^l}{dz^l}(z^2-1)^l$$

are Legendre polynomials. A frequency of electron collisions ν is inversely proportional to relaxation time. In the vicinity of the spin resonance frequency of conductivity electrons $\tau >> \tau_{\text{free}}, \tau_l$, where τ_{free} is time of a free path of an electron. Experimentally the spin waves with the spectrum (1.11) were detected in alkaline metals by Schultz and Dunifer [25]. At rather high temperatures or under reasonably high frequency of electron collisions the following physical pattern takes place in their experiments [25]. If penetrated into a metal electromagnetic field tense to damp sharply in a small region of a skin layer where it directs spin magnetic moments of electrons. Then electrons with a directed magnetic moment diffuse

from the skin layer into a metal depth. Since spin flip time τ is rather big (~10^{-6} s) magnetization caused by diffusing electrons diffuses in a metal depth at lengths which are much bigger then a depth of skin layer. Such a phenomenon occurs in a small vicinity of the spin resonance frequency. The width of a region of metal film selective transparency is determined by spin flip time [32, 36].

With temperature reduction electron momentum relaxation time increases and diffusion coefficient firstly is getting depended on a frequency at first and secondly it becomes pure imaginary in a $\omega >> \nu$ limit. The latter leads to a rising possibility of propagation of magnetization waves (the spin waves) with dispersion law (1.11) instead of magnetization diffusion. As a result transparency of metal films appears not only with spin resonance frequency but also in a neighborhood of such a frequency on spin wave frequencies.

Experiments [25] gave a possibility to assess the values of parameters B_n which are constituent the expressions (1.7). In a case of sodium it turned out [31] that $B_0 = -0{,}215 \pm 0{,}03$.

Transparency windows within which a collisionless damping of transversal spin waves is absent appear in a quantizing magnetic field in a Stoner sector. New branches of so called quantum spin waves can locate in those windows. Their spectrum is linear in a long wavelength approximation and damping is determined by electron collisions [42].

1.2. Quasi-local and magnetoimpurity states of electrons

Let us consider a conductor with one group of carriers with a dispersion law $\varepsilon(\vec{p}) = p^2 / 2m$ (m – effective mass of an electron, $\vec{p}$ - momentum), containing N_i impurity atoms randomly distributed over N lattice points.

Let's consider a concentration of impurities N_i / N little and take linear approximation over this parameter.

Let us mark potential energy of a-th electron in the field of j-th impurity center $v(\vec{r}_a - \vec{r}_j)$. Here $\vec{r}$ with or without a corresponding index is a radius-vector. Density of impurity atoms is equal to

$$\rho_i(\vec{r}) = \sum_{j=1}^{N_i} \delta(\vec{r} - \vec{r}_j). \qquad (1.12)$$

Its Fourier component is

$$\int d^3 r \rho_i(\vec{r}) \mathrm{e}^{-i\vec{q}\vec{r}} = \sum_j \mathrm{e}^{-i\vec{q}\vec{r}_j} = \rho_{\vec{q}}\,, \tag{1.13}$$

where $\vec{q}$ is a wave vector of a plane wave. Hamiltonian of electron-impurity interaction has a form $\sum_{aj} v(\vec{r}_a - \vec{r}_j)$ and Fourier component of the function $v(\vec{r})$ is equal to

$$v_{\vec{q}} = \int d^3 r v(\vec{r}) \mathrm{e}^{-i\vec{q}\vec{r}}\,. \tag{1.14}$$

Let's place the sample in a quantizing magnetic field with strength $\vec{H}$ with a vector potential in Landau gauge $\vec{A} = (0, Hx, 0)$. Then suitable quantum numbers of a electron state in the magnetic field are orbital quantum numbers $\kappa = (n, p_y, p_z)$ ($n = 0,1,2,\ldots$) and a spin quantum number $\sigma = \pm 1$ as it follows from [15].

Hamiltonian of electrons in the field of impurities and in the magnetic field in a secondary quantization representation has a form

$$\hat{H} = \sum_{\kappa\sigma} \varepsilon_{\kappa\sigma} \hat{a}^+_{\kappa\sigma} \hat{a}_{\kappa\sigma} + \frac{1}{V} \sum_{\vec{q}} v_{\vec{q}} \rho_{\vec{q}} \sum_{\kappa_1\kappa_2\sigma} I_{\kappa_1\kappa_2}(\vec{q}) \hat{a}^+_{\kappa_1\sigma} \hat{a}_{\kappa_2\sigma}\,, \tag{1.15}$$

where $\varepsilon_{\kappa\sigma}$ is an electron energy in the state with a ket vector $|\kappa\sigma\rangle$, $\hat{a}_{\kappa\sigma}$ and $\hat{a}^+_{\kappa\sigma}$ are operators of secondary quantization, V is the volume of the sample, $I_{\kappa_1\kappa_2}(\vec{q}) = \langle \kappa_1 | e^{i\vec{q}\vec{r}} | \kappa_2 \rangle$ are matrix elements of plane wave in the Landau basis $\{|\kappa\rangle\}$ [43]. Orthonormalization of spin wave functions: $\langle \sigma_1 | \sigma_2 \rangle = \delta_{\sigma_1\sigma_2}$ is taken into account.

Let's introduce Green one-particle temperature function of electrons [44, 45]:

$$G^{\sigma}_{\kappa\kappa'}(\tau) = -\langle T_\tau \{ \hat{a}_{\kappa\sigma}(\tau) \hat{\bar{a}}_{\kappa'\sigma}(0) \} \rangle\,, \tag{1.16}$$

where $-\beta \le \tau \le \beta$ ($\beta = 1/k_B T$, k_B is Boltzmann constant, T is temperature), $\langle ... \rangle = \mathrm{Sp}\{\mathrm{e}^{-\beta\hat{H}'} \ldots\} / \mathrm{Sp}\,\mathrm{e}^{-\beta\hat{H}'}$ - averaging over Gibbs large canonical ensemble ($\hat{H}' = \hat{H} - \mu\hat{N}$, μ - chemical potential, $\hat{N}$ - operator of electron number), T_τ - symbol of a chronological product of operators, $\hat{a}_{\kappa\sigma}(\tau) = \mathrm{e}^{\tau\hat{H}'} \hat{a}_{\kappa\sigma}(0) \mathrm{e}^{-\tau\hat{H}'}$ - a Matsubara operator. Symbol $\langle \ldots \rangle$ includes averaging over positions of impurity atoms as well.

Let's use a connection between a Fourier component $G^{\sigma}_{\kappa\kappa'}(i\varsigma_s)$ ($\varsigma_s = \frac{\pi}{\beta}(2s+1)$, $s = 0, \pm 1, \ldots$) of this function in the Landau basis with a scattering operator:

$$G^{\sigma}_{\kappa\kappa'}(i\varsigma_s) = G^{(0)\sigma}_{\kappa}(i\varsigma_s)\delta_{\kappa\kappa'} + G^{(0)\sigma}_{\kappa}(i\varsigma_s) T^{\sigma}_{\kappa\kappa'}(i\varsigma_s) G^{(0)\sigma}_{\kappa'}(i\varsigma_s)\,, \tag{1.17}$$

where $G^{(0)\sigma}_{\kappa}(i\varsigma_s) = 1/(i\varsigma_s - \xi_{\kappa\sigma})$ is electron Green function in a pure sample ($v_{\vec{q}} = 0$), $\xi_{\kappa\sigma} = \varepsilon_{\kappa\sigma} - \mu$, $T^{\sigma}_{\kappa\kappa'}(i\varsigma_s)$ is a Matsubara operator of scattering electrons by impurities. The last one satisfies a Lippmann-Schwinger equation which has a form in the Landau basis

$$\begin{aligned} T^{\sigma}_{\kappa\kappa'}(i\varsigma_s) &= V_{\kappa\kappa'} + \sum_{\kappa_1} V_{\kappa\kappa_1} G^{(0)\sigma}_{\kappa_1}(i\varsigma_s) V_{\kappa_1\kappa'} + \\ &+ \sum_{\kappa_1\kappa_2} V_{\kappa\kappa_1} G^{(0)\sigma}_{\kappa_1}(i\varsigma_s) V_{\kappa_1\kappa_2} G^{(0)\sigma}_{\kappa_2}(i\varsigma_s) V_{\kappa_2\kappa'} + \ldots = \\ &= V_{\kappa\kappa'} + \sum_{\kappa_1} G^{(0)\sigma}_{\kappa_1}(i\varsigma_s) V_{\kappa\kappa_1} T^{\sigma}_{\kappa_1\kappa'}(i\varsigma_s). \end{aligned} \tag{1.18}$$

Here $V_{\kappa\kappa_1}$ is matrix elements of the Hamiltonian of electron-impurity interaction. In the general case it's not possible to sum this iteration series. Let us accomplish configuration averaging [45] of its terms. Let us point out that in our case averaging concerns $T^{\sigma}_{\kappa\kappa'}$ operator or more precisely it concerns the products of Fourier-densities $\rho_{\vec{q}_1} \ldots \rho_{\vec{q}_n}$ which depend on coordinates of the impurity centers.

After averaging over the configurations of impurities

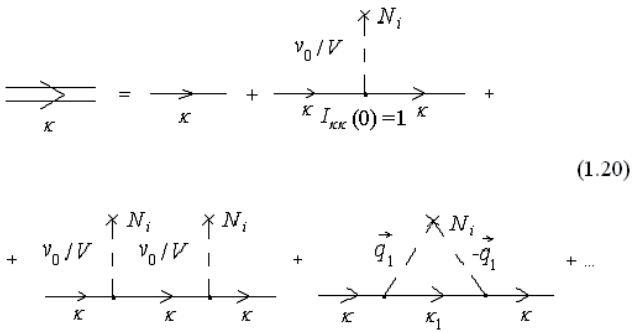
$$\langle G^{\sigma}_{\kappa\kappa'}(i\varsigma_s) \rangle_C = G^{\sigma}_{\kappa}(i\varsigma_s)\delta_{\kappa\kappa'},$$

where

$$G^{\sigma}_{\kappa}(i\varsigma_s) = \left[i\varsigma_s - \xi_{\kappa\sigma} - \sum\nolimits^{\sigma}_{\kappa}(i\varsigma_s) \right]^{-1} \tag{1.19}$$

and $\sum^{\sigma}_{\kappa}(i\varsigma_s)$ is Matsubara self-energy part.

A series for G^{σ}_{κ} can be represented in a diagram form [45]:

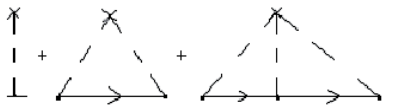


(1.20)

$G^{\sigma}_{\kappa}(i\varsigma_s)$ function is connected with a diagonal element of averaging Matsubara operator of scattering by an equation

$$G_\kappa^\sigma(i\varsigma_s) = G_\kappa^{(0)\sigma}(i\varsigma_s) + G_\kappa^{(0)\sigma}(i\varsigma_s)T_\kappa^\sigma(i\varsigma_s)G_\kappa^{(0)\sigma}(i\varsigma_s)\,. \tag{1.21}$$

Let us leave only terms which are proportional to the N_i for T_κ^σ. They descript impurity scattering of electrons by centers which act independently:



(1.22)

If an impurity is short-range, i.e. $v_{\vec{q}} \approx v_0$, we obtain:

$$T^\sigma(i\varsigma_s) \approx \frac{N_i v_0 / V}{1 - \dfrac{v_0}{V}\sum\limits_\kappa G_\kappa^{(0)\sigma}(i\varsigma_s)} \tag{1.23}$$

(a sum of geometric progression). In particular, for δ-potential

$$v(\vec{r}) = v_0\delta(\vec{r}) \tag{1.24}$$

we obtain

$$v_{\vec{q}} = \int d^3 r v(\vec{r}) e^{-i\vec{q}\vec{r}} = v_0\,. \tag{1.25}$$

Having Green temperature function G we may obtain retarded G^+ and advanced G^- Green functions by changing $i\varsigma_s$ to $\xi \pm i0$:

$$G_\sigma^\pm(\kappa,\xi) = G_\sigma^{(0)\pm}(\kappa,\xi) + \left[G_\sigma^{(0)\pm}(\kappa,\xi)\right]^2 T_\sigma^\pm(\xi)\,, \tag{1.26}$$

where

$$T_\sigma^\pm(\xi) = \frac{n_i v_0}{1 - v_0\{F_\sigma(\xi) \mp i\pi\nu_\sigma(\xi)\}}, \tag{1.27}$$

$$G_\sigma^{(0)\pm}(\kappa,\xi) = (\xi - \xi_{\kappa\sigma} \pm i0)^{-1}, \tag{1.28}$$

$$F_\sigma(\xi) = \mathrm{Re}\frac{1}{V}\sum_\kappa G_\sigma^{(0)\pm}(\kappa,\xi)\,; \tag{1.29}$$

$$\nu_\sigma(\xi) = \frac{1}{V}\sum_\kappa \delta(\xi - \xi_{\kappa\sigma}) \tag{1.30}$$

is state density (per volume unit) of electrons with the given spin orientation in a magnetic field, $n_i = N_i / V$ is impurity atom concentration. The function (1.29) is a Hilbert transform of state density.

Poles of the function (1.27) define locations and widths of impurity (local and quasi-local) electron energy levels. The local states correspond to roots of equation

$$1 - v_0 F_\sigma(\xi) = 0\,, \tag{1.31}$$

lying in a region where $\nu_\sigma = 0$. Broadening a local level is absent in the considered case. In this equation "dipping" in continuous spectrum ($\nu_\sigma \neq 0$) quasi-local states are solutions. They have finite width which is in inverse proportion to the life time of electron near the impurity [10].

In a general case the root $\varepsilon_{r\sigma}$ of this equation is lost in a continuous spectrum of positive energies. However, in some cases (low state density $\nu_\sigma(\varepsilon)$ in the neighborhood of the point $\varepsilon_{r\sigma}$, big value of $|v_0|$) the state which is about to be bound exists and has positive energy [6-9]. In a resonance way this state is combined with the states of a continuous spectrum (these states superimpose on it) with the same energy and splits into two states with close energies. Both of these states resonate in their turn with continuum states (which coincide with them in energy) and split again and so on. This process leads to some width Γ_σ of quasi-local level. Electrons may be considered to be captured for some time $\tau \sim \hbar/\Gamma$ at a quasi-local level and then they are emitted in a random direction. Wave functions making a contribution in such states are concentrated near the impurity and form almost a local state. Let us write a spectral representation of average Green function (1.26):

$$G_\sigma^{\pm}(\kappa,\xi) = \int_{-\infty}^{\infty} \frac{\rho_\sigma(\kappa,\xi')}{\xi - \xi' \pm i0} d\xi', \tag{1.32}$$

where

$$\rho_\sigma(\kappa,\xi) = \mp\frac{1}{\pi}\mathrm{Im}\, G_\sigma^{\pm}(\kappa,\xi) \tag{1.33}$$

is spectral density. For electrons in a magnetic field

$$\rho_\sigma^{(0)}(\kappa,\varepsilon) = \delta(\varepsilon - \varepsilon_{\kappa\sigma}), \tag{1.34}$$

where $\varepsilon = \xi + \mu$, $\varepsilon_{\kappa\sigma} = \xi_{\kappa\sigma} + \mu$.

Knowing the spectral density $\rho_\sigma(\varepsilon)$ of electron average Green function both in a magnetic field and a field of impurities one can determine density of states

$$g_\sigma(\varepsilon) = \frac{1}{V}\sum_\kappa \rho_\sigma(\kappa,\varepsilon). \tag{1.35}$$

Electron state density only in a magnetic field is equal to (1.30). Since the electron energy doesn't depend on p_y and $\sum_{p_y} 1 = \frac{L^2|e|H}{2\pi c\hbar}$ is a degeneracy order of the level $\varepsilon_{np_z\sigma}$ [15], $\sum_{\kappa} = \frac{|e|HL^2}{2\pi c\hbar}\sum_{np_z}$ ($|e|$ is a value of electron charge, L is a dimension of a sample). Thus

$$\nu_\sigma(\varepsilon) = \frac{|e|H}{(2\pi\hbar)^2 c}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty} dp_z \delta\left\{\varepsilon - \hbar\Omega(n+\frac{1}{2}) - \frac{p_z^2}{2m} - \frac{g}{2}\mu_0 H\sigma\right\}, \tag{1.36}$$

where $\Omega = \frac{|e|H}{mc}$ is cyclotron frequency, g is g-factor, μ_0 is Bohr magneton. From here we obtain

$$\nu_\sigma(\varepsilon) = \frac{m^{3/2}\Omega}{2^{3/2}\pi^2\hbar^2}\sum_{n=0}^{n_\sigma(\varepsilon)} \frac{1}{\sqrt{\varepsilon - \hbar\Omega(n+\frac{1}{2}) - \sigma\frac{g}{2}\mu_0 H}},$$

where $n_\sigma(\varepsilon)$ is a maximal value of the n under which the radicand is not negative.

In linear approximation over n_i

$$\rho = \rho_0 + \delta\rho, \tag{1.37}$$

where

$$\delta\rho_\sigma(\kappa,\varepsilon) = -\frac{1}{\pi}\left[G_\sigma^{(0)+}(\kappa,\varepsilon)\right]^2 \mathrm{Im}\, T_\sigma^+(\varepsilon). \tag{1.38}$$

Near the impurity level $\varepsilon_{r\sigma}$ (a root of the equation (1.31)) impurity addition to the spectral density has a form

$$\delta\rho_\sigma(\kappa,\varepsilon) \approx \frac{n_i}{|F_\sigma'(\varepsilon_{r\sigma})|}\frac{1}{(\varepsilon_{r\sigma} - \varepsilon_{\kappa\sigma})^2}\frac{1}{\pi}\frac{\Gamma_\sigma}{(\varepsilon - \varepsilon_{r\sigma})^2 + \Gamma_\sigma^2}, \tag{1.39}$$

where $F_\sigma'(\varepsilon) = dF_\sigma/d\varepsilon$ and

$$\Gamma_\sigma = \frac{\pi\nu_\sigma(\varepsilon_{r\sigma})}{|F_\sigma'(\varepsilon_{r\sigma})|} \tag{1.40}$$

is the width of quasi-local level. In the next chapter it will be shown that a feature of the function (1.39) at $\varepsilon_{\kappa\sigma} = \varepsilon_{r\sigma}$ doesn't affect the properties of new branches in a spin wave spectrum in a degenerated electron liquid. Their characteristics are connected with a sharp peak of the function (1.39) at $\varepsilon = \varepsilon_{r\sigma}$.

A contribution to the density of states near the $\varepsilon_{r\sigma}$ is obtained by summarizing the expression (1.39) over κ:

$$\delta g_\sigma(\varepsilon) \approx \frac{N_i}{V}\frac{1}{\pi}\frac{\Gamma_\sigma}{(\varepsilon-\varepsilon_{r\sigma})^2+\Gamma_\sigma^2}. \tag{1.41}$$

Near the local level $\varepsilon_{l\sigma}$ ($\Gamma_\sigma \to 0$) we find

$$\delta g_\sigma(\varepsilon) = n_i \delta(\varepsilon-\varepsilon_{l\sigma}). \tag{1.42}$$

The function $F_\sigma(\varepsilon)$ is connected with the ν_σ by a dispersion relation

$$F_\sigma(\varepsilon) = \int_{-\infty}^{\infty} d\varepsilon' \frac{\nu_\sigma(\varepsilon')}{\varepsilon-\varepsilon'}, \tag{1.43}$$

where the integral has a principal value sense. This function is equal to

$$F_\sigma(\varepsilon) = -\frac{\pi m^{3/2}\Omega}{2^{3/2}\pi^2\hbar^2}\sum_n \frac{1}{\sqrt{\varepsilon_{n\sigma}-\varepsilon}}, \tag{1.44}$$

where

$$\varepsilon_{n\sigma} = \hbar\Omega\cdot(n+\frac{1}{2}) + \sigma\frac{g}{2}\mu_0 H$$

are Landau levels.

The sum incoming into the (1.44) diverges at large n this is connected with using the δ-potential. The means of eliminating this divergence were discussed in articles [19, 46, 47].

If a distance Δ from an impurity level to a Landau level is small compared to the $\hbar\Omega$ we can leave only singular item

$$-\pi\frac{m^{3/2}\Omega}{2^{3/2}\pi^2\hbar^2}\cdot\frac{1}{\sqrt{\varepsilon_{n\sigma}-\varepsilon}}$$

in the $\sum_n$. Substituting this expression in the equation (1.31) at $v_0 < 0$ we obtain:

$$\Delta = \frac{e^2 m v_0^2 H^2}{8\pi^2 c^2 \hbar^4} \tag{1.45}$$

is a distance between a Landau level and an impurity level splitted off from it.

From the expression (1.45) it is obvious that the impurity states mentioned here are due to combined action of both an attraction impurity ($v_0 < 0$) and a magnetic field on an electron. For this reason they are called magnetoimpurity states [21, 22, 39, 48]. Proper quasi-local levels [10] which are not connected with a magnetic field can be present in the spectrum of electrons in addition to these states. They correspond to complex poles of scattering operator $\varepsilon_{r\sigma} - i\Gamma_\sigma$, where $\varepsilon_{r\sigma}$ are locations of resonances, Γ_σ are their widths. We will assume that only one (spin-splitted) resonance exists. In the next chapter we will consider its influence on a dynamic spin susceptibility of metals.

CHAPTER II. TENSOR OF DYNAMIC SPIN SUSCEPTIBILITY OF METALS WITH QUASI-LOCAL STATES OF ELECTRONS IN A MAGNETIC FIELD

2.1. Dynamic spin susceptibility of electrons

A reaction of a system on a weak variable magnetic field is characterised by a tensor of generalized magnetic susceptibility $\chi_{\mu\nu}(\vec{q},\omega)$ depending on a wave vector $\vec{q}$ and a frequency ω of the field. This tensor plays a fundamental role in the theory of magnetism. It defines a spectrum and damping of magnetic excitations of a system, a spectrum of magnetization thermal fluctuations, a cross-section of neutron magnetic scattering in a magnetics and other values. Let us consider conductivity electron response of paramagnetic metal to a variable magnetic field $\vec{H}(\vec{r},t)$. The latter induces both an orbital and a spin magnetization. Let us take just calculation of dynamic spin susceptibility. Let us use a method of Green functions for this purpose.

2.1.1. Operator of spin magnetization

Hamiltonian of interaction between spin magnetic moments of conductivity electrons and the variable magnetic field has a form:

$$\hat{V}(t) = -\int d^3 r \hat{\vec{M}}(\vec{r})\vec{H}(\vec{r},t), \tag{2.1}$$

where $\hat{\vec{M}}(\vec{r})$ is an operator of spin magnetization. Integration is carried out over the volume of a conductor. It is known that

$$\hat{\vec{M}}(\vec{r}) = -\mu_0 \sum_a \hat{\vec{\sigma}}_a \delta(\vec{r} - \vec{r}_a), \tag{2.2}$$

where $\vec{r}_a$ is a radius-vector of a-th electron, μ_0 is Bohr magneton, $\hat{\sigma}_\mu$ are Pauli matrices [15]. Further we will need a spatial Fourier component of the operator (2.2)

$$\hat{\vec{M}}(\vec{q}) = -\mu_0 \sum_a \hat{\vec{\sigma}}_a e^{-i\vec{q}\vec{r}_a}. \tag{2.3}$$

In a representation of secondary quantization the additive operator (2.3) has a form:

$$\hat{\vec{M}}(\vec{q}) = -\mu_0 \sum_{\vec{k}\alpha_1\alpha_2} (\vec{\sigma})_{\alpha_1\alpha_2} \hat{a}^+_{(\vec{k}-\vec{q})\alpha_1} \hat{a}_{\vec{k}\alpha_2}, \tag{2.4}$$

where $\hat{a}_{\vec{k}\alpha}$ and $\hat{a}^{+}_{\vec{k}\alpha}$ are operators of destruction and creation of electrons with wave vector $\vec{k}$ and spin quantum number $\alpha = \pm 1$, $(\sigma^{\mu})_{\alpha_1\alpha_2}$ are matrix elements of μ-th Pauli matrix.

2.1.2. Dynamic spin susceptibility

It is known from electrodynamics that in linear approximation over the weak field $\vec{H}$ μ-th component of the tensor of electron spin magnetization in a point $\vec{r}$ in the moment t equals to

$$M_{\mu}(\vec{r},t) = \sum_{\nu} \int d^3 r' \int_{-\infty}^{t} dt' \times$$

$$\times \chi_{\mu\nu}(\vec{r}-\vec{r}',t-t') H_{\nu}(\vec{r}',t') . \qquad (2.5)$$

Let us suppose that the medium is homogeneous and stationary so the tensor of susceptibility $\chi_{\mu\nu}$ depends only on differences of spatial $(\vec{r}-\vec{r}')$ and time $(t-t')$ coordinates. Finite upper limit in the integral over time in (2.5) reflects an existence of causality principle, i. e. the fact that the magnetization in the moment t is defined by the magnetic field only in earlier moments of time.

The Fourier transformation (2.5) leads to linear relations connecting spatial-time Fourier components of values incoming in the (2.5):

$$M_{\mu}(\vec{q},\omega) = \sum_{\nu} \chi_{\mu\nu}(\vec{q},\omega) H_{\nu}(\vec{q},\omega) . \qquad (2.6)$$

Dependence $\chi_{\mu\nu}$ on $\vec{q}$ is called spatial dispersion of susceptibility and a dependence on ω is time dispersion.

General properties of susceptibility tensor are evolved in a R.White's book [49]. Let us enumerate them.

A consequence of invariance of Shrödinger equation for electrons relatively to a time inversion operation are Onsager relations:

$$\chi_{\mu\nu}(\vec{q},\omega) = \chi_{\nu\mu}(-\vec{q},-\omega) . \qquad (2.7)$$

An expression of causality principle is Kramers-Kronig dispersion relations binding real and imaginary parts of susceptibility:

$$\mathrm{Re}\left[\chi_{\mu\nu}(\vec{q},\omega) - \chi_{\mu\nu}(\vec{q},\infty)\right] = \frac{1}{\pi} \int_{-\infty}^{\infty} d\omega' \frac{\mathrm{Im}\,\chi_{\mu\nu}(\vec{q},\omega')}{\omega'-\omega} ,$$

$$\mathrm{Im}\,\chi_{\mu\nu}(\vec{q},\omega) = -\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{d\omega'}{\omega'-\omega}\mathrm{Re}\left[\chi_{\mu\nu}(\vec{q},\omega') - \chi_{\mu\nu}(\vec{q},\infty)\right]. \qquad (2.8)$$

A connection between susceptibility and thermal fluctuations of magnetization is a content of fluctuation-dissipation theorem:

$$\int_{-\infty}^{\infty} dt e^{i\omega t}\left\langle\left\{\hat{M}_\mu(\vec{q},t)\hat{M}_\nu(-\vec{q})\right\}\right\rangle_S =$$

$$= \hbar V c t h\frac{\beta\hbar\omega}{2}\mathrm{Im}\,\chi^S_{\mu\nu}(\vec{q},\omega)\,. \qquad (2.9)$$

Here

$$\hat{\tilde{M}}(t) = \exp\left(\frac{i}{\hbar}\hat{H}'t\right)\hat{\tilde{M}}\exp\left(-\frac{i}{\hbar}\hat{H}'t\right)$$

is an operator of magnetization in Heisenberg representation, $\hat{H}'$ is Hamiltonian of electrons (without the (2.1)), $\{\hat{a}\hat{b}\} = \frac{1}{2}(\hat{a}\hat{b}+\hat{b}\hat{a})$ is a symmetrized product of operators, β is inverse temperature, V is a volume of a system. The corner brackets symbolize averaging over Gibbs large canonical ensemble. A symmetrical part of the tensor is marked by index S.

2.1.3. Kubo formula for susceptibility

In 1957 R.Kubo obtained a general expression for the tensor of qeneralized susceptibility connecting this tensor with the correlation function obtained on magnetization operators. Let us obtain the Kubo formula following the method given in works [23, 45, 50].

Spin magnetization of electron gas in variable magnetic field $\vec{H}$

$$\vec{M}(\vec{r},t) = Sp\left[\hat{\rho}\hat{\tilde{M}}_H(\vec{r},t)\right], \qquad (2.10)$$

where $\hat{\rho}$ is a statistical operator for large canonical ensemble; $\hat{\tilde{M}}_H(\vec{r},t)$ is a Heisenberg (with taking into account the (2.1)) operator of magnetization connected with an operator $\hat{\tilde{M}}_Д$ in Dirac representation by a relation:

$$\hat{M}_H(t) = \hat{S}^{-1}(t)\hat{M}_Д(t)\hat{S}(t), \qquad (2.11)$$

where

$$\hat{S}(t) = T_t\exp\left[-\frac{i}{\hbar}\int_{-\infty}^{t}dt'\hat{V}_Д(t')\right] \qquad (2.12)$$

(here T_t is a symbol of chronological arrangement of operators).

Substituting the (2.11) into the (2.10) and taking just a linear item over $\bar{H}$, for the magnetization induced by the variable field we will obtain the expression (2.5) in which

$$\chi_{\mu\nu}(\vec{r}-\vec{r}',t-t') = \frac{i}{\hbar} Sp\left\{\hat{\rho}\left[\hat{M}_{\mu}^{Д}(\vec{r},t),\hat{M}_{\nu}^{Д}(\vec{r}',t')\right]\right\}, \tag{2.13}$$

where $[\hat{a},\hat{b}] = \hat{a}\hat{b} - \hat{b}\hat{a}$ is a commutator.

As a result of Fourier transform of this expression we will obtain the Kubo formula:

$$\chi_{\mu\nu}(\vec{q},\omega) = \frac{i}{\hbar V}\int_0^{\infty} dt e^{i(\omega+i0)t} Sp\left\{\hat{\rho}\left[\hat{M}_{\mu}(\vec{q},t),\hat{M}_{\nu}(-\vec{q},0)\right]\right\}. \tag{2.14}$$

Index $Д$ is omitted here and below.

Substituting the operator (2.4) in the Kubo formula (2.14) we find:

$$\chi_{\mu\nu}(\vec{q},\omega) = -\frac{\mu_0^2}{V}\sum_{\vec{k}_1\vec{k}_2}\sum_{\alpha_1\alpha_2\alpha_3\alpha_4}\sigma^{\mu}_{\alpha_2\alpha_1}\times$$

$$\times\sigma^{\nu}_{\alpha_4\alpha_3}K^{+}_{\omega}\left[(\vec{k}_1-\vec{q})\alpha_2,\vec{k}_1\alpha_1;(\vec{k}_2+\vec{q})\alpha_4,\vec{k}_2\alpha_3\right], \tag{2.15}$$

where K^{+}_{ω} is a Fourier component of two particle electron retarded Green function which is defined by a relation:

$$K^{+}_{t}(1,2;3,4) = -\frac{i}{\hbar}\theta(t)\left\langle\left[a_1^{+}(t)a_2(t),a_3^{+}a_4\right]\right\rangle, \tag{2.16}$$

where $1 = (\vec{k}_1,\alpha_1)$, …, $\theta(t) = \begin{cases}1, t>0,\\ 0, t<0\end{cases}$ is Heavyside function.

2.1.4. Two particle Green function of free electrons

To find K^{+}_{ω} let us calculate at first two particle temperature Green function of electrons [45]:

$$K_{\tau}(1,2;3,4) = -\left\langle T_{\tau}\left[\hat{\tilde{a}}_1(\tau)\hat{a}_2(\tau)\hat{\tilde{a}}_3\hat{a}_4\right]\right\rangle, \tag{2.17}$$

where

$$0 \le \tau \le \beta,\ \hat{a}(\tau) = e^{\tau\hat{H}'}\hat{a}e^{-\tau\hat{H}'};\ \hat{\tilde{a}}(\tau) = e^{\tau\hat{H}'}\hat{a}^{+}e^{-\tau\hat{H}'},$$

T_{τ} is a symbol of chronological arrangement of operators over the variable τ.

Hamiltonian of free electrons

$$\hat{H}' = \sum_{k\alpha} \xi_{\bar{k}} \hat{a}^{+}_{\bar{k}\alpha} \hat{a}_{\bar{k}\alpha}\,, \tag{2.18}$$

where $\xi_k = \varepsilon_k - \mu$, ε_k is a particle dispersion law.

Using (2.18) it's easy to show that

$$\hat{a}(\tau) = \hat{a}e^{-\xi\tau}\,,\ \hat{\bar{a}}(\tau) = \hat{a}^{+}e^{\xi\tau}\,.$$

Let us substitute these operators in the (2.17). Then for calculating K_τ it is necessary to find an average datum $\left\langle \hat{a}_1^{+}\hat{a}_2\hat{a}_3^{+}\hat{a}_4 \right\rangle$. Average data of this type are calculated with the help of Wick theorem [45]:

$$\left\langle \hat{a}_1^{+}\hat{a}_2\hat{a}_3^{+}\hat{a}_4 \right\rangle = \left\langle \hat{a}_1^{+}\hat{a}_2 \right\rangle \left\langle \hat{a}_3^{+}\hat{a}_4 \right\rangle + \left\langle \hat{a}_1^{+}\hat{a}_4 \right\rangle \left\langle \hat{a}_2\hat{a}_3^{+} \right\rangle. \tag{2.19}$$

To a reader who is not familiar with the Wick theorem we propose to derive the (2.19) with the help of commutation relations for the operators of secondary quantization.

Substitution of the (2.19) into the (2.17) leads to the following expression for the two particle temperature Green function of free electrons:

$$K_\tau(1,2;3,4) = -\delta_{12}\delta_{34} f_1 f_2 + G_{23}(\tau) G_{41}(-\tau)\,. \tag{2.20}$$

Here

$$G_{12}(\tau) = -\left\langle T_\tau \left[\hat{a}_1(\tau) \hat{\bar{a}}_2 \right] \right\rangle \tag{2.21}$$

is one particle temperature Green function of electrons; $f_1 = (e^{\beta\xi_1} + 1)^{-1}$ is Fermi function, δ_{12} is Kroneker symbol.

It's easy to show that the first item in the right part of the (2.20) doesn't make a contribution into susceptibility. Time Fourier component of the second item equals to

$$\delta_{14}\delta_{23} \frac{1}{\beta} \sum_{S=-\infty}^{\infty} G_2(\varsigma_S) G_1(\varsigma_S - \hbar\omega_n)\,, \tag{2.22}$$

where $G_1(\varsigma_S) = (i\varsigma_S - \xi_1)^{-1}$ is Fourier component of one particle temperature Green function of free electrons,

$$\varsigma_S = \frac{\pi}{\beta}(2S+1)\,;\ \hbar\omega_n = \frac{2\pi n}{\beta}\ (n \text{ is an integer number}).$$

To calculate the sum in the (2.22) let us consider contour integral

$$\frac{1}{2\pi i} \oint_C dz f(z) (z - \xi_2)^{-1} (z - \xi_1 - i\hbar\omega_n)^{-1}\,, \tag{2.23}$$

in which $f(z)$ is Fermi function and C is a circle of a large radius with a center in the point $z = 0$. When the radius of the circle tends to infinity this integral tends to zero. On the other

hand, according to Cauchy theorem it equals to a sum of residues of the integrand function in poles. The latter has simple poles in points $\xi_1 + i\hbar\omega_n$, ξ_2, $i\varsigma_S$, where $i\varsigma_S$ are poles of Fermi function with residues - β^{-1}. As a result the expression (2.22) takes form:

$$\delta_{14}\delta_{23}\left[f(\xi_2) - f(\xi_1)\right](\xi_2 - \xi_1 - i\hbar\omega_n)^{-1}. \tag{2.24}$$

2.1.5. Dynamic spin susceptibility of free electron gas

Knowing the Fourier component $K(\omega_n)$ of the function (2.20) we can find the K^+_ω. For this reason it's necessary to make an analytical extension of the $K(\omega_n)$ from the discrete set of points $i\hbar\omega_n$ in z-plane into an upper half-plane and then to pass on a real axis [45]. In the given case this procedure comes to change the $i\omega_n$ in the (2.24) into $\omega + i0$. Substituting an expression obtained in this way in the (2.15) we obtain a dynamic spin susceptibility of free electron gas:

$$\chi_{\mu\nu} = \chi\delta_{\mu\nu}, \tag{2.25}$$

where

$$\chi(\vec{q},\omega) = -2\frac{\mu_0^2}{V}\sum_{\vec{k}}\frac{f(\varepsilon_{\vec{k}+\vec{q}}) - f(\varepsilon_{\vec{k}})}{\varepsilon_{\vec{k}+\vec{q}} - \varepsilon_{\vec{k}} - \hbar\omega - i0}.$$

In moving from the (2.15) to the (2.25) a well-known property of Pauli matrices [15] is taken into account: $\sum_{\alpha_1\alpha_2}\sigma^{\mu}_{\alpha_1\alpha_2}\sigma^{\nu}_{\alpha_2\alpha_1} = 2\delta_{\mu\nu}$.

The expression (2.25) satisfies the Onsager relations and the dispersion relations (2.8). It is useful to obtain high-temperature limit of the susceptibility (2.25) when Fermi function f may be changed into Boltzmann one.

At zero temperature for a case $\varepsilon_k = \hbar^2k^2/2m$ (m – effective mass of an electron) from the (2.25) we find

$$\begin{aligned}\mathrm{Re}\,\chi = \chi_0\Bigg\{\frac{1}{2} + \frac{1}{8\lambda}\Bigg[\left(1 - \left(\frac{w}{\lambda} - \lambda\right)^2\right)\ln\left|\frac{1 - \frac{w}{\lambda} + \lambda}{1 + \frac{w}{\lambda} - \lambda}\right| - \\ - \left(1 - \left(\frac{w}{\lambda} + \lambda\right)^2\right)\ln\left|\frac{1 - \frac{w}{\lambda} - \lambda}{1 + \frac{w}{\lambda} + \lambda}\right|\Bigg]\Bigg\};\end{aligned} \tag{2.26}$$

$$\mathrm{Im}\,\chi = \chi_0 \frac{\pi}{8\lambda}\left\{\left[1-\left(\frac{w}{\lambda}-\lambda\right)^2\right]\theta\left[1-\left(\frac{w}{\lambda}-\lambda\right)^2\right]-\left[1-\left(\frac{w}{\lambda}+\right.\right.\right.$$

$$\left.\left.\left.+\lambda\right)^2\right]\theta\left[1-\left(\frac{w}{\lambda}+\lambda\right)^2\right]\right\}, \qquad (2.27)$$

where $\lambda = q/2k_F$; $w = \hbar\omega/4\varepsilon_F$ (k_F and ε_F are Fermi wave number and Fermi energy);

$$\chi_0 = 2\mu_0^2 \frac{m^{3/2}\varepsilon_F^{1/2}}{2^{1/2}\pi^2\hbar^3}$$

is Pauli susceptibility which is obtained from the (2.26) within a limit $\omega \to 0$, $q \to 0$.

2.1.6. Susceptibility in a quantizing magnetic field

A strong magnetic field significantly changes properties of an electron gas in metals and influences dynamic spin susceptibility.

The method of calculation of the susceptibility mentioned above is applicable for the case when a constant and uniform magnetic field $\bar{H}$ is available. But only as one particle basis in the method of secondary quantization it is conveniently to choose eigenstates $|\kappa,\alpha\rangle$ of an electron in a magnetic field. Here κ is a complete set of orbital electron quantum numbers in a magnetic field [15]. Then instead of the formula (2.4) we will have:

$$\bar{M}(\vec{q}) = -\mu_0 \sum_{\kappa_1\kappa_2\alpha_1\alpha_2} (\vec{\sigma})_{\alpha_1\alpha_2} I_{\kappa_1\kappa_2}(-\vec{q}) a^+_{\kappa_1\alpha_1} a_{\kappa_2\alpha_2}, \qquad (2.28)$$

where

$$I_{\kappa_1\kappa_2}(\vec{q}) = \left\langle \kappa_1 \left| e^{i\vec{q}\vec{r}} \right| \kappa_2 \right\rangle \qquad (2.29)$$

is matrix elements of a plane wave in the Landau basis.

The formula (2.15) connecting the tensor of susceptibility with two particle Green function of electrons must be converted into the following:

$$\chi_{\mu\nu}(\vec{q},\omega) = -\frac{\mu_0^2}{V}\sum_{1234}(\sigma^\mu)_{\alpha_2\alpha_1}(\sigma^\nu)_{\alpha_4\alpha_3} \times$$

$$\times I_{\kappa_2\kappa_1}(-\vec{q}) I_{\kappa_4\kappa_3}(\vec{q}) K^+_\omega(2,1;4,3), \qquad (2.30)$$

where now $1 = (\kappa_1,\alpha_1)$, ... Substituting the above-obtained expression for the K^+_ω here we will find a tensor of susceptibility in a quantizing magnetic field:

$$\chi_{\mu\nu}(\vec{q},\omega) = -\frac{\mu_0^2}{V}\sum_{\kappa_1\kappa_2\alpha_1\alpha_2}(\sigma^{\mu})_{\alpha_2\alpha_1}(\sigma^{\nu})_{\alpha_1\alpha_2}\left|I_{\kappa_1\kappa_2}(\vec{q})\right|^2 \times$$

$$\left(f_{\kappa_1\alpha_1} - f_{\kappa_2\alpha_2}\right)\left(\varepsilon_{\kappa_1\alpha_1} - \varepsilon_{\kappa_2\alpha_2} - \hbar\omega - i0\right)^{-1}. \qquad (2.31)$$

Here $\varepsilon_{\kappa\alpha}$ is an energy of an electron in the state $\left|\kappa,\alpha\right\rangle$. In the case of isotropic quadratic dispersion law it equals to [15]:

$$\varepsilon_{nk_z\alpha} = \hbar\omega_C(n+\frac{1}{2}) + \frac{\hbar^2 k_z^2}{2m} + \alpha\mu_0 H, \qquad (2.32)$$

where n is an oscillator quantum number, $\hbar k_z$ is a projection of an electron momentum onto a magnetic field $\vec{H}$, ω_C is cyclotron frequency.

Neglecting a spatial dispersion of susceptibility i.e. putting $\vec{q} = 0$ in the (2.31) we will obtain:

$$\chi_{\mu\nu}(\omega) = -\mu_0^2\sum_{\alpha_1\alpha_2}(\sigma^{\mu})_{\alpha_2\alpha_1}(\sigma^{\nu})_{\alpha_1\alpha_2} \times$$

$$\times\frac{n_{\alpha_1} - n_{\alpha_2}}{\mu_0 H(\alpha_1 - \alpha_2) - \hbar\omega - i0}, \qquad (2.33)$$

where $n_\alpha = \frac{1}{V}\sum_{\kappa} f_{\kappa\alpha}$ is a concentration of electrons with a spin quantum number α.

It is conveniently to calculate a "circular" component of susceptibility χ_{-+} containing "increasing" and "depressing" Pauli matrices $\sigma_{\pm} = \frac{1}{2}(\sigma_x \pm i\sigma_y)$ in the (2.31) and the (2.33). From the (2.33) for it we will obtain an expression

$$\chi_{-+}(\omega) = -\mu_0^2\frac{n_1 - n_{-1}}{2\mu_0 H - \hbar\omega - i0}. \qquad (2.34)$$

Transversal components of susceptibility may be found with the help of relations

$$\chi_{xx}(\omega) = \chi_{yy}(\omega) = \chi_{-+}(\omega) + \chi_{-+}^*(-\omega),$$

$$\chi_{xy}(\omega) = -\chi_{yx}(\omega) = -i\left[\chi_{-+}(\omega) - \chi_{-+}^*(-\omega)\right]$$

which are easily obtainable from the (2.33) and the (2.34).

2.2. An influence of quasi-local electron states on dynamic spin susceptibility of metals in a magnetic field

In this paragraph a dynamic spin susceptibility of conductivity electrons in metals with quasi-local electron states in presence of quantizing magnetic field [51-56] is discussed.

Using the temperature Green function method described in the paragraphs 1.2 and 2.1 we arrive to a formula for a tensor of electron dynamic spin susceptibility of nonferromagnetic metal with quasi-local states of electrons

$$\chi_{ik}(\vec{q},\omega) = -\frac{\mu^2}{V}\sum_{\kappa_1\kappa_2\sigma_1\sigma_2}\sigma^i_{\sigma_2\sigma_1}\sigma^k_{\sigma_1\sigma_2}\left|I_{\kappa_1\kappa_2}(\vec{q})\right|^2\int_{-\infty}^{\infty}d\varepsilon_1\int_{-\infty}^{\infty}d\varepsilon_2\times$$
$$\times\rho_{\kappa_2\sigma_2}(\varepsilon_2)\rho_{\kappa_1\sigma_1}(\varepsilon_1)\frac{f(\varepsilon_1)-f(\varepsilon_2)}{\varepsilon_1-\varepsilon_2-\hbar\omega-i0}. \qquad (2.35)$$

This formula is analogous to the expression (2.31) for the χ_0 of a pure sample.

With taking into account the expansion (1.37) the tensor of dynamic spin susceptibility will be presented in the form of a sum

$$\chi = \chi_0 + \delta\chi\,, \qquad (2.36)$$

where χ_0 is the ideal contribution of electrons in impurityless sample, $\delta\chi$ is impurity addition.

Taking into account the (1.34) and the (1.37) we obtain

$$\chi_{ik}(\vec{q},\omega) = -\frac{\mu^2}{V}\sum_{\kappa_1\kappa_2\sigma_1\sigma_2}\int_{-\infty}^{\infty}d\varepsilon\delta\rho_{\sigma_1}(\kappa_1,\varepsilon)\left|I_{\kappa_1\kappa_2}(\vec{q})\right|^2[f(\varepsilon)-f(\varepsilon_{\kappa_2\sigma_2})]\times$$
$$\times\left\{\frac{\sigma^i_{\sigma_2\sigma_1}\sigma^k_{\sigma_1\sigma_2}}{\varepsilon-\varepsilon_{\kappa_2\sigma_2}-\hbar\omega-i0}+\frac{\sigma^i_{\sigma_1\sigma_2}\sigma^k_{\sigma_2\sigma_1}}{\varepsilon-\varepsilon_{\kappa_2\sigma_2}+\hbar\omega+i0}\right\},$$

where f is Fermi function. From this we notice that $\mathrm{Re}\,\delta\chi_{ik}$ is an even function of a frequency and $\mathrm{Re}\,\delta\chi_{ik}$ is an odd one. From this expression we find circular components of a susceptibility tensor:

$$\chi_{\pm} = \chi_{xx} \pm i\chi_{yx},$$

$$\delta\chi_{\pm} = -\frac{2\mu^2}{V}\sum_{\kappa_1\kappa_2\sigma_1\sigma_2}\int_{-\infty}^{\infty}d\varepsilon\delta\rho_{\sigma_1}(\kappa_1,\varepsilon)[f(\varepsilon)-f(\varepsilon_{\kappa_2\sigma_2})]\left|I_{\kappa_1\kappa_2}(\vec{q})\right|^2\times$$

$$\times\left\{\frac{\sigma^{\mp}_{\sigma_2\sigma_1}\sigma^{\pm}_{\sigma_1\sigma_2}}{\varepsilon-\varepsilon_{\kappa_2\sigma_2}-\hbar\omega-i0}+\frac{\sigma^{\mp}_{\sigma_1\sigma_2}\sigma^{\pm}_{\sigma_2\sigma_1}}{\varepsilon-\varepsilon_{\kappa_2\sigma_2}+\hbar\omega+i0}\right\}.$$

Summing over spin indexes is easily performed. Then

$$\delta\chi_{-}(\vec{q},\omega)=-\frac{2\mu^2}{V}\sum_{\kappa_1\kappa_2}\int_{-\infty}^{\infty}d\varepsilon\delta\rho_{\uparrow}(\kappa_1,\varepsilon)[f(\varepsilon)-f(\varepsilon_{\kappa_2\downarrow})]\left|I_{\kappa_1\kappa_2}(\vec{q})\right|^2\times$$

$$\times(\varepsilon-\varepsilon_{\kappa_2}+\mu H-\hbar\omega-i0)^{-1}-\frac{2\mu^2}{V}\sum_{\kappa_1\kappa_2}\int_{-\infty}^{\infty}d\varepsilon\delta\rho_{\downarrow}(\kappa_1,\varepsilon)\times \tag{2.37}$$

$$\times[f(\varepsilon)-f(\varepsilon_{\kappa_2\uparrow})]\left|I_{\kappa_2\kappa_1}(\vec{q})\right|^2\times(\varepsilon-\varepsilon_{\kappa_2}-\mu H+\hbar\omega+i0)^{-1}.$$

Here $\varepsilon_{\kappa\sigma}=\varepsilon_{\kappa}+\sigma\frac{\hbar\Omega_0}{2}$, Ω_0 is a frequency of spin resonance, the indexes $\uparrow\downarrow$ correspond to $\sigma=\pm1$. We are interested in a contribution of electron impurity states in the $\delta\chi$ that is why in the formula (2.37) the $\delta\rho$ can be taken in the form (1.39). As a result we obtain

$$\delta\chi_{\mp}(\vec{q},\omega)=-\frac{2\mu^2}{V}n_i r_{\substack{\uparrow\\\downarrow}}\sum_{\kappa_1\kappa_2}\left|I_{\kappa_1\kappa_2}(\vec{q})\right|^2\frac{P}{(\varepsilon_r-\varepsilon_{\kappa_1})^2}\int_{-\infty}^{\infty}d\varepsilon\frac{1}{\pi}\frac{\Gamma_{\substack{\uparrow\\\downarrow}}}{(\varepsilon-\varepsilon_{r_{\substack{\uparrow\\\downarrow}}})^2+\Gamma^2_{\substack{\uparrow\\\downarrow}}}\times$$

$$\times[f(\varepsilon)-f\left(\varepsilon_{\kappa_2\substack{\downarrow\\\uparrow}}\right)]\frac{1}{\varepsilon-\varepsilon_{\kappa_2}\pm\mu H-\hbar\omega-i0}- \tag{2.38}$$

$$-\frac{2\mu^2}{V}n_i r_{\substack{\downarrow\\\uparrow}}\sum_{\kappa_1\kappa_2}\left|I_{\kappa_1\kappa_2}(\vec{q})\right|^2\frac{P}{(\varepsilon_r-\varepsilon_{\kappa_1})^2}\int_{-\infty}^{\infty}d\varepsilon\frac{1}{\pi}\frac{\Gamma_{\substack{\downarrow\\\uparrow}}}{\left(\varepsilon-\varepsilon_{r_{\substack{\downarrow\\\uparrow}}}\right)^2+\Gamma^2_{\substack{\downarrow\\\uparrow}}}[f(\varepsilon)-$$

$$-f\left(\varepsilon_{\kappa_2\substack{\uparrow\\\downarrow}}\right)]\frac{1}{\varepsilon-\varepsilon_{\kappa_2}\mp\mu H+\hbar\omega+i0},$$

where

$$r_{\sigma}=\frac{1}{\left|F'_{\sigma}(\varepsilon_{r\sigma})\right|} \tag{2.39}$$

is a residue of amplitude of electron impurity scattering in the pole $\varepsilon_{r\sigma}-i\Gamma_{\sigma}$. In the similar way a contribution of impurity states in a longitudinal component of susceptibility can be obtained:

$$\delta\chi_{zz}(\vec{q},\omega)=-\frac{\mu^2}{V}n_i\sum_{\kappa_1\kappa_2}\left|I_{\kappa_1\kappa_2}(\vec{q})\right|^2\frac{P}{(\varepsilon_r-\varepsilon_{\kappa_1})^2}\sum_{\sigma}r_{\sigma}\int_{-\infty}^{\infty}d\varepsilon\frac{1}{\pi}\frac{\Gamma_{\sigma}}{(\varepsilon-\varepsilon_{r\sigma})^2+\Gamma^2_{\sigma}}\times$$

$$\times[f(\varepsilon)-f(\varepsilon_{\kappa_2\sigma})]\left(\frac{1}{\varepsilon-\varepsilon_{\kappa_2\sigma}-\hbar\omega-i0}+\frac{1}{\varepsilon-\varepsilon_{\kappa_2\sigma}+\hbar\omega+i0}\right). \tag{2.40}$$

In derivating these relations properties of symmetry of matrix elements $I_{\kappa_1\kappa_2}(-\vec{q}) = I^*_{\kappa_2\kappa_1}(\vec{q})$ were taken into account.

The contribution we are interested in is connected with integration in the neighborhood of poles of the Lorentz functions incoming into the (2.38) and the (2.40). Let us note that a pole of the second order in a point $\varepsilon = \varepsilon_{r\sigma}$ doesn't come to the region of integration and thus it is not dangerous. The contribution of the poles of Lorentz function is found equal to

$$\delta\chi_{\mp}(\vec{q},\omega) = -\frac{2\mu^2 n_i}{V} r_{\substack{\uparrow\\\downarrow}} \sum_{\kappa_1\kappa_2} \left|I_{\kappa_2\kappa_1}(\vec{q})\right|^2 \frac{P}{(\varepsilon_r - \varepsilon_{\kappa_1})^2} \times$$

$$\times \frac{f(\varepsilon_{r_{\substack{\uparrow\\\downarrow}}}) - f(\varepsilon_{\kappa_{2\substack{\downarrow\\\uparrow}}})}{\varepsilon_{r_{\substack{\uparrow\\\downarrow}}} - i\Gamma_{\substack{\uparrow\\\downarrow}} - \varepsilon_{\kappa_2} \pm \mu H - \hbar\omega} - \tag{2.41}$$

$$-\frac{2\mu^2 n_i}{V} r_{\substack{\downarrow\\\uparrow}} \sum_{\kappa_1\kappa_2} \left|I_{\kappa_2\kappa_1}(\vec{q})\right|^2 \frac{P}{(\varepsilon_r - \varepsilon_{\kappa_1})^2} \frac{f(\varepsilon_{r_{\substack{\downarrow\\\uparrow}}}) - f(\varepsilon_{\kappa_{2\substack{\uparrow\\\downarrow}}})}{\varepsilon_{r_{\substack{\downarrow\\\uparrow}}} + i\Gamma_{\substack{\downarrow\\\uparrow}} - \varepsilon_{\kappa_2} \mp \mu H + \hbar\omega},$$

$$\delta\chi_{zz}(\vec{q},\omega) = -\frac{\mu^2 n_i}{V} \sum_{\kappa_1\kappa_2} \left|I_{\kappa_2\kappa_1}(\vec{q})\right|^2 \frac{P}{(\varepsilon_r - \varepsilon_{\kappa_1})^2} \sum_{\sigma} r_\sigma \left[f(\varepsilon_{r\sigma}) - \right.$$

$$\left. - f(\varepsilon_{\kappa_2\sigma})\right] \left\{ \frac{1}{\varepsilon_{r\sigma} - i\Gamma_\sigma - \varepsilon_{\kappa_2\sigma} - \hbar\omega} + \frac{1}{\varepsilon_{r\sigma} + i\Gamma_\sigma - \varepsilon_{\kappa_2\sigma} + \hbar\omega} \right\}. \tag{2.42}$$

It may be obtained immediately from the formula (2.37) if we take into account the availability of poles of spectral density of Green function (these poles correspond to the quasi-local states) and if we use Cauchy theorem about residues. The result derived in this way contains characteristics of the quasi-local state i.e. location of the resonance in a complex plane of energy $\varepsilon_{r\sigma} \pm i\Gamma_\sigma$ and a residue r_σ of amplitude of electron impurity scattering in the pole. These values can be calculated if we give concrete expression to scattering potential and a spectrum of electrons in the pure sample or they can be determined through comparison of theory and experiment.

Let us complete an expansion of the expressions (2.41) and (2.42) in a series over the q powers in the long-wavelength approximation. With this aim let us use the relation [57]

$$\sum_{k_{y_1} k_{y_2}} \left|I_{\kappa_1\kappa_2}(\vec{q})\right|^2 = \frac{L^2}{2\pi l^2} \delta_{k_{z_2}, k_{z_1} + q_z} C_{n_1 n_2}(q_\perp),$$

where

$$C_{n_1 n_2}(q_\perp) = \left| \int_{-\infty}^{\infty} d\xi e^{i\sqrt{\frac{\hbar}{m\Omega}} q_\perp \xi} h_{n_1}(\xi) h_{n_2}(\xi) \right|^2, \tag{2.43}$$

$$h_n(\xi) = \frac{1}{\pi^{1/4}\sqrt{2^n n!}} e^{-\xi^2/2} H_n(\xi)$$

are eigenfunctions (normalized on the unit) of one-dimensional oscillator, $l = \sqrt{c\hbar/|e|H}$ is a magnetic length, $q_\perp = \sqrt{q_x^2 + q_y^2}$; $V = L^3$ is normalizing volume. If $\bar{q} = q_\perp l << 1$ an expansion of the function (2.43) has a form [57]

$$C_{nn} = 1 - (n + 1/2)\bar{q}^2 + O(\bar{q}^4),$$

$$C_{n,n+1} = (n+1)\frac{1}{2}\bar{q}^2 + O(\bar{q}^4),$$

$$C_{n,n-1}(q_\perp) = C_{n-1,n}(q_\perp) = n\frac{1}{2}\bar{q}^2 + O(\bar{q}^4),$$

$$C_{n,n+m} = \frac{1}{m!} C_{n+m}^n (\bar{q}^2/2)^m.$$

Taking into account these expansions and taking q^2-order terms we obtain:

$$\delta\chi_\pm(\vec{q},\omega) = -\frac{2\mu^2 n_i}{V}\sum_n \frac{L^2}{2\pi l^2}\sum_{k_z}\left\{ r_{\substack{\downarrow \\ \uparrow}} \frac{f(\varepsilon_{r_\downarrow}) - f(\varepsilon_{nk_z\uparrow_\downarrow})}{\varepsilon_r - i\Gamma_{\substack{\downarrow \\ \uparrow}} - \varepsilon_{nk_z} \mp 2\mu H - \hbar\omega} + \right.$$

$$\left. + r_{\substack{\uparrow \\ \downarrow}} \frac{f(\varepsilon_{r_\uparrow}) - f(\varepsilon_{nk_z\downarrow_\uparrow})}{\varepsilon_r + i\Gamma_{\substack{\uparrow \\ \downarrow}} - \varepsilon_{nk_z} \pm 2\mu H + \hbar\omega} \right\} \times \tag{2.44}$$

$$\times P\left\{ \frac{1}{(\varepsilon_r - \varepsilon_{nk_z})^2} + \frac{\hbar^2 q_z^2/m}{(\varepsilon_r - \varepsilon_{nk_z})^3} + \frac{3\hbar^4 k_z^2 q_z^2/m^2}{(\varepsilon_r - \varepsilon_{nk_z})^4} + \right.$$

$$\left. + \frac{\bar{q}^2}{2}\frac{n+1}{(\varepsilon_r - \hbar\Omega - \varepsilon_{nk_z})^2} + \frac{\bar{q}^2}{2}\frac{n}{(\varepsilon_r + \hbar\Omega - \varepsilon_{nk_z})^2} - \frac{\bar{q}^2(n+1/2)}{(\varepsilon_r - \varepsilon_{nk_z})^2} \right\},$$

$$\delta\chi_{zz}(\vec{q},\omega) = -\frac{\mu^2 n_i}{V}\sum_n \frac{L^2}{2\pi l^2}\sum_{k_z}\sum_\sigma r_\sigma \left[f(\varepsilon_{r\sigma}) - f(\varepsilon_{nk_z\sigma}) \right] \times$$

$$\times \left\{ \frac{1}{\varepsilon_{r\sigma} - i\Gamma_\sigma - \varepsilon_{nk_z\sigma} - \hbar\omega} + \frac{1}{\varepsilon_{r\sigma} + i\Gamma_\sigma - \varepsilon_{nk_z\sigma} + \hbar\omega} \right\} \times \tag{2.45}$$

$$\times P\left\{\frac{1}{(\varepsilon_r-\varepsilon_{nk_z})^2}+\frac{\hbar^2q_z^2/m}{(\varepsilon_r-\varepsilon_{nk_z})^3}+\frac{3\hbar^4k_z^2q_z^2/m^2}{(\varepsilon_r-\varepsilon_{nk_z})^4}+\right.$$

$$\left.+\frac{\overline{q}^2}{2}\frac{n+1}{(\varepsilon_r-\hbar\Omega-\varepsilon_{nk_z})^2}+\frac{\overline{q}^2}{2}\frac{n}{(\varepsilon_r+\hbar\Omega-\varepsilon_{nk_z})^2}-\frac{\overline{q}^2(n+1/2)}{(\varepsilon_r-\varepsilon_{nk_z})^2}\right\}.$$

Let us use a rule of summarizing over Landau quantum numbers:

$$\frac{1}{V}\sum_{nk_yk_z}\psi_n(\varepsilon_{nk_z})=\int_0^\infty d\varepsilon\sum_n\nu_n(\varepsilon)\psi_n(\varepsilon),$$

where $\psi_n(\varepsilon)$ is certain function and

$$\nu_n(\varepsilon)=(m/2)^{3/2}\frac{\Omega}{\pi^2\hbar^2}\cdot\frac{1}{\sqrt{\varepsilon-\varepsilon_n}}\theta(\varepsilon-\varepsilon_n)$$

is density of electron states in n-th Landau zone. With taking into account this rule we obtain:

$$\delta\chi_\pm(\vec{q},\omega)=-2\mu^2n_i(m/2)^{3/2}\frac{\Omega}{\pi^2\hbar^2}\sum_n\int_{\varepsilon_n}^\infty d\varepsilon\left\{r_{\downarrow\atop\uparrow}\frac{f(\varepsilon_{r_\downarrow})-f(\varepsilon\pm\mu H)}{\varepsilon_r-i\Gamma_{\downarrow\atop\uparrow}-\varepsilon\mp2\mu H-\hbar\omega}+\right.$$

$$\left.+r_{\uparrow\atop\downarrow}\frac{f(\varepsilon_{r_\uparrow})-f(\varepsilon\mp\mu H)}{\varepsilon_r+i\Gamma_{\uparrow\atop\downarrow}-\varepsilon\pm2\mu H+\hbar\omega}\right\}\frac{1}{\sqrt{\varepsilon-\varepsilon_n}}P\left\{\frac{1}{(\varepsilon_r-\varepsilon)^2}+\frac{\hbar^2q_z^2/m}{(\varepsilon_r-\varepsilon)^3}+\right.$$

$$+\frac{6\hbar^2q_z^2/m}{(\varepsilon_r-\varepsilon)^4}(\varepsilon-\varepsilon_n)+\frac{\overline{q}^2}{2}\cdot\frac{n+1}{(\varepsilon_r-\varepsilon-\hbar\Omega)^2}+ \qquad (2.46)$$

$$\left.+\frac{\overline{q}^2}{2}\cdot\frac{n}{(\varepsilon_r-\varepsilon+\hbar\Omega)^2}-\frac{\overline{q}^2(n+1/2)}{(\varepsilon_r-\varepsilon)^2}\right\},$$

$$\delta\chi_{zz}=-\mu^2n_i(m/2)^{3/2}\frac{\Omega}{\pi^2\hbar^2}\sum_n\sum_\sigma r_\sigma\int_{\varepsilon_n}^\infty d\varepsilon\left[f(\varepsilon_{r\sigma})-f(\varepsilon+\sigma\mu H)\right]\times$$

$$\times\left\{\frac{1}{\varepsilon_r-i\Gamma_\sigma-\varepsilon-\hbar\omega}+\frac{1}{\varepsilon_r+i\Gamma_\sigma-\varepsilon+\hbar\omega}\right\}\times \qquad (2.47)$$

$$\times\frac{1}{\sqrt{\varepsilon-\varepsilon_n}}P\left\{\frac{1}{(\varepsilon_r-\varepsilon)^2}+\frac{\hbar^2q_z^2/m}{(\varepsilon_r-\varepsilon)^3}+\frac{6\hbar^2q_z^2/m}{(\varepsilon_r-\varepsilon)^4}(\varepsilon-\varepsilon_n)+\right.$$

$$\left.+\frac{\bar{q}^2}{2}\cdot\frac{n+1}{(\varepsilon_r-\varepsilon-\hbar\Omega)^2}+\frac{\bar{q}^2}{2}\cdot\frac{n}{(\varepsilon_r-\varepsilon+\hbar\Omega)^2}-\frac{\bar{q}^2(n+1/2)}{(\varepsilon_r-\varepsilon)^2}\right\}.$$

Here $\varepsilon_n=\hbar\Omega(n+1/2)$, $n=0,1,2,...$

In these formulae if one neglects the width of a quasi-local level ($\Gamma_\sigma=0$) or applies them to a local level and also if one uses a symbolic identity $\frac{1}{x\pm i0}=P\frac{1}{x}\mp i\pi\delta(x)$ the imaginary part of susceptibility turns out to be equal to:

$$\mathrm{Im}\delta\chi_{\pm}(\vec{q},\omega)=-2\pi\mu^2n_i(m/2)^{3/2}\frac{\Omega}{\pi^2\hbar^2}\sum_n\left\{r_{\substack{\downarrow\\\uparrow}}\frac{\theta(\varepsilon_r-\varepsilon_n\mp2\mu H-\hbar\omega)}{\sqrt{\varepsilon_r-\varepsilon_n\mp2\mu H-\hbar\omega}}\times\right.$$

$$\times\left[f(\varepsilon_{r_{\substack{\downarrow\\\uparrow}}})-f(\varepsilon_r\mp\mu H-\hbar\omega)\right]\left[\frac{1}{(\pm2\mu H+\hbar\omega)^2}+\frac{\hbar^2q_z^2/m}{(\pm2\mu H+\hbar\omega)^3}+\right.$$

$$+\frac{6\hbar^2q_z^2/m}{(\pm2\mu H+\hbar\omega)^4}(\varepsilon_r-\varepsilon_n\mp2\mu H-\hbar\omega)+\frac{\bar{q}^2(n+1)}{2(\pm2\mu H+\hbar\omega-\hbar\Omega)^2}+$$

$$\left.+\frac{\bar{q}^2n}{2(\pm2\mu H+\hbar\omega+\hbar\Omega)^2}-\frac{\bar{q}^2(n+1/2)}{(\pm2\mu H+\hbar\omega)^2}\right]- \qquad (2.48)$$

$$-r_{\substack{\uparrow\\\downarrow}}\frac{\theta(\varepsilon_r-\varepsilon_n\pm2\mu H+\hbar\omega)}{\sqrt{\varepsilon_r-\varepsilon_n\pm2\mu H+\hbar\omega}}\left[f(\varepsilon_{r_{\substack{\uparrow\\\downarrow}}})-f(\varepsilon_r\pm\mu H+\hbar\omega)\right]\times$$

$$\times\left[\frac{1}{(\pm2\mu H+\hbar\omega)^2}-\frac{\hbar^2q_z^2/m}{(\pm2\mu H+\hbar\omega)^3}+\frac{6\hbar^2q_z^2/m}{(\pm2\mu H+\hbar\omega)^4}(\varepsilon_r-\varepsilon_n\pm\right.$$

$$\pm2\mu H+\hbar\omega)+\frac{\bar{q}^2(n+1)}{2(\pm2\mu H+\hbar\omega+\hbar\Omega)^2}+\frac{\bar{q}^2n}{2(\mp2\mu H-\hbar\omega+\hbar\Omega)^2}-$$

$$\left.\left.-\frac{\bar{q}^2(n+1/2)}{(\pm2\mu H+\hbar\omega)^2}\right]\right\},$$

$$\mathrm{Im}\delta\chi_{zz}(\vec{q},\omega)=-\pi\mu^2n_i(m/2)^{3/2}\frac{\Omega}{\pi^2\hbar^2}\times$$

$$\times\sum_{n\sigma}r_\sigma\left\{\left[\theta(\varepsilon_r-\varepsilon_n-\hbar\omega)/\sqrt{\varepsilon_r-\varepsilon_n-\hbar\omega}\right]\times\right.$$

$$\left.\times\left[f(\varepsilon_{r\sigma})-f(\varepsilon_r-\hbar\omega+\sigma\mu H)\right]\left[\frac{1}{(\hbar\omega)^2}+\frac{\hbar^2q_z^2/m}{(\hbar\omega)^3}+\right.\right.$$

$$+\frac{6\hbar^2 q_z^2/m}{(\hbar\omega)^4}(\varepsilon_r-\varepsilon_n-\hbar\omega)+\frac{\bar{q}^2(n+1)}{2(\hbar\omega-\hbar\Omega)^2}+\frac{\bar{q}^2 n}{2(\hbar\omega+\hbar\Omega)^2}-\frac{\bar{q}^2(n+1/2)}{(\hbar\omega)^2}\Bigg]-$$

$$-\frac{\theta(\varepsilon_r-\varepsilon_n+\hbar\omega)}{\sqrt{\varepsilon_r-\varepsilon_n+\hbar\omega}}\left[f(\varepsilon_{r\sigma})-f(\varepsilon_r+\hbar\omega+\sigma\mu H)\right]\times \tag{2.49}$$

$$\times\Bigg[\frac{1}{(\hbar\omega)^2}-\frac{\hbar^2 q_z^2/m}{(\hbar\omega)^3}+\frac{6\hbar^2 q_z^2/m}{(\hbar\omega)^4}(\varepsilon_r-\varepsilon_n+\hbar\omega)+$$

$$+\frac{\bar{q}^2(n+1)}{2(\hbar\omega+\hbar\Omega)^2}+\frac{\bar{q}^2 n}{2(\hbar\Omega-\hbar\omega)^2}-\frac{\bar{q}^2(n+1/2)}{(\hbar\omega)^2}\Bigg]\Bigg\},$$

where the discontinuous Heaviside function θ restricts a summarizing over that n wherein the radicals are real.

From these formulae it is evident that dynamic spin susceptibility has root singularities on frequencies of electron resonance transitions between quasi-local level and Landau levels. These transitions are accompanied by spin-flip $\downarrow\rightarrow\uparrow$ in the case of $\delta\chi_-$ and $\uparrow\rightarrow\downarrow$ in the case of $\delta\chi_+$ and they are not accompanied by spin-flip in the case of $\delta\chi_{zz}$. This is due to features of electron state density on Landau levels taking part in the transitions.

The real part of susceptibility can be obtained from the formulae (2.46), (2.47) or from the dispersion relations (2.8) connecting the real and the imaginary parts. It is easy to make sure that the real part of susceptibility also has root singularities on resonance frequencies.

The imaginary parts of susceptibility (2.48) and (2.49) determine energy absorbed by the system of conductivity electrons [23, 58, 59], a differential cross-section of neutron scattering on oscillations of spin magnetization in conductors with quasi-local states of electrons [60]. From the formula (2.48) it is seen that the first item in the right part of the $\mathrm{Im}\delta\chi_-$ has root singularities on frequencies

$$\omega_{rn}=\frac{1}{\hbar}(\varepsilon_r-\varepsilon_n+2\mu H) \tag{2.50}$$

of resonance electron transitions from Landau levels on a quasi-local level with the spin-flip $\downarrow\rightarrow\uparrow$. The second item has a feature of the same type on frequencies

$$\omega_{rn}=\frac{1}{\hbar}(\varepsilon_n-\varepsilon_r+2\mu H) \tag{2.51}$$

of the trasitions $\downarrow\rightarrow\uparrow$ from quasi-local level on Landau levels.

If frequency ω lies near one of the resonance frequencies obtained above, in the sum over n incoming in susceptibility one can determine a resonance item and the rest of the sum can be changed into an integral. It contributes to susceptibility in the absence of a magnetic field

[61]. This contribution leads to threshold effects in the absorption of electromagnetic waves by electrons. These effects are accompanied by umklapps of electrons (localized on impurities) in a conduction band. It is easy to make sure that in the resonance contribution in susceptibility the finite width of the quasi-local level can be taken into account by the change

$$\omega \to \omega + i\Gamma / \hbar . \tag{2.52}$$

As a result near resonance frequencies in the long-wavelength case the components of susceptibility $\delta\chi$ have a form listed in the appendix I. Here resonance frequencies of electron transitions are marked ω_{rn}. The values $a_n(\vec{q})$ incoming in the (I.1)-(I.3) play the role of oscillator forces of resonance transitions. They depend on temperature, strength of a magnetic field, wave vector and are proportional to the concentration of impurity atoms. The differences of Fermi functions incoming in these oscillator forces provide realization of the Pauli principle.

The expressions (I.1)-(I.3) must be taken into account in the dispersion equation for a spectrum of spin waves in a Fermi-liquid of metals. They have features of the

$$\left(\omega - \frac{\varepsilon_r - \varepsilon_n + 2\mu H}{\hbar} + i\frac{\Gamma_{\uparrow}}{\hbar} \right)^{-1/2} \tag{2.53}$$

type or

$$\left(\omega - \frac{\varepsilon_n - \varepsilon_r + 2\mu H}{\hbar} + i\frac{\Gamma_{\downarrow}}{\hbar} \right)^{-1/2} \tag{2.54}$$

on frequencies of the transitions (2.50) and (2.51) respectively.

2.3. Dynamic spin susceptibility of metals with anisotropic Fermi surface

In the previous section the case when prime electron energy spectrum of a non-ferromagnetic metal is isotropic and quadratic was considered. Here dynamic spin susceptibility of electrons in metals whose Fermi surface has a form of revolution ellipsoid is investigated. Results obtained in this paragraph were published in [62].

Let us consider an electron with anisotropic quadratic dispersion law:

$$\varepsilon(\vec{p}) = \frac{1}{2}\sum_{ik} m_{ik}^{-1} p_i p_k , \tag{2.55}$$

where $m_{ki} = m_{ik}$ is a tensor of effective mass. At presence of a magnetic field $\vec{H} \| z$ its Hamiltonian has a form:

$$H_0 = \frac{1}{2}\sum_{ik} m_{ik}^{-1}(-i\hbar\nabla_i + \frac{|e|}{c}A_i)(-i\hbar\nabla_k + \frac{|e|}{c}A_k),$$

where $\vec{H} = rot\vec{A}$; $\vec{A} = (0, Hx, 0)$ is a vector potential, $e = -|e|$ is a charge of an electron.

If isoenergetic surfaces have a form of revolution ellipsoids, a tensor of inverse effective mass is diagonal in principal axes:

$$\left(m_{ik}^{-1}\right)_{pr.axes} = \begin{pmatrix} m_\perp^{-1} & 0 & 0 \\ 0 & m_\perp^{-1} & 0 \\ 0 & 0 & m_\parallel^{-1} \end{pmatrix},$$

where $m_\perp$ and $m_\parallel$ are transversal and longitudinal effective mass.

An orientation of principal axes in accordance with the laboratory system of coordinates is the following. The axes p_y and p'_y coincide. Let us mark θ as an angle between the p'_z and p_z, i.e. between the ellipsoid axis of revolution and a magnetic field. With such a choice of the axes

$$p'_x = p_x\cos\theta - p_z\sin\theta,\ p'_y = p_y,\ p'_z = p_x\sin\theta + p_z\cos\theta. \qquad (2.56)$$

An inverse transformation is:

$$p_x = p'_x\cos\theta + p'_z\sin\theta,\ p_y = p'_y,$$

$$p_z = -p'_x\sin\theta + p'_z\cos\theta.$$

Let us substitute (2.56) in the dispersion law

$$\varepsilon(\vec{p}') = \frac{p_x'^2 + p_y'^2}{2m_\perp} + \frac{p_z'^2}{2m_\parallel}$$

and compare this expression with (2.55). We shall obtain the tensor m_{ik}^{-1} in the laboratory system connected with the magnetic field:

$$m_{ik}^{-1} = \begin{bmatrix} \frac{\cos^2\theta}{m_\perp} + \frac{\sin^2\theta}{m_\parallel} & 0 & \frac{1}{2}(\frac{1}{m_\parallel} - \frac{1}{m_\perp})\sin 2\theta \\ 0 & m_\perp^{-1} & 0 \\ \frac{1}{2}(\frac{1}{m_\parallel} - \frac{1}{m_\perp})\sin 2\theta & 0 & \frac{\sin^2\theta}{m_\perp} + \frac{\cos^2\theta}{m_\parallel} \end{bmatrix}.$$

In this system

$$\hat{H}_0 = \frac{1}{2}\left[\left(\frac{\cos^2\theta}{m_\perp} + \frac{\sin^2\theta}{m_\parallel}\right)\hat{p}_x^2 + \frac{1}{m_\perp}(\hat{p}_y + \frac{|e|}{c}Hx)^2 + \left(\frac{\sin^2\theta}{m_\perp} + \right.\right.$$

$$\left.\left. + \frac{\cos^2\theta}{m_\parallel}\right)\hat{p}_z^2 + (\frac{1}{m_\parallel} - \frac{1}{m_\perp})\sin 2\theta \hat{p}_x \hat{p}_z\right],$$

where $\hat{\vec{p}} = -i\hbar\nabla$.

A solution of the Schrödinger equation with such a Hamiltonian

$$\hat{H}_0\psi = \varepsilon\psi$$

we search in the form [62]

$$\psi(x,y,z) = \chi(x)\exp\left\{ik_y y + ik_z\left[z - \frac{x}{2}\left(\frac{m_\perp - m_\parallel}{M}\right)\sin 2\theta\right]\right\},$$

where $M = m_\perp \sin^2\theta + m_\parallel \cos^2\theta$.

The equation for the $\chi(x)$ has a form:

$$\frac{d^2\chi}{dx^2} + \frac{2m_\perp m_\parallel}{\hbar^2 M}\left\{\varepsilon - \frac{\hbar^2 k_z^2}{2M} - \frac{e^2H^2}{2m_\perp c^2}(x - x_0)^2\right\}\chi = 0,$$

where $x_0 = -\frac{c\hbar k_y}{|e|H}$.

Comparing it with an equation for the oscillator [15] we make sure that the $m_\perp m_\parallel / M$ plays a role of a mass of the oscillator, frequency of the oscillator equals to

$$\frac{|e|H}{m_\perp c}\sqrt{M/m_\parallel}$$

and a radius of a minimal "orbit" is $l_* = \sqrt{\frac{c\hbar}{|e|H}\sqrt{\frac{M}{m_\parallel}}}$.

Substituting these values in a wave function of a stationary state of the oscillator we will find

$$\psi_{nk_yk_z}(x,y,z) = \frac{1}{\sqrt{A_n}}\frac{1}{L}\exp\left\{ik_y y + ik_z\left[z - \frac{x}{2}\left(\frac{m_\perp - m_\parallel}{M}\right)\sin 2\theta\right]\right\}\times$$

$$\times\exp\left[-\frac{1}{2l_*^2}(x - x_0)^2\right]H_n\left[\frac{x - x_0}{l_*}\right], \qquad (2.57)$$

where $A_n = \sqrt{\pi}\,2^n n! l_*$.

An electron energy equals to

$$\varepsilon_{nk_z} = \hbar\omega_C(n+1/2) + \frac{\hbar^2 k_z^2}{2M},$$

where $\omega_C = \frac{|e|H}{m_\perp c}\sqrt{\frac{M}{m_\parallel}}$ is cyclotron frequency [32].

Using (2.57) let us obtain matrix elements of a plane wave in the Landau basis:

$$I_{\kappa\kappa'}(\pm\vec{q}) = \left\langle \kappa \left| e^{\pm i\vec{q}\vec{r}} \right| \kappa' \right\rangle = \delta_{\pm q_y, k_y - k'_y}\delta_{\pm q_z, k_z - k'_z} e^{\pm\frac{i}{2}(x_0+x'_0)\tilde{q}_x}\sqrt{\frac{n!}{n'!}}\times$$

$$\times\exp\left\{-\frac{1}{4}\left[\left(\frac{x_0 - x'_0}{l_*}\right)^2 + (l_*\tilde{q}_x)^2\right]\right\}\times$$

$$\times\left\{\frac{1}{\sqrt{2}l_*}(x_0 - x'_0) \pm \frac{i}{\sqrt{2}}l_*\tilde{q}_x\right\}^{n'-n}\times$$

$$\times L_n^{n'-n}\left(\frac{1}{2}\left[\left(\frac{x_0 - x'_0}{l_*}\right)^2 + (l_*\tilde{q}_x)^2\right]\right)$$

$$(n' \geq n),$$

where

$$\tilde{q}_x = q_x + q_z\frac{m_\perp - m_\parallel}{2M}\sin 2\theta;$$

$$L_n^\alpha(x) = \frac{1}{n!}e^x x^{-\alpha}\frac{d^n}{dx^n}(e^{-x}x^{n+\alpha})$$

are generalized Laguerr polynomials, δ_{k_y,k'_y} is Kronecker symbol.

Calculations analogous to those given in the previous paragraph lead to such a result (see appendix II).

From the (II.1) and (II.2) it is easy to find circular components of susceptibility near the resonance frequencies.

For simplicity we neglect spatial dispersion of susceptibility and restrict to resonance contributions due to transitions QL $\rightarrow$ LL. To take into account a contribution of transitions LL $\rightarrow$ QL it is necessary to change the σ_1 in the F'_{σ_1} and Γ_{σ_1} into the σ_2 and the σ_2 into the σ_1.

Thus, resonance contribution in components of dynamic spin susceptibility tensor of electrons, whose Fermi surface has a form of revolution ellipsoid depends on an angle

between a direction of a magnetic field in which a sample is placed and an axis of ellipsoid revolution.

The formulae of appendix II can be used for calculation of characteristics of spin waves and a cross-section of neutron magnetic scattering in metals with anisotropic Fermi surface.

2.4. A limit case H=0

As it was pointed out above spin susceptibility of metals dependent on frequency and a wave vector defines a series of observable values: a differential cross-section of neutron magnetic scattering by electrons, fluctuation spectrum of spin magnetization and others [23,49,78]. Being sensitive to dynamics of conductivity electrons it experiences an influence of impurity atoms which are present in the sample. The last ones not only restrict length of a carrier free path but also change their energy spectrum.

Under certain conditions in conductors impurity electron states (local and resonance) appear [1,10] that must affect dynamic susceptibility and the values connected with it. As a result a possibility to study impurity electron states appears in the experiments of neutron magnetic scattering.

As a rule local electron states do not appear in metals [13]. Resonance states were discovered in Al with impurity of Cu and with impurities of transition metals [6-9,14]. In these systems impurity atoms lead to d-resonances, localized magnetic moments being absent.

A contribution of resonance electron states on impurity atoms in the dynamic spin susceptibility of simple metals in absence of magnetic field can be obtained from the formulae (2.41) and (2.42) by a limiting transition $H \to 0$. But we would like to revise a method of calculation used in the paragraph 2.2 supposing $H = 0$ [61] from the beginning.

In this section simple metals with non-magnetic impurities are considered. An electron dispersion law of matrix metal is supposed to be parabolic $\varepsilon_{\vec{k}} = k^2 / 2m$ where m and $\vec{k}$ are effective mass and a momentum of an electron. In Al such approximation leads to the error which doesn't exceed 3% [14]. In the proposed theory a pole part of electron scattering amplitude is important. This part doesn't depend on a particular form of the impurity potential [15]. Only characteristics of resonance states (energy and widths of resonances, residues of scattering amplitude in poles) depend on it. In our consideration these values are parameters which can be found by comparison with an experiment. On this reason

here a particular form of scattering potential is not important. The only thing which is important is that its intensity is sufficient for the resonance appearing. One can make a scattering potential self-consistent after subjecting it to the rule of Friedel sums [6-9,14]. Pole structure of scattering amplitude can be taken into account precisely approximating impurity potential by a separable operator [10,17] $|\varphi\rangle u_0 \langle\varphi|$ (u_0 is a constant characterizing intensity of interaction, $\varphi(\vec{r}) = \langle \vec{r}|\varphi\rangle$ is arbitrary function) used in the theory of pseudopotential. This potential can be used for both *s*–resonance description and a description of resonances with non-zero orbital moment. In the last case the function φ must not be invariant relatively to rotations. Consideration of separable potential using for description of diluted alloy properties is contained in [10]. We will calculate dynamic spin susceptibility in I.M.Lifshits model [1,10]. In particular, this model was used for Pauli susceptibility calculation of simple metal alloys with 3*d*-elements at low temperatures when local magnetic moments of impurity atoms are absent [6-9]. It was used also for description of magnetic systems [12].

Dynamic spin susceptibility tensor $\chi_{\mu\nu}(\vec{q},\omega)$ of a conductor coincides with Fourier component of a retarded Green function assembled on spin magnetization operators [23]. For calculation of the last one let us introduce Green temperature function [47]. Writing down magnetization operators in the secondary quantization representation and approximating configuration average mean of two Green electron function product by the product of average then we obtain

$$\chi_{\mu\nu}(\vec{q},\omega) = -\mu_0^2 \int \frac{d^3\vec{k}}{(2\pi)^3} \sum_{\alpha_1\alpha_2} \sigma^{\mu}_{\alpha_1\alpha_2} \sigma^{\nu}_{\alpha_2\alpha_1} \frac{1}{\beta} \sum_{s} G_{\alpha_2}(\vec{k}+\vec{q},\varepsilon_s) \times$$

$$\times G_{\alpha_1}(\vec{k},\varepsilon_s - \omega_n), \qquad (2.58)$$

where $G_{\alpha}(\vec{k},\varepsilon_s)$ is average one-particle temperature Green function of electrons; $\vec{\sigma}_{\mu}$ are Pauli matrices; μ_0 is Bohr magneton; ω_n and ε_s are Matsubara frequencies of bosons and fermions; α is spin quantum number; $\beta = T^{-1}$ is inverse temperature, $k_B = \hbar = 1$.

Function G incoming in (2.58) is connected with the transition operator R by a relation [10] $G = G_0 + G_0 R G_0$ (G_0 is Green function of free electrons). In a case of small concentration of impurity centers the full operator of transition R can be represented into a sum of one-center operators. The last ones can be found precisely [10]. As a result we obtain $G = G_0 + \delta G$, where δG is an addition to the Green function, which is linear over impurity concentration.

An approximation adapted here is equivalent to an account of a set of diagrams with one cross which describe multiple scattering of electrons and holes by an impurity center in electron lap for the (2.58). It means that the frequency ω and a wave number q of external magnetic field satisfy a condition [45] $|\omega_- - qv_F|\tau >> 1$. Here v_F is Fermi velocity; τ is time of electron free path; $\omega_\pm = \omega \pm \varepsilon_q$. In other words, q and ω denote a point (q,ω) on the plane "transferred momentum – transferred energy" which lies far from the region located between the parabolas $\omega = \pm qv_F + \varepsilon_q$ and axis q. In this region the conservation laws of energy and momentum allow forming electron-hole pares by quantum of external field.

For calculation (2.58) it is convenient to use spectral representation of Green function [45] allowing to express χ through spectral density. The last one coincides with imaginary part of retarded Green function. Substituting spectral expansion of G in (2.58), making summarizing over Matsubara frequencies and analytical extension we obtain $\chi = \chi_0 + \delta\chi$, where $\chi_0(q,\omega)$ is spin susceptibility of pure conductor; $\delta\chi$ is impurity addition. It consists of two items: $\delta\chi = \delta\chi_l + \delta\chi_c$ ($\delta\chi_l$ is a contribution of localized electrons)

$$\delta\chi^{(c)}_{\mu\nu}(q,\omega) = \mu_0^2 \int \frac{d^3\vec{k}}{(2\pi)^3} \sum_{\alpha_1\alpha_2} \sigma^{\mu}_{\alpha_1\alpha_2} \sigma^{\nu}_{\alpha_2\alpha_1} \int_0^\infty d\varepsilon [f(\varepsilon) - f(\varepsilon_{\vec{k}\alpha_2})] \times$$

$$\times \frac{\delta\rho_{\alpha_1}(\vec{k}+\vec{q},\varepsilon)}{\varepsilon_{\vec{k}\alpha_2} - \varepsilon + \omega + i0} + (\omega - i0 \to -\omega - i0, \mu \rightleftarrows \nu) \qquad (2.59)$$

is a contribution of transitions in continuous spectrum. Here $\delta\rho_\alpha(\vec{k},\varepsilon)$ is impurity contribution to spectral density of Green function; $f(\varepsilon)$ is Fermi function.

In isotropic paramagnetic the tensor $\chi_{\mu\nu}$ is diagonal. It has both real χ' and imaginary χ'' parts. If $q \to 0$ and $\omega \to 0$ the imaginary part being odd function of frequency turns into zero and the real part coincides with the Pauli susceptibility $\chi_P = \chi_P^{(0)} + \delta\chi_P$ where $\chi_P^{(0)} = 2\mu_0^2 \nu_0(\varepsilon_F)$ is spin susceptibility of pure conductor ($\nu_0(\varepsilon_F)$ is dissolvent density of states on Fermi level ε_F); $\delta\chi_P$ is impurity addition. If the resonance in the electron spectrum is absent or its width Γ is big in comparison with the width of the Fermi distribution thermal disassembly for strongly degenerated electrons we obtain $\delta\chi_P = 2\mu_0^2 \delta\nu(\varepsilon_F)$ where $\delta\nu$ is impurity addition to the state density. In the case of a sharp resonance ($\Gamma << T << \varepsilon_F$) an additional item appears. It equals to

$$\delta\chi_r = 2\mu_0^2 n_i \frac{1}{4T} ch^{-2} \frac{\varepsilon_r - \varepsilon_F}{2T}. \tag{2.60}$$

Here n_i is density of impurity atoms; ε_r is the resonance energy. It is the root of the equation [1,10] $1 - u_0 F_\alpha(\varepsilon) = 0$ getting into the region of continuous spectrum. The contribution (2.60) is evident if ε_r and ε_F are located in a zone of the Fermi distribution disassembly.

Let us consider the imaginary part of susceptibility of isotropic paramagnetic connected directly with the differential cross-section of neutron magnetic scattering by electrons [23]. In the region of frequencies and wave vectors evolved $\chi_0'' = 0$ therefore the impurity part $\delta\chi''$ dominates. If $aq << 1$ (a is radius of scattering potential) a Fourier component of the function $\varphi(\vec{r})$ can be considered as a constant φ_0. In this case one can neglect spatial dispersion of the susceptibility. As a result from the (2.59) we obtain

$$\delta\chi''(\omega) = \frac{m\mu_0^2 u_0 \varphi_0^2 n_i}{\pi^2 \omega^2} \int_0^\infty d\varepsilon k(\varepsilon)[-\operatorname{Im} D^{-1}(\varepsilon + \omega)][f(\varepsilon) -$$

$$- f(\varepsilon + \omega)]\theta(\varepsilon + \omega) - (\omega \to -\omega), \tag{2.61}$$

where $D(\varepsilon)$ is resonance denominator occurring in the theory of quasi-local states [10]; θ is Heaviside function, $k(\varepsilon) = (2m\varepsilon)^{1/2}$.

In the Born approximation over electron-impurity interaction at $\omega << T << \varepsilon_F$ from the (2.61) an expression

$$\delta\chi''(\omega) = 4\pi\mu_0^2 u_0^2 \nu_0^2(\varepsilon_F) n_i / \omega, \tag{2.62}$$

follows valid at $\omega\tau >> 1$.

In the case of the sharp resonance ($\Gamma << T$) under the integral sign in the (2.61) one can suppose that

$$-\operatorname{Im} D^{-1}(\varepsilon) = \pi(u_0 |F'(\varepsilon_r)|)^{-1} \delta(\varepsilon - \varepsilon_r),$$

where derivative over energy in the point ε_r is marked by the prime. As a result $\delta\chi'' = \delta\chi_{r1}'' + \delta\chi_{r2}''$:

$$\delta\chi_{r1}''(\omega) = \frac{m\mu_0^2 \varphi_0^2 n_i}{\pi |F'(\varepsilon_r)| \omega^2} k(\varepsilon_r + \omega)[f(\varepsilon_r) - f(\varepsilon_r + \omega)] \tag{2.63}$$

is contribution of electron transitions from a resonance level into the zone and

$$\delta\chi_{r2}''(\omega) = \frac{m\mu_0^2 \varphi_0^2 n_i}{\pi |F'(\varepsilon_r)| \omega^2} \theta(\varepsilon_r - \omega) k(\varepsilon_r - \omega)[f(\varepsilon_r - \omega) - f(\varepsilon_r)] \tag{2.64}$$

is contribution of zone electron transitions into the resonance level. The expressions (2.63) and (2.64) are proportional to the zone state density and possess typical factors (in square brackets) due to the Pauli principle. If the resonance level is located below the Fermi level only the first item (2.63) remains. At $\omega >> \varepsilon_F - \varepsilon_r$ it decreases according to $\omega^{-3/2}$ law. If then $\varepsilon_r > \varepsilon_F$ the second item (2.64) remains. It has threshold frequency ε_r connected with the transitions of zone bottom electrons into a resonance level. Near threshold $\delta\chi_{r2} \sim (\varepsilon_r - \omega)^{1/2}$. In both cases at $T \to 0$ lower threshold $|\varepsilon_F - \varepsilon_r|$ exists due to the Pauli principle. Taking into account thermal motion of electrons leads to disassembly of this red bound. Let us point out that the maximum items (2.63) and (2.64) exist at the background of the smooth dependence (2.62) due to potential scattering of electrons.

The expressions (2.63) and (2.64) can be checked in the experiments measuring neutron magnetic scattering cross-section in Al with impurity atoms mentioned above. For checking the formula (2.63) one should use diluted alloys of Al with those $3d$-elements (Cr, …, Cu) which are located in Mendeleev periodic chart after the Chromium. They lead to the d-resonance located below the Fermi level [6-9,14]. In alloys of Al with Ti and V the resonance level is located above the Fermi boundary [6-9,14] which corresponds to the formula (2.64). The method of inelastic neutron magnetic scattering was used for measuring dynamic spin susceptibility of Ni above the Curie point [88]. Experimental data are listed at a fig. 3 in [88]. It is not inconceivable that evident non-monotony in distribution of experimental points in interval $\omega = (0{,}05 - 0{,}1)$ eV is connected with the effect of resonance state activation descripted here.

Let us consider static case $\omega \to 0$. Since the imaginary part of the susceptibility turns into zero in a point $\omega = 0$ we can deal with linear term of expansion over ω powers beyond Kohn threshold where $q >> 2k_F$. In the Born approximation at $\omega \to 0$, $q >> 2k_F$, $T << \varepsilon_F$ from (2.59) we obtain

$$\delta\chi'' = \pi\mu_0^2 u_0^2 \nu_0^2(\varepsilon_F) n_i \omega / \varepsilon_q^2 . \tag{2.65}$$

At presence of sharp resonance one should add a resonance contribution

$$\delta\chi_r'' = \chi_P^{(0)} \frac{2n_i \Gamma \omega}{\nu_0(\varepsilon_F)\varepsilon_q^2} \frac{1}{4T} ch^{-2} \frac{\varepsilon_F - \varepsilon_r}{2T} \tag{2.66}$$

to the smooth part (2.65) which is due to potential scattering of electrons.

This expression differs from zero in the case where there is a possibility of thermal excitation of resonance level ($|\varepsilon_F - \varepsilon_r| \lesssim T$). At $T \to 0$ the expression (2.66) comes to a δ-shaped splash in the point $\varepsilon_F = \varepsilon_r$.

Let us list a final expression for the contribution into dynamic spin susceptibility of transitions "local level-zone":

$$\delta\chi_l''(q,\omega) = \chi_P^{(0)} \frac{\pi\varphi_0^2 n_i k_l(\omega)}{k_F |F'(\varepsilon_l)|} \theta(\omega - \omega_g)[f(\varepsilon_l) - f(\varepsilon_l + \omega)] \times$$

$$\times \left[\omega_+^2 - \left(\frac{k_l q}{m} \right)^2 \right]^{-1}, \qquad (2.67)$$

where ε_l is energy of local state; $k_l(\omega) = [2m(\omega - \omega_g)]^{1/2}$; $\omega_g = |\varepsilon_l|$ is threshold frequency. This expression differs from zero at $\omega > \omega_g$, while near the threshold $\delta\chi_l'' \sim (\omega - \omega_g)^{1/2}$. If $T \to 0$ then the threshold moves to a point $\omega_g + \varepsilon_F$. With the increase of frequency the expression (2.67) passes through the maximum and at $\omega >> \omega_g$ decreases proportionally to $\omega^{-3/2}$. As electron local levels in metals are non-known, one should refer to experiments on neutron magnetic scattering in semiconductors for examining the formula (2.67).

CHAPTER III. SPIN WAVES IN NON-FERROMAGNETIC METALS WITH THE QUASI-LOCAL ELECTRON STATES IN THE MAGNETIC FIELD

3.1. A new type of spin waves in metals with the quasi-local states of electrons

It is necessary to take into account the contributions (I.1)-(I.3) in the dispersion equation for the transversal spin wave spectrum. Let us consider spin waves with the right circular polarization "minus" propagating along the magnetic field. The materials of this paragraph are listed at the works [51,53,55,63,65,66,69-72].

Let us take into account an electron-electron interaction in the random phase approximation [73-76] as it was done in [37]. The Landau quantization in this approximation was taken into account in [75,77,78] and the potential scattering of electrons by the impurity atoms (not considering the quasi-local states) was in [79,80]. Dispersion equation for the waves near the frequencies of resonance electron transitions from quasi-local level with the spin "down" (QL$\downarrow$) into the Landau levels with the spin "up" (LL $\uparrow$) has a form [75]:

$$1-\frac{I}{2\mu^2}\left[\chi_-^{(0)}(q,\omega)+\delta\chi_-^{(n)}(q,\omega)\right]=0, \tag{3.1}$$

where I is a constant of Fermi-liquid interaction [44].

At fig. 2 the scheme of these transitions is listed. Spin-splitted Landau levels are painted in solid lines and the quasi-local level is dotted. The transitions $\mathrm{QL}\downarrow\rightarrow\mathrm{LL}\uparrow$ on the resonance frequencies (2.50) are shown by vertical arrows.

The last ones can be rewritten in such a form

$$\omega_{rn}=\omega_0+\Omega_0+n\Omega, \tag{3.2}$$

where Ω_0 is the spin resonance frequency, $n=0,1,...$ is the resonance number, Ω is the cyclotron frequency, $\hbar\omega_0=\varepsilon_L-\varepsilon_r$ the distance between the quasi-local level and the nearest Landau one ε_L lied above. One should mean that mutual location of the quasi-local level

ε_r and the nearest Landau level ε_L can be different (see fig. 2). In the case b) the resonance

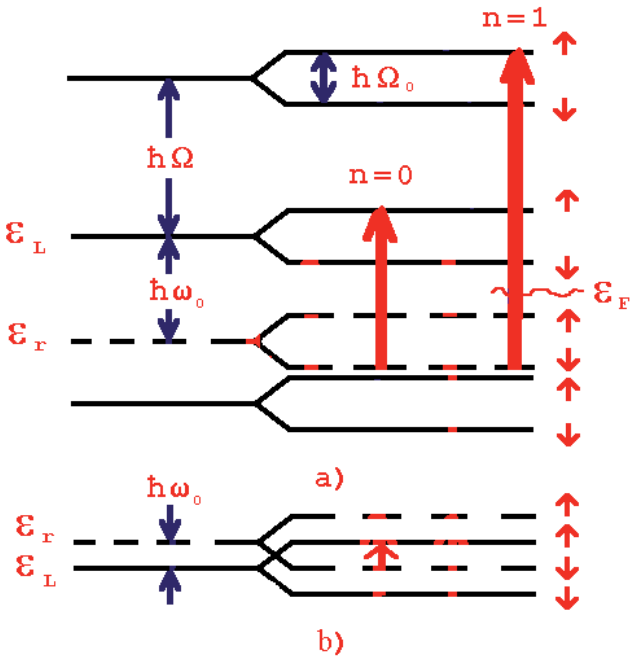


Fig.2. Resonance transitions QL$\downarrow \rightarrow$LL$\uparrow$:

a) scheme of transitions for the case $\varepsilon_L > \varepsilon_r$;

b) the case $\varepsilon_r > \varepsilon_L$.

frequency equals to (3.2) in which it is necessary to suppose $\omega_0 = -|\omega_0|$ where $\hbar|\omega_0| = \varepsilon_r - \varepsilon_L$.

Due to the Fermi function difference in (I.2,b) such transitions are possible if the Fermi boundary ε_F is located between the first and the final levels taking part in the transitions. This difference equals to 1 if ε_F lies far from the initial and the final levels (the width from ε_F to these levels must exceed the disassembly of Fermi step kT). If the field H is reduced some low-frequency resonances are "switched off" due to the difference of Fermi functions, i.e. the Pauli principle.

From the expression (2.31) with taking into account electron scattering in the case of longitudinal propagation ($\vec{q} \| \vec{H}$) of long-wavelength excitations when qv_F and relaxation frequency ν_2 of transversal magnetization are small in comparison with $|\omega - \Omega_0|$ we obtain

$$\chi_{-}^{(0)}(q,\omega)=\chi_P\left\{\frac{\Omega}{\Omega_0-\omega}+\frac{1}{3}\frac{\omega(qv_F)^2}{(\Omega_0-\omega)^3}+i\nu_2\frac{\Omega_0}{(\omega-\Omega_0)^2}\right\}, \tag{3.3}$$

where χ_P is Pauli susceptibility. The part with ν_2 is due to the potential electron scattering on impurities and the other scatters. The Landau quantization in this form is not taken into account since $\hbar\Omega << \varepsilon_F$.

Near the frequencies (3.2) the component (I.2,b) has the resonance. Let us write it in a form

$$\delta\chi_{-}(q,\omega)=\chi_P a_n^{-}(q)i\left(\frac{\omega_{rn}}{\omega-\omega_{rn}+i\Gamma_{-}/\hbar}\right)^{1/2}, \tag{3.4}$$

where Γ_{-} is a half-width of quasi-local level. If one neglects spatial dispersion of the susceptibility, the oscillator forces incoming in this expression will be equal to

$$a_n^{-}=\frac{\pi\hbar\Omega r_{\downarrow}n_i}{2(\hbar\omega_n)^2(\varepsilon_F\hbar\omega_{rn})^{1/2}}\left[f(\varepsilon_{r\downarrow})-f(\varepsilon_{r\downarrow}+\hbar\omega_{rn})\right]. \tag{3.5}$$

Here $\omega_n=\omega_0+n\Omega$.

The real part (3.4) is responsible for the dispersion of spin waves and the imaginary part – for their damping. The root singularities of the imaginary part of (3.4) at $\Gamma_{-}=0$, $\omega\to\omega_{rn}+0$ reproduce the peculiarities of electron state density at Landau levels taking part in the transitions. From the equations (2.8) it follows that the real part of susceptibility will have the same features on the other side of the resonance. In the region $\omega>\omega_{rn}$ the imaginary part is big, i.e. strong damping of spin waves due to resonance transitions $\mathrm{QL}\downarrow\to\mathrm{LL}\uparrow$ takes place. The new branches of the spin wave spectrum can be formed only in the region $\omega\lesssim\omega_{rn}$ where their damping is small.

Substituting (3.3) and (3.4) in the dispersion equation (3.1) we obtain

$$\frac{\Omega_0}{\Omega_0-\omega}\left[1+\frac{1}{3}\frac{\omega}{\Omega_0}\left(\frac{qv_F}{\Omega_0-\omega}\right)^2+i\frac{\nu_2}{\Omega_0-\omega}\right]+$$

$$+a_n^{-}\left(\frac{\omega_{rn}}{\omega_{rn}-\omega-i\Gamma_{-}/\hbar}\right)^{1/2}=\frac{1}{|B_0|}, \tag{3.6}$$

where $|B_0|=Ig_F$ is a parameter describing the Fermi-liquid interaction, g_F is the state density at the Fermi boundary. It is known [28] that $|B_0|<1$. Neglecting the little values ν_2 and Γ_{-} in the (3.6) and assuming $q=0$ we obtain the equation for the limit frequencies, i.e. proper oscillation frequencies of spin magnetization with taking into account the quasi-local state:

$$\frac{\Omega_0}{\Omega_0-\omega}+a_n^-\sqrt{\frac{\omega_{rn}}{\omega_{rn}-\omega}}=\frac{1}{|B_0|}.$$

This equation can be written in the form

$$a_n^-\sqrt{\frac{\omega_{rn}}{\omega_{rn}-\omega}}=[\omega-\Omega_0(1-|B_0|)][|B_0|(\omega-\Omega_0)]^{-1}. \qquad (3.7)$$

From this it is evident that the solutions of this equation can exist in the regions $\omega>\Omega_0$ and $\omega<\Omega_0(1-|B_0|)$.

Thus, new branches of spin waves can be formed both above the corner of the Stoner sector Ω_0 and below the limit frequency of the Silin wave [28]. In the last case the transparency band of the new spin waves lays on the spectrum of the Silin waves.

We solve the dispersion equation (3.6) in the region $\omega\lesssim\omega_{rn}$. Near the ω_{rn} one can assume $\omega=\omega_{rn}$ everywhere except the root in the denominator:

$$\frac{\Omega_0}{-\omega_n}\left[1+\frac{1}{3}\frac{\omega_{rn}}{\Omega_0}\left(\frac{qv_F}{\omega_n}\right)^2-i\frac{\nu_2}{\omega_n}\right]+a_n^-\left(\frac{\omega_{rn}}{\omega_{rn}-\omega-i\Gamma_-/\hbar}\right)^{1/2}=\frac{1}{|B_0|},$$

$$\frac{\omega_{rn}-\omega-i\Gamma_-/\hbar}{\omega_{rn}}=\frac{\left(a_n^-\right)^2}{\left\{\frac{1}{|B_0|}+\frac{\Omega_0}{\omega_n}\left[1+\frac{1}{3}\frac{\omega_{rn}}{\Omega_0}\left(\frac{qv_F}{\omega_n}\right)^2-\frac{i\nu_2}{\omega_n}\right]\right\}^2}.$$

Let us linearize the dispersion equation over the small imaginary additions which lead to the spin wave damping:

$$1-i\frac{\Gamma_-}{\hbar\omega_{rn}}-\left(\frac{a_n^-}{\frac{1}{|B_0|}+\frac{\Omega_0}{\omega_n}}\right)^2\left[1+\frac{1}{3}\frac{\omega_{rn}/\omega_n}{\frac{1}{|B_0|}+\frac{\Omega_0}{\omega_n}}\left(\frac{qv_F}{\omega_n}\right)^2\right]^{-2}-\left(\frac{1}{|B_0|}+\frac{\Omega_0}{\omega_n}\right)^{-3}\times$$

$$\times\frac{\left(a_n^-\right)^2}{\left[1+\frac{1}{3}\frac{\omega_{rn}/\omega_n}{\frac{1}{|B_0|}+\frac{\Omega_0}{\omega_n}}\left(\frac{qv_F}{\omega_n}\right)^2\right]^3}2i\frac{\nu_2\Omega_0}{\omega_n^2}=\frac{\omega}{\omega_{rn}}.$$

It is well seen that the solution of this equation has a form

$$\omega = \omega_n(q) - i\gamma_n(q), \tag{3.8}$$

where

$$\omega_n(q) = \omega_{rn}\left\{1 - \left[a_n^- b_n \frac{\omega_n/\omega_{rn}}{1+\frac{1}{3}b_n(qv_F/\omega_n)^2}\right]^2\right\} \tag{3.9}$$

is the dispersion law of the spin waves and

$$\gamma_n(q) = \frac{\Gamma_-}{\hbar} + 2\nu_2(a_n^-)^2 b_n^3 \frac{\Omega_0\omega_n}{\omega_{rn}^2}\left[1+\frac{1}{3}b_n\left(\frac{qv_F}{\omega_n}\right)^2\right]^{-3} \tag{3.10}$$

is wave damping decrement. Here a designation

$$b_n = \frac{\omega_{rn}|B_0|}{\omega_n + \Omega_0|B_0|}$$

is introduced.

In the long-wavelength limit with taking into account the item with νq^2 we obtain

$$\gamma_n(q) = \frac{\Gamma_-}{\hbar} + 2\nu_2(a_n^-)^2 b_n^3 \frac{\omega_n}{\omega_{rn}} + \frac{2}{3}\nu\left(\frac{qv_F}{\omega_n}\right)^2 (a_n^-)^2 b_n^2 \frac{\omega_n}{\omega_{rn}},$$

where ν is the frequency of electron collisions accompanied by a momentum relaxation and the part with $\nu_2 q^2$ is omitted.

In the fig. 3 the new branch of the spin wave spectrum is shown at $n = 0$. The branches with $n = 1,2,...$ are located at higher frequencies.

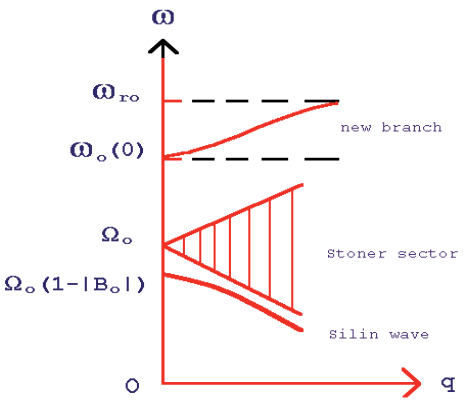


Fig.3. The dispersion curve (3.9) location at $n = 0$ relatively to the Stoner sector and the Silin wave spectrum.

The relationship $\gamma_n(0) - \Gamma_- / \hbar$ to the first item in the right part of (3.10) equals to:

$$\frac{\gamma_n(0) - \Gamma_- / \hbar}{\Gamma_- / \hbar} = \frac{2\nu_2 \hbar (a_n^-)^2 b_n^3 \Omega_0 \omega_n}{\Gamma_- \omega_{rn}^2}. \tag{3.11}$$

The estimations listed below show that this expression is usually little in comparison with the 1. Presence of small values ν_2 and Γ_- in the damping decrement (3.10) means that the spin waves slightly damp in transparency bands $[\omega_n(0), \omega_{rn}]$ located between the resonance frequencies and the proper oscillation frequencies.

The limit frequency $\omega_0(0)$ can be obtained directly from the equation (3.7):

$$\omega_0(0) = \omega_{r0}\left[1 - \left(\frac{a_0^- \omega_0 |B_0|}{\omega_0 + \Omega_0 |B_0|}\right)^2\right].$$

The width of the transparency band for the waves with $n = 0$ equals to

$$\Delta\omega_0 = \omega_{r0} - \omega_0(0) = \omega_{r0}\left(\frac{a_0^- \omega_0 |B_0|}{\omega_0 + \Omega_0 |B_0|}\right)^2.$$

In the general case:

$$\Delta\omega_n = \omega_{rn}(a_n^- b_n \omega_n / \omega_{rn})^2. \tag{3.12}$$

The dispersion of these waves is normal. They will be slightly damped if the width of transparency band (3.12) exceeds the damping decrement (3.10). If

$$\frac{\Gamma_-}{\hbar} >> 2\nu_2 (a_n^-)^2 b_n^3 \frac{\Omega_0 \omega_n}{\omega_{rn}^2},$$

this condition has a form

$$\frac{\hbar \Delta\omega_n}{\Gamma_-} = \frac{\pi^2}{4} b_n^2 \left(\frac{\Omega}{\omega_{rn}}\right)^2 \frac{r_\downarrow^2 n_i^2}{\varepsilon_F \Gamma_- \omega_n^2 \hbar^2} > 1. \tag{3.13}$$

With increasing n the widths (3.12) and the oscillator forces (3.5) decrease proportionally to n^{-4} and $n^{-5/2}$ respectively. At

$$n \gtrsim n_m = \left(\frac{\pi^2 I^2 g_F^2 r_\downarrow^2 n_i^2}{\hbar^2 \varepsilon_F \Omega^2 \Gamma_-} \right)^{1/4} \tag{3.14}$$

the band width is compared to $\Gamma_- / \hbar$ and one could not resolve it.

The condition (3.13) allows to define the minimal concentration of impurity starting with which the width of n-th transparency band exceeds $\Gamma_- / \hbar$:

$$n_i^{\min} = \frac{2}{\pi} \cdot \frac{\omega_n}{\Omega} \cdot \frac{\hbar(\omega_n + \Omega_0 |B_0|)}{r_\downarrow |B_0|} (\varepsilon_F \Gamma_-)^{1/2}. \tag{3.15}$$

Dynamic spin susceptibility χ_- also has the root singularities at the frequencies of resonance transitions $\downarrow \rightarrow \uparrow$ between the Landau levels and the quasi-local level shown at fig. 4.

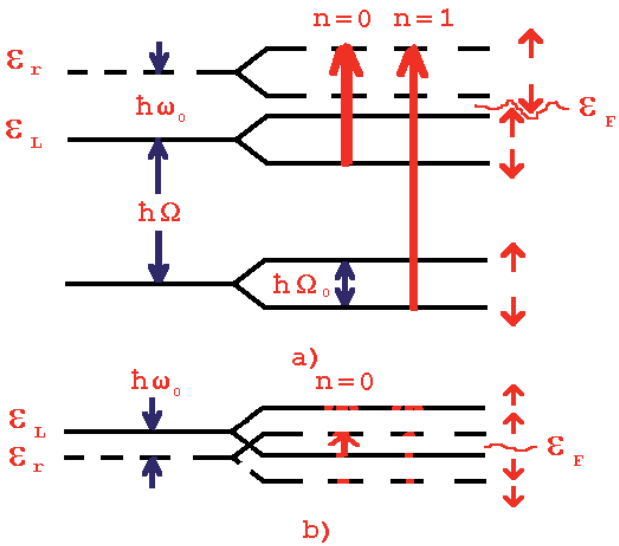


Fig. 4. Resonance transitions LL$\downarrow \rightarrow$QL$\uparrow$:

a) transition scheme for the case $\varepsilon_r > \varepsilon_L$;

b) the case $\varepsilon_r < \varepsilon_L$.

It is well shown from this figure that the resonance frequencies are equal to $\omega_{rn} = \omega_0 + \Omega_0 + n\Omega$ as in the previous case but now $\hbar\omega_0 = \varepsilon_r - \varepsilon_L$ and n is limited from above by the number of the filled Landau levels. In this case the resonance part of susceptibility and the oscillator force equal to

$$\delta\chi_-(q,\omega) = \chi_P a_n^-(q) i\left(\frac{\omega_{rn}}{\omega_{rn} - \omega - i\Gamma_-/\hbar}\right)^{1/2}$$

and

$$a_n^- = \frac{\pi\hbar\Omega r_\uparrow n_i}{2(\hbar\omega_n)^2(\varepsilon_F\hbar\omega_{rn})^{1/2}}\left[f(\varepsilon_{r\uparrow} - \hbar\omega_{rn}) - f(\varepsilon_{r\uparrow})\right]$$

respectively. Weak damping solution of the equation (3.1) exists now only at $n = 0$ if $\omega_0 < 0$ ($\varepsilon_r < \varepsilon_L$ but $\varepsilon_{r\uparrow} > \varepsilon_{L\downarrow}$) and $|\omega_0| < \Omega_0|B_0|$. This case is shown on the fig. 4, b. The dispersion law of the spin waves in the vicinity of the frequency $\omega_{r0} = \Omega_0 - |\omega_0|$ has a form

$$\omega_0(q) = \omega_{r0}\left\{1 + \left[a_0^- b_0 \frac{\omega_0/\omega_{r0}}{1 + \frac{1}{3}b_0(qv_F/\omega_0)^2}\right]^2\right\}, \tag{3.16}$$

and damping decrement is:

$$\gamma_0(q) = \frac{\Gamma_+}{\hbar} + 2\nu_2(a_0^-)^2 b_0^3 \frac{\Omega_0|\omega_0|}{\omega_{r0}^2}\left[1 + \frac{1}{3}b_0\left(\frac{qv_F}{\omega_0}\right)^2\right]^{-3}. \tag{3.17}$$

Here

$$b_0 = \frac{\omega_{r0}|B_0|}{\Omega_0|B_0| - |\omega_0|}.$$

The transparency width for these waves is located between the central Stoner sector and the branch of the quasi-classical Silin wave. Dispersion curve (3.16) is shown schematically on the fig. 5. The dispersion of the wave is anomalous in contrast to (3.9). The width of the transparency band is defined by the expression

$$\Delta\omega_0 = \omega_{r0}(a_0^- b_0|\omega_0|/\omega_{r0})^2.$$

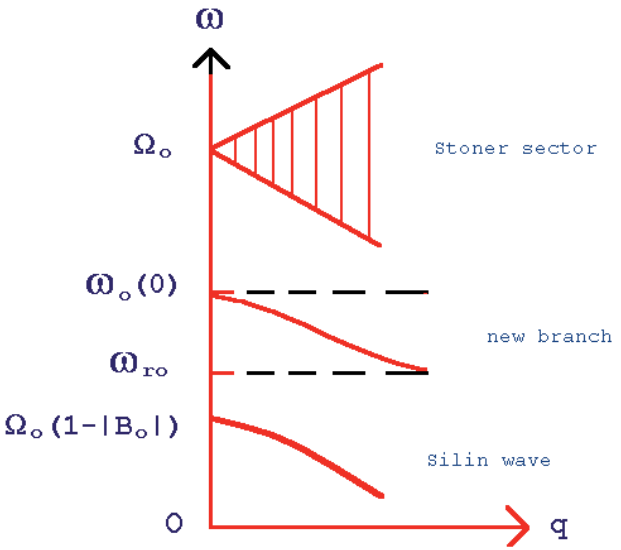


Fig.5. The location of the dispersion curve
(3.16) relatively to the Stoner sector
and the Silin wave spectrum.

The quasi-local electron states in the absence of the magnetic field were discovered in Bismuth with the impurities of IV and VI qroups of elements [81]. For example, the impurities of Sn, Pb and Se, Te lead to the quasi-local levels located below and above the Fermi boundary respectively. For the estimations we use the residue $r_{\downarrow}$ calculated in the model of Gauss separable potential [46,82]. Substituting in the formulae (3.2), (3.11)-(3.15) the values $\varepsilon_F = 4{,}8 \cdot 10^{-14}$ erg, $\varepsilon_r = 4{,}6 \cdot 10^{-14}$ erg, $m = 10^{-29}$ g, $\Gamma_- / \hbar\omega_0 = 10^{-3}$, $|B_0| = 0{,}1$, $n_i = 10^{-2}$ at. %, $v_2 = 10^6$ s^{-1}, $H = 10^3$ Oe, we obtain $\omega_{r0} = 1{,}5 \cdot 10^{12}$ s^{-1}, $\Delta\omega_0 / \omega_{r0} = 7{,}7 \cdot 10^{-3}$, $\hbar\omega_0 / \Gamma_- = 8{,}7$, $n_m = 4$, $n_i^{\min} = 1{,}9 \cdot 10^{18}$ sm^{-3}, $(\gamma_0(0) - \Gamma_- / \hbar) / \frac{\Gamma_-}{\hbar} = 1{,}7 \cdot 10^{-6}$.

Thus, in Bi with impurities of Sn or Pb one can observe a few branches of the spin waves with the spectrum (3.9), for example, in the experiments with slow neutrons.

3.2. Spin waves propagating at the angle to the magnetic field

In the previous paragraph the spectrum and the damping decrement of spin waves propagating along quantizing magnetic field were calculated. In this paragraph the waves whose wave vector is oriented arbitrarily in relation to the field direction are considered [71].

Circularly polarized spin waves with the right circular polarization weakly damp near the frequencies of electron transitions $\downarrow\rightarrow\uparrow$ from the quasi-local level into the Landau level and are propagated at the angle α to the magnetic field. Their spectrum is defined from the equation

$$\frac{\Omega_0}{\Omega_0-\omega}+i\omega\frac{\nu_2+q^2D_-}{(\omega-\Omega_0)^2}+a_n^-\left(\frac{\omega_{rn}}{\omega_{rn}-\omega-i\Gamma_-/\hbar}\right)^{1/2}=\frac{1}{|B_0|}, \tag{3.18}$$

in which

$$D_-=i\frac{v_F^2}{3}\left[\frac{\cos^2\alpha}{\omega+i\nu-\Omega_0}+\frac{\sin^2\alpha(\omega+i\nu-\Omega_0)}{(\omega+i\nu-\Omega_0)^2-\Omega^2}\right],$$

ν is a frequency of electron momentum relaxation. As regards the dispersion of $\delta\chi_-$ one can neglect it since q^2 incomes as a factor at a little impurity concentration n_i.

From the dispersion equation (3.18) near the resonance frequency (3.2) of the above mentioned transitions we obtain the wave dispersion law:

$$\omega_n(\vec{q})=\omega_{rn}\left\{1-\frac{(a_n^-)^2b_n^2(\omega_n/\omega_{rn})^2}{\left[1+\frac{1}{3}b_n(qv_F/\omega_n)^2A_n(\alpha)\right]^2}\right\}, \tag{3.19}$$

where

$$A_n(\alpha)=\frac{1-(\Omega/\omega_n)^2\cos^2\alpha}{1-(\Omega/\omega_n)^2}.$$

From the formula (3.19) it is well shown that there is the critical angle α_c at which the normal dispersion of the waves changes into the anomalous one. It is defined from an equation

$$\cos\alpha_c=\frac{\omega_0}{\Omega}. \tag{3.20}$$

If $\alpha<\alpha_c$ the dispersion of the wave with the spectrum (3.19) is normal. At $\alpha>\alpha_c$ the dispersion becomes anomalous. The wave dispersion in the bands $n=1,2,...$ remains normal at any value of α. In the case of the waves near the frequencies of transitions $\downarrow\rightarrow\uparrow$ from the Landau level into the quasi-local level, which were considered in the previous paragraph, the

situation is reverse. Their dispersion is anomalous at $\alpha < \alpha_c$. From the formula (3.20) it is well shown that with increasing the magnetic field the angle α_c decreases if one neglects a dependence of the quasi-local level location on the magnetic field. In the case of the magnetoimpurity states $\omega_0 \sim H^2$, i.e. the angle α_c also decreases with increasing magnetic field. Let us point out that the critical angle also exists in the Silin theory [28]. It is defined by the parameters of the Fermi-liquid interaction and does not depend on a magnetic field.

In the long-wavelength limit the damping decrement of the waves with the spectrum (3.19) equals to

$$\gamma_n(\vec{q}) = \frac{\Gamma_-}{\hbar} + 2\nu_2 (a_n^-)^2 b_n^3 \frac{\omega_n}{\omega_{rn}} + \frac{2}{3}\nu \left(\frac{qv_F}{\omega_n}\right)^2 (a_n^-)^2 b_n^3 \frac{\omega_n}{\omega_{rn}} C_n(\alpha),$$

where

$$C_n(\alpha) = \cos^2\alpha + \sin^2\alpha \frac{1+(\Omega/\omega_n)^2}{\left[1-(\Omega/\omega_n)^2\right]^2}.$$

Function C_0 and damping decrement γ_0 have the minimum at $\alpha = \pi/2$ if $\Omega/\omega_0 < \sqrt{3}$. The widths of the transparency bands for the spin waves do not depend on α. As in (3.12) they are equal to

$$\Delta\omega_n = \omega_{rn}\left(a_n^- \frac{\omega_n |B_0|}{\omega_n + \Omega_0 |B_0|}\right)^2.$$

Let us consider the resonance transitions of electrons from the quasi-local level into the Landau levels. These transitions are accompanied by the spin-flip $\uparrow \to \downarrow$. The scheme of those transitions is listed at a fig. 6. In this case the frequencies ω_{rn} of electron transitions are

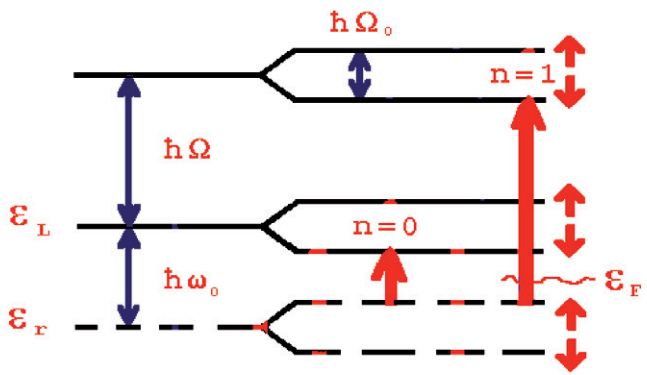


Fig. 6. The resonance transitions QL$\uparrow \to$ LL$\downarrow$.

$$\omega_{rn} = \omega_0 - \Omega_0 + n\Omega, \tag{3.21}$$

where ω_0 is the distance between the quasi-local level ε_r non-splitted due to the spin and the next Landau level located above it; $n = n_1,...$ (n_1 is the number of the Landau levels between ε_r and the Fermi boundary). The circular component of magnetic electron susceptibility χ_+ has the root singularities due to the state density at the Landau levels. From the expression for χ_0 in the long-wavelength limit ($qv_F << 1$, $\nu_2 << |\omega - \Omega_0|$) in the case $\vec{q} \| \vec{H}$ we obtain [28]

$$\chi_+^{(0)}(q,\omega) = \chi_P \left\{ \frac{\Omega}{\Omega_0 + \omega} + i\omega \frac{\nu_2 + q^2 D_+}{(\omega + \Omega_0)^2} \right\}, \tag{3.22}$$

where

$$D_+ = i\frac{v_F^2}{3}\left[\frac{\cos^2\alpha}{\omega + \Omega_0 + i\nu} + \frac{\sin^2\alpha(\omega + \Omega_0 + i\nu)}{(\omega + \Omega_0 + i\nu)^2 - \Omega^2} \right].$$

On the frequencies (3.21) the component

$$\delta\chi_+(q,\omega) = \chi_P a_n^+ i \left(\frac{\omega_{rn}}{\omega - \omega_{rn} + i\Gamma_+/\hbar} \right)^{1/2}, \tag{3.23}$$

has the resonance. In this expression the oscillator forces of the resonance transitions in the neglecting of spatial dispersion of susceptibility $\delta\chi_+$ equal to

$$a_n^+ = \frac{\pi\hbar\Omega r_\uparrow n_i}{2(\hbar\omega_n)^2(\varepsilon_F \hbar\omega_{rn})^{1/2}} \left[f(\varepsilon_{r\uparrow}) - f(\varepsilon_{r\uparrow} + \hbar\omega_{rn}) \right].$$

The dispersion equation for the spin waves with taking into account the contributions (3.22) and (3.23) has a form

$$\frac{\Omega_0}{\omega + \Omega_0} + i\omega \frac{\nu_2 + q^2 D_+}{(\omega + \Omega_0)^2} + a_n^- \left(\frac{\omega_{rn}}{\omega_{rn} - \omega - i\Gamma_+/\hbar} \right)^{1/2} = -\frac{1}{|B_0|}.$$

Its analyses shows that there are the transparency bands for slightly-damping transversal spin waves with the "left" circular polarization below frequencies (3.21). The dispersion law of these waves equals to

$$\omega_n(q) = \omega_n(0) \left\{ 1 + \frac{2}{3}\frac{A_n}{a_n^+} \cdot \frac{\omega_{rn}}{\omega_n} \left(\frac{\Delta\omega_n}{\omega_{rn}} \right)^{3/2} \left(\frac{qv_F}{\omega_n} \right)^2 \right\} \tag{3.24}$$

and the damping decrement in the long-wavelength limit has a form

$$\gamma_n(q) = \frac{\Gamma_+}{\hbar} + \frac{2}{a_n^+}\left(\frac{\omega_{rn}}{\omega_n}\right)^2\left(\frac{\Delta\omega_n}{\omega_{rn}}\right)^{3/2}\left[\nu_2 + \frac{1}{3}\nu C_n\left(\frac{qv_F}{\omega_n}\right)^2\right], \tag{3.25}$$

where $\Delta\omega_n = \omega_{rn}(a_n^+)^2 b_n^2(\omega_n/\omega_{rn})^2$ are the transparency band widths. Here

$$b_n = \frac{|B_0|\omega_{rn}}{\omega_n - |B_0|\Omega_0}.$$

If one consider reversal transitions from the Landau level into the quasi-local level with the spin-flip $\uparrow\rightarrow\downarrow$ it turns out that the dispersion equation does not have slightly-damping solutions. The spin waves near the frequencies of these transitions are absent.

The expressions (3.24) and (3.25) are also valid in the case of the magnetoimpurity levels splitted from the each Landau level with the value $\hbar\omega_0$. Now the scattering amplitude residue of electrons by the impurity atom in the pole $\varepsilon_r - i\Gamma$ equals to [82]

$$r = \frac{2\pi\hbar^2}{\Omega}(2\hbar\omega_0/m)^{3/2}$$

and oscillator forces contain an additional summarizing over the numbers of the magnetoimpurity levels which take part in electron transitions between the magnetoimpurity levels and the Landau levels on the given frequency [37].

3.3. Quantum spin waves in the non-ferromagnetic metals with the quasi-local states of electrons

Quantum spin waves in the non-ferromagnetic metals without taking into account quasi-local states of electrons are considered in the works [42,77,78] and the same waves with taking into account these states are described in the works [54,56,83].

Let us consider transversal quantum spin waves propagating along the magnetic field. The oscillations of the electron spin magnetization occur in the plane (x, y) which is normal to the magnetic field. If one neglects the orbital quantization of the electron motion, the spin waves with the "left" circular polarization whose spectrum is defined by the dispersion equation (3.1) will experience collisionless damping in the region of the plane (q,ω) limited by the parabolas

$$\omega = \Omega_0 + qv_F + \omega_q,$$

$$\omega = \Omega_0 - qv_F + \omega_q$$

(for the electrons $\Omega_0 > 0$). Here $\omega_q = \hbar q^2 / 2m$.

The Landau quantization leads to the fact that inside of this region transparency windows appear, in which quantum spin waves can propagate [77,78]. Let us make sure of this.

Let us restrict ourselves by the consideration of the spin waves with the "left" circular polarization propagating along the magnetic field. In the absence of impurities they damp due to Cherenkov absorption of spin waves by electrons [29]. To find regions of damping on the plane (q,ω) let us use the laws of energy

$$\hbar\omega + \varepsilon_n^{\sigma}(k_z) = \varepsilon_{n'}^{\sigma'}(k_z') \tag{3.26}$$

and z-momentum component conservation

$$\hbar q + \hbar k_z = \hbar k_z' \tag{3.27}$$

at the magnon absorption by the electron. Here $\varepsilon_n^{\sigma}(k_z)$ is electron energy (2.32) before absorption, $\varepsilon_{n'}^{\sigma'}(k_z')$ the energy after it. According to conservation laws let us take into account Pauli principle:

$$\begin{aligned} &\varepsilon_n^{\sigma}(k_z) \le \varsigma, \\ &\varepsilon_{n'}^{\sigma'}(k_z') \ge \varsigma, \end{aligned} \tag{3.28}$$

where ς is chemical potential of electrons in the magnetic field at zero temperature ($T = 0$). Let us take into account the selection rules [42]

$$\begin{aligned} &\Delta n = n' - n = 0, \\ &\Delta\sigma = \sigma' - \sigma = 2 \end{aligned} \tag{3.29}$$

for the transitions $\downarrow \to \uparrow$ between the states within one Landau zone.

Form the relations (3.26)-(3.29) it follows that the Landau damping exists in the hatching regions at the fig. 7. For simplicity we restrict ourselves the case of one filled Landau zone with $n = 0$. At this figure v_n^{σ} is a velocity projection of the electron which belongs to the n^{σ}-th Landau tube on the magnetic field, $k_n^{\sigma} = m v_n^{\sigma} / \hbar$, $\sigma = \pm 1$. Indices + and – correspond the electron spin orientation along and against the magnetic field respectively.

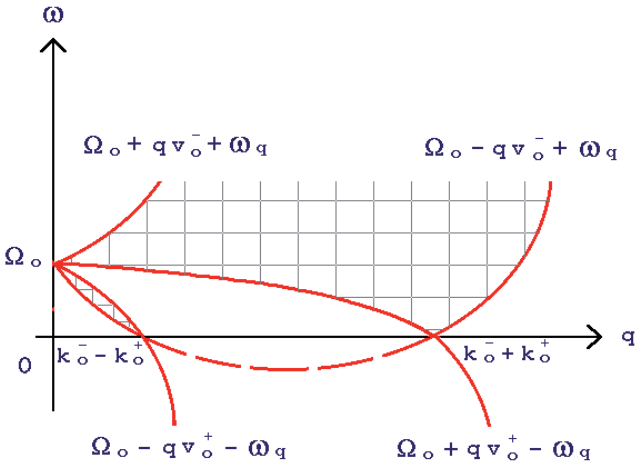


Fig. 7. The regions of collisionless wave damping in the case of one filled Landau zone.

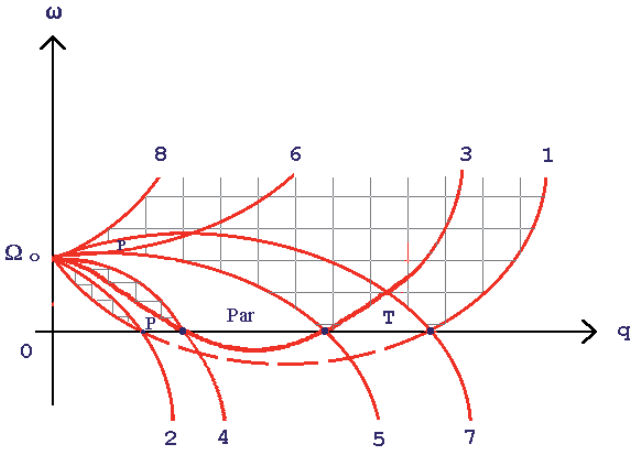
At fig. 8 the Landau damping regions are hatched in the case of two filled Landau zones. Besides it is supposed that $\Omega > \Omega_0$, i.e. $v_{n+1}^- < v_n^+$. From the fig. 8 it is well-shown that three types of the transparency windows (P – petalous, Par – parabolic, T – triangular) exist.

At fig. 9 the Landau damping regions are shown (hatched) at little q. By $N+1$ the number of filled Landau levels is designated. The magnetic field is selected in such a way that the number of filled spin-splitted Landau sublevels is even. In the case of odd number of filled Landau sublevels the parabolic window at fig. 8 is partially closed. In this case the Landau pipe N^- is filled and the N^+-th one is empty.

Let us find the solutions of dispersion equation (3.1) in the transparency windows. The real part of $\chi_-^{(0)}$ in these regions is given by the first item in (2.36). Low frequency solution (2.36) of the dispersion equation can be obtained in the quasi-classical limit $\hbar\Omega << \varsigma$:

$$\omega(q) = \Omega_0(1+B_0)\left[1+\frac{1}{3B_0}\left(\frac{qv_F}{\Omega_0}\right)^2\right],$$

where $B_0 < 0$. This solution corresponds to the quasi-classical Silin spin wave.



1: $\Omega_0 - qv_0^- + \omega_q$ 5: $\Omega_0 + qv_1^+ - \omega_q$

2: $\Omega_0 - qv_0^+ - \omega_q$ 6: $\Omega_0 + qv_1^- + \omega_q$

3: $\Omega_0 - qv_1^- + \omega_q$ 7: $\Omega_0 + qv_0^+ - \omega_q$

4: $\Omega_0 - qv_1^+ - \omega_q$ 8: $\Omega_0 + qv_0^- + \omega_q$

Fig.8. The Landau damping regions and transparency windows for the spin waves in the case of the two filled Landau zones.

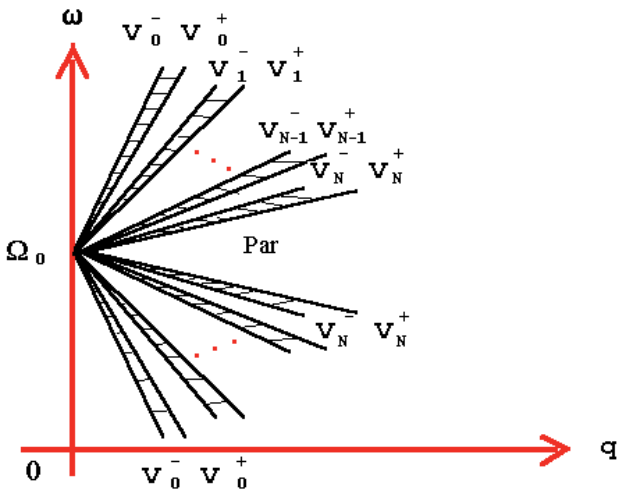


Fig.9. The Landau damping regions in the long-wavelength limit.

If only two Landau sublevels 0^- and 0^+ are filled, then in the sum $\sum_n$ in χ_-^0 (2.35) it is necessary to remain only one item with $n=0$. Then the dispersion equation can be solved precisely:

$$\omega(q)=\Omega_0-\frac{q}{2}(v_0^- - v_0^+)cth\,\alpha+\left\{\frac{q^2}{4}(v_0^- - v_0^+)^2 cth^2\alpha+\right.$$

$$\left.+q^2 v_0^- v_0^+ - q\omega_q(v_0^- + v_0^+)cth\,\alpha+\omega_q^2\right\}^{1/2}, \qquad (3.30)$$

where $\alpha=\frac{(2\pi\hbar)^2 c\hbar q}{2mI|e|H}$. The dispersion curve corresponding to this solution lies in the parabolic transparency window (fig. 10). In the long-wavelength limit from (3.30) we obtain

$$\omega(q)=\Omega_0+\frac{2\pi^2 c\hbar^3 q^2}{mI|e|H(\frac{1}{v_0^+}-\frac{1}{v_0^-})}.$$

Let us notice that the spectrum of this quantum wave is quadratic in the long-wavelength limit. The spin wave spectrum in petalous transparency windows is linear at $q\to 0$.

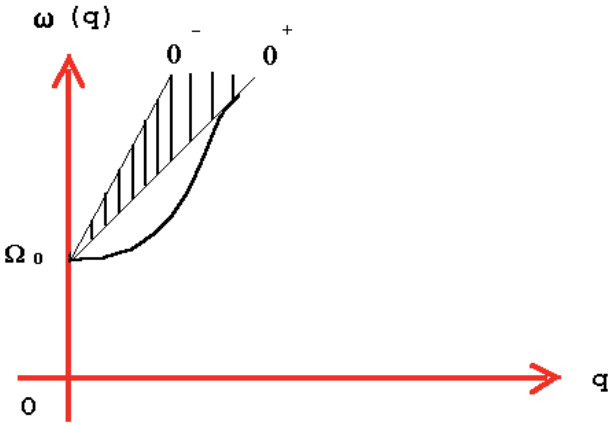


Fig.10. The dispersion curve of the quantum spin waves in the parabolic transparency window without taking into account the quasi-local states at two filled Landau sublevels $0^{\pm}$.

Let us consider the case of slow quantum spin waves in the parabolic transparency window for which $\frac{\omega-\Omega_0}{q}<<v_N^{\pm}$. Performing expansion of the logarithm in (2.26) over the powers of

$\frac{\omega-\Omega_0}{qv_N^{\pm}}<<1$ we find the spectrum of the slow quantum spin waves in the parabolic transparency window in the case of arbitrary even number of the filled Landau sublevels:

$$\omega(q)=\Omega_0\left\{1+\frac{\hbar q^2}{mI\Omega_0(g_+-g_-)}\right\},$$

where g_σ is the state density of Fermi electrons in the magnetic field.

Let us find out how the quasi-local electron states influence the properties of the quantum spin waves in the parabolic transparency window. The main cause of the existence of quantum spin waves in degenerated electron liquid of non-ferromagnetic metals in the magnetic field is the quantization of Fermi electron velocity projections on the field direction. As a result of the quantization in the continuous region of the wave quasiclassical collisionless damping the transparency windows, in which the damping is absent, appear. The boundaries of these windows can be found from the conservation laws of energy and z-th momentum component $(z\|\vec{H})$ in the process of magnon absorption by electron with taking into account the Pauli principle and the selection rules. In the windows the new branches of the spin wave spectrum can be located. These waves were named "the quantum waves". The variety in the system of conductivity electrons, which are at the different Landau levels, leads to the phenomenon that the spectrum of the transversal quantum spin waves appears to be basically linear in the long-wavelength limit. The dispersion curves of these waves begin at the frequency of the spin resonance Ω_0 and phase velocities are defined by the Landau pipe lengths. If the electron cyclotron frequency Ω exceeds the frequency of the spin transitions Ω_0 and the even number of the spin-splitted Landau sublevels are located below the Fermi boundary, the wide transparency window for the transversal quantum spin waves with the left circular polarization propagating along the magnetic field exists in the neighborhood of Ω_0. Its boundaries are parabolas

$$\omega=\Omega_0\pm qv_N^{+}-\omega_q, \tag{3.31}$$

where N is the number of the last filled Landau level, $v_N^{\pm}$ are velocities of the Fermi electrons on the Landau sublevels $N^{\pm}$. In this window the dispersion curve of the slow quantum spin waves whose spectrum is quadratic in the long-wavelength limit is located.

The circular components of the high-frequency electron spin susceptibility $\chi_\pm$ have the features on the frequencies of the resonance electron transitions between the quasi-local levels

and the Landau levels with the spin flip [54]. It is necessary to take into account these features in the dispersion equation (3.1). It turns out that intersection of the quantum spin wave dispersion curve with the resonance frequency of these transitions leads to the cross situation which is analogous to that found in studying sound propagation in dielectrics with quasi-local oscillations of the crystalline lattice [84]. Instead of one spectrum branch of the quantum spin waves in each transparency window two branches – low and high frequency - appear. They are due to the interaction of electron spin magnetization oscillations at the resonance transitions with oscillations in the process of the spin wave propagation.

Let us consider a conductor with the isotropic quadratic spectrum of the carriers with randomly distributed insulated impurity atoms on which the quasi-local states are formed. Let us find out how they influence the quantum spin waves.

The results listed below are expressed through the characteristics of the quasi-local states, i.e. energy $\varepsilon_r^{\pm}$ of resonances, their half-widths $\Gamma_{\pm}$ and the residues $r_{\pm}$ of the electron impurity scattering amplitude in the pole $\varepsilon_r^{\pm} - i\Gamma_{\pm}$. These values can be calculated giving concrete expression to scattering potential or they can be obtained from the experiment. The sample temperature and also the widths of the levels participating in the transitions induced by the variable magnetic field of the spin wave are supposed to be little in comparison with the transition energy. Quantizing magnetic field influences the electrons. It is selected in such a way that below the Fermi boundary the even number of spin-splitted Landau sublevels is located. Then between the parabolas (3.31) the transparency window for slow quantum spin wave with the quadratic spectrum exists. If inequality

$$\omega - \Omega_0 << qv_N^+ \tag{3.32}$$

is obeyed its spectrum has a form[1)]

$$\omega(q) = \Omega_0 \left[1 + \frac{q^2}{m\Omega_0 I} (g_+ - g_-)^{-1} \right], \tag{3.33}$$

[1)] Here and then in this paragraph $\hbar = 1$.

where $g_{\pm}$ is state density of the electrons with the spin quantum number $\pm 1/2$ on the Fermi level; I the parameter of electron-electron interaction. It is connected with the constant B_0 which appears in the theory of Fermi liquid by the relation $|B_0| = Ig_F$. If the sublevels $N^{\pm}$ are located on the different side from the Fermi boundary, electron transitions between them will lead to appearance of the Landau damping region limited by parabolas

$$\omega = \Omega_0 \pm qv_N^- + \omega_q .$$

Inside this region the propagation of the wave with the spectrum (3.33) is impossible.

The method of calculation of electron dynamic spin susceptibility tensor $\chi_{ik}(q,\omega)$ with taking into account the quasi-local states is listed above. In the transparency windows where collisionless damping of the spin waves is absent this tensor can be expanded in series over the powers of impurity atom concentration n_i:

$$\chi = \chi_0 + \delta\chi ,$$

where χ_0 is a well-known contribution calculated without taking into account quasi-local states and $\delta\chi$ is the impurity addition which is due to these states. To obtain it we should take into account the resonance structure of electron scattering operator by the impurity atoms. With the help of this operator Green functions incoming in the electron lap for the high frequency spin susceptibility are expressed.

The circular component of susceptibility χ_- has the root singularities on the frequencies ω_n of the electron transitions from the quasi-local level into the Landau levels which are accompanied by the spin-flip $\downarrow\rightarrow\uparrow$. Near the resonance frequency ($|\omega-\omega_n| << \Omega$) it equals to

$$\delta\chi_n(q,\omega) = a_n(q)\left(\frac{\omega_n}{\omega_n-\omega-i\Gamma_-}\right)^{1/2}, \qquad (3.34)$$

where the values a_n play the role of oscillator forces of the resonance transitions. In the long-wavelength limit at $\vec{q}\|\vec{H}$ they are equal to

$$a_n(q) = \frac{m^{3/2}\mu^2\Omega r_\downarrow n_i}{2^{1/2}\pi\omega_n^{5/2}}\left[f(\varepsilon_{r\downarrow}) - f(\varepsilon_{r\downarrow}+\omega_n)\right]\times$$

$$\times\left(1-\frac{\Omega_0}{\omega_n}\right)^{-2}\left[1+\frac{q^2}{m\omega_n}(1-\Omega_0/\omega_n)^{-1}\right], \qquad (3.35)$$

where μ is electron magnetic moment; $\omega_n = \varepsilon_{n\uparrow} - \varepsilon_{r\downarrow} = \Omega_0 + \varepsilon_n - \varepsilon_r$ are the resonance frequencies (ε_n and ε_r are the locations of the n-th Landau level and the resonance level without taking into account spin splitting); f is Fermi function. The radical in (3.34) is connected with the singularity of the electron state density on the Landau level participating in the transitions, and the difference of the Fermi functions in (3.35) takes into account the Pauli principle.

The expressions (3.34) and (3.35) are obtained on assuming that in the electron spectrum one quasi-local state exists. Its characteristics can be calculated in the frames of a certain model of the priming electron spectrum and the impurity potential. For example, in the case of the resonance on the short-range impurity the residue of S-scattering amplitude in the pole $\varepsilon_r - i\Gamma$ equals to [15]

$$r = \frac{2\pi^3 g_r \Gamma}{m^3 \varepsilon_r},$$

where g_r is the state density of free electrons in the point ε_r. In the case of electron magnetoimpurity states formed on the short-range donor impurity in the magnetic field, the residue is listed in the end of the p. 3.2. It consists $\Delta = ½\Omega(a/l)^2$, i.e. the distance between the Landau level and the magnetoimpurity level splitted off from it (a is scattering length, l is magnetic length). In this case the width of the n-th magnetoimpurity level equals to [21,22]

$$\Gamma_n = 4\Delta\left(\frac{\Delta}{\Omega}n\right)^{1/2} \qquad (n >> 1).$$

The expressions (3.34) and (3.35) also remain valid for the magnetoimpurity states. But the oscillator forces (3.35) contain an additional summarizing over the numbers of magnetoimpurity levels participating in the transitions on the frequency ω_n [82].

The contribution (3.34) will be taken into account in the dispersion equation for the spectrum of the quantum spin waves. Let us find spectrum and damping the transversal spin waves with the left circular polarization which are propagating along the magnetic field. The dispersion equation for them in the random phase approximation has the form (3.1). Electron-electron interaction is supposed to be contact. This approximation is equivalent to inclusion of the first item in the expansion of the exchange part of the function of Landau Fermi-liquid interaction in the series over the Legendre polynomials [44]. We are interested in the solution (4.40) of the dispersion equation (3.1) near the resonance frequency $\omega_r = \Omega_0 + \omega_0$ which exceeds the spin resonance frequency with the value $\omega_0 = \varepsilon_{N+1} - \varepsilon_r$ which equals to the distance between the resonance level ε_r and the (N+1)-th Landau level. The magnetic field is selected so that the Fermi boundary is located between the levels $\varepsilon_r^{\downarrow}$ and $\varepsilon_{N+1}^{\uparrow}$.

Let us consider an influence of the quasi-local state on the properties of slow quantum spin wave with the spectrum (3.33). In the region of intersection of the straight line $\omega = \omega_r$ with

the dispersion curve (3.33) of this wave the inequality (3.32) is obeyed, that is why the dispersion equation in this region can be written in a form

$$(1-Ax)(1-x)^{1/2} = B, \tag{3.36}$$

where

$$x = \frac{\omega - \Omega_0}{\omega_0},\ A = \frac{m\omega_0 I}{q^2}(g_+ - g_-),$$

$$B = \left(\frac{m}{2}\right)^{3/2} \frac{I\Omega r_\downarrow n_i}{\pi\omega_0^{5/2}}[f(\varepsilon_{r\downarrow}) - f(\varepsilon_{r\downarrow} + \omega_r)].$$

From the equation (3.36) it is well shown that the parameter B plays the role of the coupling constant of electron spin magnetization oscillations at the transitions $\varepsilon_r^\downarrow \to \varepsilon_{N+1}^\uparrow$ and in the process of the spin wave propagation. In the absence of such a coupling ($B = 0$) the functions (3.33) are the solutions of the equation (3.36) and $\omega = \omega_r$.

At the presence of the quasi-local state the frequency change of the spectrum of electron magnetization oscillations occurs. Two branches of the spin wave spectrum - low $\omega_d(q)$ and high $\omega_u(q)$ frequency appear. They are shown on the fig. 11 where only one sector in which the Landau damping exists is depicted (hatched).

Low frequency solution of the equation (3.36) depends on the parameter $q_0 = [m\omega_0 I(g_+ - g_-)]^{1/2}$. If $q < q_0$, this solution has a form

$$x = \frac{1}{3}[1 + \frac{2}{A} - 2(1 - \frac{1}{A})\cos\frac{\pi - \varphi}{3}], \tag{3.37}$$

where $\cos\varphi = 1 - \frac{27}{2}\left(\frac{B}{A}\right)^2 (1 - 1/A)^{-3}$.

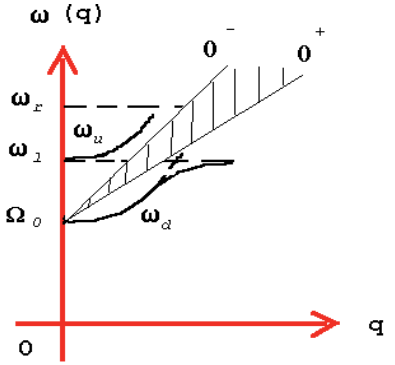


Fig. 11. Two branches of the quantum spin wave spectrum.

If $q > q_0$ we find

$$x = \frac{1}{3}\left[1 + \frac{2}{A} + 2(1-1/A)ch\frac{\varphi}{3}\right], \tag{3.38}$$

where now

$$ch\varphi = 1 + \frac{27}{2}\left(\frac{B}{A}\right)^2\left(\frac{1}{A}-1\right)^{-3}.$$

In a long-wavelength limit ($q << q_0$) from the expression (3.37) we obtain

$$\omega_d(q) = \Omega_0\left[1 + \frac{(1-B)q^2}{m\Omega_0 I}(g_+ - g_-)^{-1}\right]. \tag{3.39}$$

The quasi-local state leads to reducing frequency and group velocity of the quantum spin wave with the spectrum (3.33). If $q >> q_0$, then with increasing q the dispersion curve (3.38) asymptotically approaches to a limit frequency

$$\omega_l = \omega_r\left[1 - \left(\frac{m^{3/2} I\Omega r_\downarrow n_i}{2^{3/2}\pi\omega_0^2\omega_r^{1/2}}\right)^2\right].$$

Low frequency branch of the quantum spin wave spectrum is located in the band $[\Omega_0, \omega_l]$ where the Landau damping is absent. The damping of this wave is defined by the collisions of electrons with impurity atoms. The damping decrement equals to:

$$\gamma_d = \left[\nu(1-x) + \frac{\Gamma_-}{2}\left(\frac{1}{A}-x\right)\right]\left[1 - x + \frac{1}{2}\left(\frac{1}{A}-x\right)\right]^{-1}, \tag{3.40}$$

where x is equal to (3.37) or (3.38) in accordance with q/q_0 and ν is a relaxation frequency of the electron spin magnetization. This frequency is due to the potential scattering on the impurities. In the long-wavelength limit ($q << q_0$) from the formula (3.40) we obtain

$$\gamma_d(q) = \nu + \frac{q^2 B}{2m\omega_0 I}(\nu - \Gamma_-)(g_+ - g_-)^{-1}.$$

Small values ν and Γ_- provide the trifle of the decrement in comparison with the wave frequency (3.39).

The spectrum of the high frequency wave in the long-wavelength limit $qv_F << \omega_0$ can be obtained in the quasi-classical approximation for χ_0:

$$\omega_u(q) = \omega_r\left\{1 - \left[a_0 b_0 \frac{\omega_0}{\omega_r}\left(1 + \frac{1}{3}b_0\frac{q^2 v_F^2}{\omega_0^2}\right)^{-1}\right]^2\right\}, \tag{3.41}$$

where

$$b_0 = \frac{|B_0|\omega_r}{\omega_0 + |B_0|\Omega_0},$$

a_0 is the value (3.35) at $n = N+1$ and $q = 0$. The decrement of this wave equals to

$$\gamma_u(q) = \Gamma_- + 2\nu a_0^2 b_0^3 \frac{\omega_0}{\omega_r}.$$

At $q \sim \omega_0 / v_0^-$ the dispersion curve (3.41) crosses the Landau damping boundary beyond which the wave damps fast. If $\omega > \omega_r$, the contribution (3.34) becomes imaginary in general. It means that the high frequency spectrum almost coincides with (3.33). With the increase of q the dispersion curve of this wave approaches to the Landau damping boundary $\omega = \Omega_0 + qv_N^+ + \omega_q$. In the region $\omega \gtrsim \omega_r$ this wave experiences strong damping with decrement

$$\gamma_u = \frac{B}{A}\left(\frac{\omega - \omega_r}{\omega_0}\right)^{-1/2}, \qquad (3.42)$$

due to the resonance electron transitions $\varepsilon_r^{\downarrow} \to \varepsilon_{N+1}^{\uparrow}$ induced by the magnetic field of the spin wave.

The characteristics of a new type of spin waves discussed in this chapter depend on the parameters of the quasi-local states – the resonance locations and widths, the residues of the electron scattering amplitude by the insulated impurity atoms in the pole.

In the next chapter we will consider the neutron magnetic scattering method which is convenient for the experimental detection of these waves.

CHAPTER IV. NEUTRON MAGNETIC SCATTERING IN NON-FERROMAGNETIC METALS WITH THE QUASI-LOCAL STATES OF ELECTRONS

4.1. Connection between the cross-section of neutron magnetic scattering and the tensor of dynamic spin susceptibility

Spin waves to be considered here as well as the waves in the magneto-ordered crystals [23,49,50,60,85-90] can be found in the experiments with slow neutrons and also in the experiments on measuring light cross-section [91-93]. In this chapter we will find out how new types of the spin waves in the non-ferromagnetic metals described in chapter III become apparent in a neutron scattering cross-section.

Neutron magnetic scattering discussed in this chapter is due to the interaction of a neutron magnetic moment with the current of spin magnetization of collectivized electrons [49,88,90]. Let $\hat{V}$ be the Hamiltonian of this interaction. The latter causes the neutron transitions $(\vec{k}m_s) \to (\vec{k}'m_s')$ from the initial state with the wave vector $\vec{k}$ and spin quantum number m_s in a final state $(\vec{k}'m_s')$. In Born approximation the probability of the transition in a unit of time equals to:

$$W_{\vec{k}m_s \to \vec{k}'m_s'} = \frac{2\pi}{\hbar} \sum_{nn'} \rho_n \left| \left\langle n' \middle| V_{\vec{k}'\vec{k}} \middle| n \right\rangle \right|^2 \delta(E_n + \in_{\vec{k}m_s} - E_{n'} - \in_{\vec{k}'m_s'}), \tag{4.1}$$

where n is index of the scatter stationary state with energy E_n, ρ_n is probability of the state $|n\rangle$, $V_{\vec{k}'\vec{k}}$ is a matrix element of the operator $\hat{V}$ assembled on neutron proper states, $\in_{\vec{k}m_s}$ is neutron energy.

If one takes into account the δ-function expansion in the Fourier integral, the expression (4.1) can be represented in a form

$$W_{\vec{k}m_s \to \vec{k}'m_s'} = \frac{1}{\hbar^2} \int_{-\infty}^{\infty} dt e^{\frac{i}{\hbar}(\in_{\vec{k}'m_s'} - \in_{\vec{k}m_s})t} \left\langle \hat{V}^{+}_{\vec{k}'m_s', \vec{k}m_s} \hat{V}_{\vec{k}'m_s', \vec{k}m_s}(t) \right\rangle, \tag{4.2}$$

where

$$\hat{V}(t) = e^{\frac{i}{\hbar}\hat{H}t} \hat{V}(0) e^{-\frac{i}{\hbar}\hat{H}t}$$

is the interaction Hamiltonian in Heisenberg representation, $\hat{H}$ is the scatter Hamiltonian ($H|n\rangle = E_n|n\rangle$), averaging with the density matrix of the scatter and also configuration averaging over locations of impurity atoms in the sample are designated by the angular

brackets. To obtain twice differential neutron scattering cross-section $\frac{d^2\sigma}{dO'd\in'}$ in a solid angle dO' and the energy interval $d\in'$ it is necessary to multiply the probability (4.2) on the number

$$\frac{Vm_n k'dO'd\in'}{8\pi^3\hbar^2}$$

of neutron states with the wave vector $\vec{k}'$ in a solid angle dO' and with the energy $\in'$ in the interval $d\in'$ (m_n is neutron mass, V is the volume of the sample) and divide into the density of falling neutron beam $\hbar k/m_n V$:

$$\frac{d^2\sigma}{dO'd\in'} = \frac{(m_n V)^2 k'}{(2\pi)^3\hbar^5 k}\sum_{m_s m_s'} P_{m_s}\int_{-\infty}^{\infty} dt \exp\left[\frac{i}{\hbar}(\in_{\vec{k}'m_s'} - \in_{\vec{k}m_s})t\right]\times$$

$$\times\left\langle \hat{V}^{+}_{\vec{k}'m_s',\vec{k}m_s}\hat{V}_{\vec{k}'m_s',\vec{k}m_s}(t)\right\rangle, \qquad (4.3)$$

where P_{m_s} is probability of neutron spin projection in a falling beam equals to m_s. As usual, in (4.3) the averaging over neutron spin states in falling beam and summarizing over spin states in a scattered beam are accomplished. It is well seen that (4.3) contains the Fourier component of the correlator assembled on the operators of neutron interaction with the target particles. The formula (4.3) is also applicable at presence of the magnetic field H. In this case

$$\in_{\vec{k}m_s} = \frac{\hbar^2 k^2}{2m_n} + \gamma\frac{|e|\hbar}{2m_n c}m_s H,$$

where $m_s = \pm 1$, $\gamma = 1{,}913$ is gyromagnetic relation for a neutron. The interaction energy of a neutron with the matter consists of two main parts: the energy of nuclear interaction with the nuclei of the atoms and the energy of magnetic interaction with electrons [85-89]. Let us consider magnetic interaction of the magnetic field made by a neutron magnetic moment with a spin magnetization current of conductivity electrons. The vector potential of neutron magnetic field in the point of electron location $\vec{r}_e$ equals to [88]

$$\vec{A}(\vec{r}_e) = \frac{[\vec{\mu}_n\times(\vec{r}_e - \vec{r})]}{|\vec{r}_e - \vec{r}|^3},$$

where $\vec{\mu}_n$ is a magnetic moment of the neutron located in the point $\vec{r}$. It equals to

$$\vec{\mu}_n = -\gamma \frac{|e|\hbar}{2m_n c}\vec{\sigma}_n,$$

where $\vec{\sigma}_n$ is the ort directed against $\vec{\mu}_n$. The energy operator of neutron interaction with the spin electron current equals to [88]

$$\hat{V} = -\frac{1}{c}\int d^3 r_e \hat{\vec{J}}(\vec{r}_e)\vec{A}(\vec{r}_e),$$

where $\hat{\vec{J}}$ is the operator of magnetization current density. Passing to Fourier components over $\vec{r}_e$ we obtain

$$\hat{V} = -\frac{1}{cV}\sum_{\vec{q}} \hat{\vec{J}}\vec{A}(-\vec{q}),$$

where

$$\vec{A}(-\vec{q},\vec{r}) = \int d^3 r_e e^{i\vec{q}\vec{r}_e}\frac{[\vec{\mu}_n(\vec{r}_e - \vec{r})]}{|\vec{r}_e - \vec{r}|^3}.$$

Let us calculate matrix elements of the operator $\hat{V}$ between the states of neutron $\vec{k}$, $\vec{k}'$ and the scatter n, n'. Let us take into account the matrix element of the spin current operator [15]:

$$\vec{J}_{nn'} = -c\mu rot(\psi_n^* \hat{\vec{\sigma}}\psi_{n'}),$$

where ψ_n is a wave function of the target, μ is an electron magnetic moment, $\hat{\vec{\sigma}}$ are Pauli matrices. As a result of not complicated calculations we obtain:

$$\left\langle n'\left|V_{\vec{k}'\vec{k}}\right|n\right\rangle = -\frac{4\pi\hbar^2\gamma_0}{m_n}\left\langle n'\left|\sum_e e^{i\vec{q}\vec{r}_e}\hat{\vec{S}}_e\right|n\right\rangle[\vec{S}_n - \vec{e}\times(\vec{e}\vec{S}_n)], \tag{4.4}$$

where

$$r_0 = \frac{e^2}{mc^2}$$

is a classical radius of the electron, $\sum_e$ is summarizing over electrons, $\vec{S}_e$ and $\vec{S}_n$ are spin moments of electron and neutron respectively, $\vec{q} = \vec{k} - \vec{k}'$ is a neutron scattering vector, $\vec{e} = \vec{q}/q$. Substituting the matrix element (4.4) in the formulae (4.1) and (4.3) we find

$$\frac{d^2\sigma}{dO'd\in'} = \frac{(\gamma r_0)^2 k'}{8\pi\hbar\mu^2 k}\sum_{\alpha\beta}(\delta_{\alpha\beta} - e_\alpha e_\beta)S_{\alpha\beta}(\vec{q},\omega), \tag{4.5}$$

where

$$S_{\alpha\beta}(\vec{q},\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} \left\langle M_\alpha(\vec{q},t) M_\beta(-\vec{q},0) \right\rangle,$$

$\vec{M}(\vec{q})$ is a spatial Fourier component of the electron spin magnetization operator (2.4), $\hbar\omega = \epsilon - \epsilon'$. The formula (4.5) is also applicable in the case of a magneto-ordered crystal with localized magnetic moments. If $|\vec{k}| = |\vec{k}'|$ one can obtain Bragg scattering cross-section which equals to

$$(\gamma r_0)^2 n^2 \sum_{\vec{b}} (1 - e_z^2) \delta_{\vec{q}\vec{b}} \delta(\hbar\omega).$$

Here $n = [n_+(0) - n_-(0)]/2$, $n_\sigma(\vec{q})$ is a Fourier component of electron density with the spin projection $\sigma = \pm 1$, $\vec{b}$ is a vector of reciprocal magnetic lattice.

To connect the cross-section (4.5) with the tensor of dynamic spin susceptibility of electrons $\chi_{\alpha\beta}(\vec{q},\omega)$ let us notice that the correlator $S_{\alpha\beta}$ in (4.5) can be changed into a symmetrized expression

$$S^S_{\alpha\beta} = \frac{1}{2}(S_{\alpha\beta} + S_{\beta\alpha}).$$

Comparing (4.5) with the Kubo formula (2.14) we find the connection of cross-section of neutron magnetic scattering with the tensor $\chi_{\alpha\beta}$:

$$\frac{d^2\sigma}{dO' d\epsilon'} = \frac{(\gamma r_0)^2 V k'}{4\pi\mu^2 k} (1 - e^{-\beta\hbar\omega})^{-1} \sum_{\alpha\beta} (\delta_{\alpha\beta} - e_\alpha e_\beta) \operatorname{Im} \chi^S_{\alpha\beta}(\vec{q},\omega). \qquad (4.6)$$

Here β is an inverse temperature. Thus if the scattering vector $\vec{q}$ is normal to the magnetic field [$\vec{e} = (1,0,0)$] it is easy to obtain

$$\sum_{\alpha\beta} (\delta_{\alpha\beta} - e_\alpha e_\beta) \chi^S_{\alpha\beta} = \chi^S_{yy} + \chi^S_{zz}.$$

If $\vec{q} \| \vec{H}$ [$\vec{e} = (0,0,1)$] then

$$\sum_{\alpha\beta} (\delta_{\alpha\beta} - e_\alpha e_\beta) \chi^S_{\alpha\beta} = \chi_+ + \chi_-,$$

where

$$\chi_\pm = \chi_{xx} \pm i\chi_{yx}$$

are circular components of a susceptibility tensor. So, from the formula (4.6) in the case $\vec{q} \| \vec{H}$ we find:

$$\frac{d^2\sigma}{dO'd\in'} = \frac{(\gamma r_0)^2 Vk'}{4\pi\mu^2 k}(1-e^{-\beta\hbar\omega})^{-1}\,\mathrm{Im}[\chi_+(\vec{q},\omega)+\chi_-(\vec{q},\omega)]. \qquad (4.7)$$

The features of susceptibility described in chapter II can be discovered in the experiments with slow neutrons.

4.2. Neutron magnetic scattering on spin waves in non-ferromagnetic metals with quasi-local electron states

The results listed in this paragraph were published in the work [52].

In the previous paragraph it was shown that twice differential cross- section of inelastic magnetic neutron scattering by an electron subsystem of a conductor contains a spin contribution:

$$\frac{d^2\sigma}{dO'd\in'} = \frac{1}{4\pi}\left(\frac{\gamma r_0}{\mu}\right)^2 V(n_\omega+1)\frac{p'}{p}\,\mathrm{Im}[\chi_+(\vec{q},\omega)+\chi_-(\vec{q},\omega)], \qquad (4.8)$$

where

$$\hbar\vec{q} = \vec{p} - \vec{p}',$$

$$\hbar\omega = \frac{1}{2m_n}(p^2 - p'^2)$$

are changes of a neutron momentum and energy as a result of scattering, χ_+ and χ_- are circular components of electron dynamic spin susceptibility,

$$n_\omega = \frac{1}{e^{\hbar\omega/k_BT}-1}$$

is Planck function, V is a scatter volume. Scattering vector $\vec{q}$ is supposed to be parallel to $\vec{H}$. The components χ_+ and χ_- correspond to the electron transitions with the spin-flip $\uparrow\to\downarrow$ and $\downarrow\to\uparrow$ respectively.

Susceptibility features considered in the chapter II become apparent directly in the cross-section. We are interested in behavior of the cross-section as a function of ω near the frequencies of the electron transitions between the quasi-local level and Landau levels. At the random phase approximation (RPA) a circular component of dynamic spin susceptibility equals to:

$$\frac{\chi_-(\vec{q},\omega)}{1-\frac{I}{2\mu^2}\chi_-(\vec{q},\omega)}. \qquad (4.9)$$

The function $\chi_-(\vec{q},\omega)$ with taking into account the quasi-local states and the magnetic field was calculated in the par. 2.2. From the formula (4.9) it is well seen that the features of susceptibility can be of two types. The first type is presented also in the case $I=0$. It is due to the numerator in (4.9). Due to the numerator we obtain the contribution of one-particle excitations with the spin-flip in the neutron cross-section. Further we will consider this contribution near the frequencies of the transitions between the quasi-local levels and the Landau levels. The other type of susceptibility (4.9) features is connected with the zeros of the denominator. They correspond to the collective electron excitations, i.e. the spin waves. Let us calculate the cross-section (4.8) in the regions of existence of spin waves considered in the par. 3.1, 3.2.

The following results from (4.9)

$$\mathrm{Im}\frac{\chi}{1-\frac{I}{2\mu^2}\chi}=\left[\chi'\frac{I}{2\mu^2}\chi''+\chi''(1-\frac{I}{2\mu^2}\chi')\right]\times$$

$$\times\left[(1-\frac{I}{2\mu^2}\chi')^2+(\frac{I}{2\mu^2}\chi'')^2\right]^{-1}, \qquad (4.10)$$

where $\chi=\chi'+i\chi''$ and indices "+" and "-" of susceptibility are omitted. This expression has a sharp maximum in the region of existence of slightly damping spin waves where

$$1-\frac{I}{2\mu^2}\chi'_-(\vec{q},\omega)=0 \qquad (4.11)$$

and χ''_- is small. The solutions $\omega(q)$ of the dispersion equation (4.11) was considered in par. 3.1, 3.2. The left part of the equation (4.11) near zero of $\omega(q)$ can be represented in the form

$$1-\frac{I}{2\mu^2}\chi'_-(q,\omega)\approx[\omega-\omega(q)]\frac{I}{2\mu^2}\left[-\frac{d}{d\omega}\chi'_-(q,\omega)\right]\Bigg|_{\omega=\omega(q)}.$$

Substituting this expression in (4.10) near the point $\omega=\omega(q)$ we obtain

$$\mathrm{Im}\,\chi^{\mathrm{RPA}}(q,\omega)\approx\frac{\chi''(q,\omega(q))}{\left(-\frac{I}{2\mu^2}\frac{d}{d\omega}\chi'\right)^2_{\omega_q}\left[(\omega-\omega(q))^2+\left(\frac{\chi''}{\frac{d}{d\omega}\chi'}\right)^2_{\omega_q}\right]},$$

where (4.11) is taking into account. This expression can be written like this:

$$\mathrm{Im}\,\chi_{-}^{\mathrm{RPA}}(q,\omega) \approx \frac{4\mu^4}{I^2} \cdot \frac{1}{\left|\left(\frac{d\chi'_{-}}{d\omega}\right)_{\omega_q}\right|} \cdot \frac{\gamma_q}{(\omega-\omega_q)^2+\gamma_q^2}, \tag{4.12}$$

where

$$\gamma_q = \frac{\chi''_{-}[q,\omega(q)]}{\left|\left(\frac{d}{d\omega}\chi'_{-}(q,\omega)\right)_{\omega_q}\right|} \tag{4.13}$$

is a damping decrement of the spin wave. Substituting (4.12) in the formula (4.8) we will obtain differential cross-section of neutron scattering accompanied by a magnon production with the energy $\hbar\omega(q)$ and also by an electron spin-flip $\downarrow\rightarrow\uparrow$. In the same way one can consider the contribution of the processes with the spin-flip $\uparrow\rightarrow\downarrow$. Let us find out how the cross-section (4.8) behaves in the region of existence of new spectrum branches of the spin waves considered in par. 3.1, 3.2.

As it has been noticed before the magnetic scattering cross-section of slow-neutrons by a spin current in normal metals is connected with dynamic spin susceptibility of conductivity electrons [49,88,90]. Susceptibility of the non-ferromagnetic metals with the quasi-local electron states on insulated nonmagnetic impurity atoms in the absence of the magnetic field was considered in the work [61]. It was shown that electron transitions between the quasi-local and zone states caused by the variable magnetic field lead to the features of dynamic spin susceptibility. In a quantizing magnetic field these features become stronger [37]. The electron resonance transitions from the quasi-local levels into the Landau levels lead to the root singularities of susceptibility, which reproduce the features of electron state density. The new spectrum branches of collective excitations of metal spin system are based on these root singularities. The features of susceptibility must arise in the cross-section of neutron inelastic magnetic scattering. In the energy spectrum of scattered neutrons the maxima must be observed when the energy transmitted by a neutron equals to the resonance frequency. It is true not only for the metals with the proper quasi-local electron states [10] also existing in the absence of the magnetic field but also with the magnetoimpurity levels [37].

The calculation results of inelastic magnetic scattering cross-section of slow neutrons by a spin current in normal metals with the quasi-local electron states in the presence of a quantizing magnetic field are listed below. The model and the method of the calculation are described in [37,61]. It is supposed that $\hbar\omega$ exceeds summary width of the levels

participating in the transitions and a scattering vector $\vec{q}$ is parallel to the constant magnetic field $\vec{H}$. The scatter temperature is supposed to be low in comparison with transition energy.

The frequencies of the electron resonance transitions between the quasi-local level $\varepsilon_{r\downarrow} = \varepsilon_r - \mu H$ (indices $\uparrow\downarrow$ correspond to the spin orientation along and against the field $\vec{H}$) and the Landau levels accompanied by the spin-flip equal to

$$\omega_{rn} = \omega_0 + \Omega_0 + n\Omega, \tag{4.14}$$

where $\omega_0 = \varepsilon_L - \varepsilon_r$ ($\varepsilon_{L\uparrow} = \varepsilon_L + \mu H$ is the nearest to the Fermi energy ε_F free Landau level), Ω_0 is the spin resonance frequency, Ω is the cyclotron frequency, $n = 0,1,...$ is the resonance number. In the neighborhood of the frequency (4.14) the circular component of susceptibility $\chi_-(q,\omega)$ equals to $\chi = \chi_0 + \delta\chi$ where χ_0 is a well-known contribution [78] and $\delta\chi$ is a resonance part connected with electron transitions evolved. It equals to (3.4). This expression leads to the additional maxima in the energy spectrum of scattered neutrons. These maxima lie above the Stoner sectors. These maxima are due to one-particle excitations of localized electrons with the spin-flip. The dimensionless twice differential scattering cross-section in the solid angle dO and the energy interval $d\omega$

$$h = \frac{d^2\sigma}{dOd\omega} \cdot \frac{4\pi}{\chi_P}\left(\frac{\mu}{1{,}9r_0}\right)^2 \tag{4.15}$$

as a function of ω has maxima at the frequencies (4.14). The value of the n-th maximum equals to

$$h_n = a_n\left(\frac{\hbar\omega_{rn}}{2\Gamma_-}\right)^{1/2}. \tag{4.16}$$

The maxima are asymmetric and displaced in the high frequency region. With the increase of n the value (4.16) decreases proportionally to n^{-2}. Let us notice that the electron resonance transitions $\downarrow\rightarrow\uparrow$ between the nearest to ε_F filled Landau level and free quasi-local level located above ε_F lead at $\varepsilon_r < \varepsilon_L$ (but $\varepsilon_{r\uparrow} > \varepsilon_{L\downarrow}$) to the analogous cross-section maximum at frequency $\Omega_0 - |\varepsilon_r - \varepsilon_L|$ lying below a central Stoner sector. This maximum is displaced in the low frequency region.

Let us take into account electron-electron interaction in the random phase approximation. In this approximation the new spectrum branches of collective excitations of non-ferromagnetic metal spin system appear outside the Stoner sectors. They are analogous to the transversal magnetoimpurity spin waves [37]. These waves slightly damp due to electron collisions in

narrow transparency bands lying below the resonance frequencies (4.14). The dispersion law and damping decrement of the waves in the n-th band equal to (3.9) and (3.10).

The neutron cross-section with the excitation of the quantum of the spin wave with the spectrum (3.9) as a function of ω has Lorentz maxima at spin wave frequencies. In the neighborhood of the n-th maximum

$$\frac{d^2\sigma_n}{dOd\omega} = \frac{(1{,}9r_0)^2 \Delta\omega_n}{I^2 g_F a_n} \left(\frac{\Delta\omega_n}{\omega_{rn}} \right)^{1/2} \frac{1}{\pi} \times$$

$$\times \gamma_n(q) \{ [\omega - \omega_n(q)]^2 + [\gamma_n(q)]^2 \}^{-1}, \qquad (4.17)$$

where $\Delta\omega_n$ is the transparency band width (3.12). The maximal value of the cross-section (4.17) equals to

$$h_n = \frac{2}{(Ig_F)^2 a_n} \left(\frac{\Delta\omega_n}{\omega_{rn}} \right)^{1/2} \frac{\Delta\omega_n}{\gamma_n}. \qquad (4.18)$$

The width of the maximum coincides with the damping decrement of the spin waves (3.10). With the increase of n the value of the maximum (4.18) decreases proportionally to n^{-4} and its width tends to a constant limit Γ_-.

As it has already been mentioned the quasi-local electron states below the Fermi boundary in the absence of the magnetic field were observed in Bi with the impurities of Sn and Pb [81]. Using the parameters of the Bi spectrum and also the characteristics of quasi-local states listed in [81] we find the ratio of the value of the first ($n = 0$) maximum (4.16) at $n_i = 10^{-2}$ at. %, $H = 10^3$ Oe to maximal value of $\mathrm{Im}(\chi_0 / \chi_P)$ at the spin resonance: $24{,}2(\nu_2 / \Omega_0)$. If one takes $Ig_F = 0{,}1$, the ratio of h_0 (4.18) to the maximal value of the cross-section with excitation of the quasi-classical Silin waves [90] will equal $142{,}4(\nu_2 / \Omega_0)$. The ratio of the value of the first maximum (4.18) to (4.16) turns out to be equal to 5,4. For calculations we used the residue r in the model of Gauss separable potential [82].

Let us consider the case when the scattering vector $\vec{q}$ is oriented at an angle to $\vec{H}$. As it has already been pointed out the differential cross-section of the neutron magnetic scattering on the spin magnetization fluctuations of collectivized electrons is connected to the anti-Hermitian part of susceptibility tensor [87-90]. Susceptibility features become apparent

directly in the scattered neutron spectrum. On the curve of cross-section dependence on transmission of neutron energy $\hbar\omega = \varepsilon - \varepsilon'$ the maxima due to one-particle Stoner excitations of electrons are present. They are located near energy $\hbar\omega = \hbar(\Omega_0 + n\Omega)$ and restricted by the Stoner sectors [90]. In the metals with the quasi-local states there exist additional maxima connected with electron (localized on the impurities) excitations by the neutron magnetic field. The maxima due to the electron transitions $\downarrow\rightarrow\uparrow$ are located in the points (4.14) the value ω_0 higher than the Stoner sector corners. They are asymmetric. The character of the maximum asymmetry depends on the transition type. For example, in the case of the transitions from the quasi-local level into Landau levels the maxima are displaced in the high energy region and in the case of the transitions from the Landau levels into the quasi-local level – in the region of low energies. The series of the maxima due to the transitions $\uparrow\rightarrow\downarrow$ is located in the points

$$\omega_{rn} = \omega_0 - \Omega_0 + n\Omega\,. \tag{4.19}$$

Depending on the ratio ω_0 / Ω_0 they can be located both above and below the Stoner sectors.

The differential cross-section of the neutron scattering in the energy interval $d\varepsilon'$ in the solid angle dO' related to the unit volume of a sample in the neighborhood of the energy $\hbar\omega_{rn}$ (4.19) equals to

$$\frac{d^2\sigma_n}{d\varepsilon' dO'} = \frac{\chi_P}{8\pi}\left(\frac{1{,}9r_0}{\mu}\right)^2 \frac{k'}{k}(n_\omega + 1)(1 + \cos^2\alpha)a_n^+ \times$$

$$\times \mathrm{Re}\left(\frac{\omega_{rn}}{\omega - \omega_{rn} + i\Gamma_+/\hbar}\right)^{1/2}, \tag{4.20}$$

where α is the angle of the neutron scattering.

On the fig.12 the dependence of dimensionless scattering cross-section

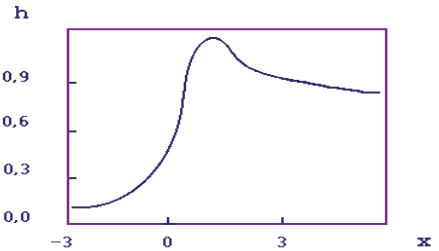


Fig. 12. The dependence of the neutron scattering cross-section on the value of energy loss near the frequency ω_{rn}.

$$h = \frac{d^2\sigma_n}{d\varepsilon' dO'}\left[\frac{\chi_P}{4\pi}\left(\frac{1{,}9r_0}{\mu}\right)^2 a_n^+\left(\frac{\omega_{rn}}{2\Gamma_+/\hbar}\right)^{1/2}\right]^{-1}$$

on $x = \hbar(\omega - \omega_{rn})/\Gamma_+$ at $\alpha = 0$ is presented. The scattering cross-section near frequencies of the transitions from the Landau levels into the quasi-local level can be obtained by the change of the resonance denominator in the expression (4.20) into $(\omega_{rn} - \omega - i\Gamma_-/\hbar)$, indices "+,↑" into "-,↓" at the characteristics of the quasi-local state and by the change of the oscillator forces described in [71].

The neutron scattering cross-section also has the maxima at the points

$$\omega_n = \omega_0 + n\Omega\,.$$

They are due to the features of the longitudinal susceptibility on the frequencies of the transitions without the electron spin orientation change. Near these frequencies due to the transitions from the quasi-local level into the Landau levels we obtain

$$\frac{d^2\sigma_n}{d\varepsilon' dO'} = \frac{\chi_P}{8\pi}\left(\frac{1{,}9r_0}{\mu}\right)^2 \frac{k'}{k}(n_\omega + 1)\sin^2\alpha \times$$

$$\times\,\mathrm{Re}\sum_{\pm} a_n^{\pm}\left(\frac{\omega_n}{\omega - \omega_n + i\Gamma_\pm/\hbar}\right)^{1/2}, \qquad (4.21)$$

where

$$a_n^{\pm} = \frac{\pi\Omega r_{\substack{\uparrow\\\downarrow}} n_i}{2\varepsilon_F^{1/2}\omega_n^{5/2}}[f(\varepsilon_{r_{\substack{\uparrow\\\downarrow}}}) - f(\varepsilon_{r_{\substack{\uparrow\\\downarrow}}} + \hbar\omega_n)].$$

The maxima in the expression (4.21) are displaced in the region of high frequencies. The ratio of the maximal value of the cross-section at $n = 0$ to the maximum of the cross-section on the quasi-classical Silin waves [90] equals to $a_0(\nu_2/\Omega_0)(\hbar\omega_0/\Gamma)^{1/2}$. For the magnetoimpurity states in bismuth with the impurities of Te at $n_i/n_e = 10^{-2}$, $\nu_2 = 10^8$ s^{-1}, $H = 10^4$ Oe this ratio equals to 0,013.

With the increase of n the maxima values of cross-sections defined by the expressions (4.20), (4.21) decrease proportionally to n^{-2}. In the case of the transitions from the Landau levels into the quasi-local level it is necessary to change the sign of the resonance denominator in (4.21) that will lead to the series of the maxima displaced in the low energy region.

Besides the maxima due to the one-particle excitations in energy spectrum of scattered neutrons the Lorentz lines connected with scattering on the spin waves are present. The cross-

section of scattering with emission of a quantum of the transversal spin waves considered in ch. III equals to

$$\frac{d^2\sigma_n}{d\varepsilon' dO'} = \frac{\chi_P}{4\pi}\left(\frac{1{,}9r_0}{\mu I g_F}\right)^2 \frac{k'}{k}(n_\omega +$$

$$+1)(1+\cos^2\alpha)\frac{\omega_{rn}}{a_n}\left(\frac{\Delta\omega_n}{\omega_{rn}}\right)^{3/2}\gamma_n(q)\times \qquad (4.22)$$

$$\times\{[\omega-\omega_n(q)]^2+\gamma_n^2(q)\}^{-1}.$$

The maxima of this expression are located inside of the transparency bands and their widths coincide with the wave damping decrement. Along with the maxima in the region $\omega<0$ connected with the wave absorption they form satellites located symmetrically relative to the unshifted line $\omega=0$. The experimental examining of these satellites would allow to define $\omega_n(q)$ and $\gamma_n(q)$ of new waves. The ratio of the expression (4.21) maximum value to the maximum value of the scattering cross-section with excitation of the Silin waves [90] in the case of narrow quasi-local level ($\Gamma << (\omega_{rn}/\omega_n)^2(\Delta\omega_n/\omega_{rn})^{3/2}\frac{\hbar v_2}{a_n}$) equals to

$$C_n = \omega_n^2/\Omega_0\omega_{rn}I^2g_F^2.$$

If the typical values

$$Ig_F = 0{,}3;\ \omega_0/\Omega_0 = 0{,}1$$

are substituted here, we obtain $C_0 = 0{,}1$. With the increase of n the ratio C_n decreases proportionally to n^{-2}.

The listed values show that several branches of spin waves in the non-ferromagnetic metals predicted in the work can be discovered in the experiments with slow neutrons. As for the new branches of the wave spectrum they can exist in the ferromagnetic metals either. The consistent theory of these waves with taking into account the Fermi liquid effects (without the quasi-local electron states) is evolved in the works [94-96].

CHAPTER V. SILIN SPIN WAVES IN TWO-DIMENSIONAL ELECTRON GAS

5.1. Dynamic spin susceptibility of two-dimensional electron gas with impurity electron states

A reaction of the two-dimensional electron gas [97] on a slight variable magnetic field is characterised by a tensor of dynamic spin susceptibility $\chi(\vec{q},\omega)$ depending on the wave vector $\vec{q}$ and the field frequency ω. The features of susceptibility on a complex plane of the frequency ω determine the spectrum and damping of magnetic excitations of a system. Susceptibility allows to obtain the fluctuation spectrum of the spin magnetization of the two-dimensional electron gas, the cross-section of the neutron magnetic scattering by the spin magnetization current of conductivity electrons and the other values.

A big deal of the works are dedicated to a calculation of spin susceptibility of two-dimensional electron systems. The calculation results of free electron gas static susceptibility in a magnetic field which is normal to a plane of electron motion, are listed in a work [98]. Coulomb electron interaction is taken into account in [99]. A precise expression for dynamic spin susceptibility and also the functions of a reaction density-density of free degenerated two-dimensional electron gas were listed in the works [97,100]. The quantizing magnetic field was taken into account in [101]. High frequency asymptotics of spin susceptibility of two-dimensional Fermi liquid was obtained in a work [102]. An influence of the impurity atoms (which potentially scatter conductivity electrons) on susceptibility was considered in [103]. The review of properties of two-dimensional disordered systems in a magnetic field was given in [104].

Being sensitive to the dynamics of conductivity electrons, spin susceptibility experiences an influence of impurity atoms in the system. In particular, the electron impurity states must affect on susceptibility and the values connected with it. To include these states is actual since in the two-dimensional case the impurity atom which attracts the electrons in any slight way forms a bound state. The corresponding local level is located at a lower edge of two-dimensional conductivity zone. In a magnetic field the “multiplication” of the local levels takes place. They exist in the field of both attracting and repulsing scatters. The local levels are located between the Landau levels [105].

Here an influence of the local states on high-frequency dynamic spin susceptibility of the two-dimensional electron gas is considered at low temperatures. The method of the local excitations [10] applied earlier [106] for the calculation of susceptibility tensor is used. The

field frequency ω is supposed to be high in comparison with the frequency of the electron collisions.

As in the par. 2.2 for the calculation of spin susceptibility tensor let us use the Kubo formula

$$\chi_{\alpha\beta}(\vec{q},\omega) = i\int_0^\infty dt e^{i\omega t}\left\langle\left[M_\alpha(\vec{q},t), M_\beta(-\vec{q},0)\right]\right\rangle, \tag{5.1}$$

here $\vec{M}(\vec{q},t)$ is spatial Fourier component of Heisenberg operator of the two-dimensional electron spin magnetization; a commutator of operators is designated by the brackets and the Gibbs averaging and averaging over the configurations of the impurity atoms are marked by corner brackets; α, $\beta = x, y$; the sample area and quantum constant are equal to 1. The operator of the spin magnetization in the representation of the secondary quantization has a form

$$M_\alpha(\vec{q}) = -\mu\sum_{\vec{p}ss'}\sigma^\alpha_{s's}a^+_{(\vec{p}-\vec{q})s'}a_{\vec{p}s}, \tag{5.2}$$

where μ is electron magnetic moment; $\vec{p}$ and s are momentum and spin quantum number; $a_{\vec{p}s}$ and $a^+_{\vec{p}s}$ the annihilation and production operators of electrons in the state $|\vec{p}s\rangle$; σ^α is the Pauli matrices. Substituting the expression (5.2) in the formula (5.1) we obtain a connection between susceptibility tensor and the Fourier component of the retarded two-electron Green function. Let us use the method of the temperature Green functions [45] for calculation of the last one.

In the one-electron approximation the two-particle Green function comes to one-particle Green function product averaged over the impurity atom configurations. If one neglects the vertex corrections [45] then this mean value will come to the product of the mean values of one-particle Green functions. Using their spectral representations [45] we obtain for the tensor (5.1) the expression

$$\chi_{\alpha\beta}(\vec{q},\omega) = -\mu^2\sum_{\vec{p}ss'}\sigma^\alpha_{s's}\sigma^\beta_{ss'}\times$$

$$\times\int_{-\infty}^{\infty}d\varepsilon\int_{-\infty}^{\infty}d\varepsilon'\frac{f(\varepsilon)-f(\varepsilon')}{\varepsilon-\varepsilon'-\omega-i0}\rho_s(\vec{p},\varepsilon)\rho_{s'}(\vec{p}-\vec{q},\varepsilon'), \tag{5.3}$$

in which $f(\varepsilon)$ is the Fermi function; $\rho_s(\vec{p},\varepsilon)$ is spectral density of the one-electron Green function averaged over the impurity configurations. In a pure sample it equals to

$$\rho_0(\vec{p},\varepsilon) = \delta(\varepsilon-\varepsilon_{\vec{p}}),$$

where $\varepsilon_{\vec{p}} = p^2/2m$, m is the effective electron mass.

The one-particle Green function G is connected with the scattering operator T of electrons by the impurity centers [10]:

$$G = G_0 + G_0 T G_0, \tag{5.4}$$

where G_0 is Green function of the free electrons. The exact expression for the mean value of electron scattering operator by short-range acting impurity atoms in the one-center approximation is well-known [10]. Therefore, the spectral density of the mean Green function (5.4) can be represented in the form $\rho = \rho_0 + \delta\rho_i$ where $\delta\rho_i$ is the impurity addition. In the linear approximation over the impurity atom density n_i it is proportional to n_i. As a result $\chi = \chi_0 + \delta\chi_i$ where χ_0 is tensor of spin susceptibility of a pure sample and $\delta\chi_i$ is impurity contribution. It equals to $\delta\chi_{\alpha\beta}^{(i)} = \delta\chi_i \delta_{\alpha\beta}$ where

$$\delta\chi_i(\vec{q},\omega) = -2\mu^2 \sum_{\vec{p}} \int_{-\infty}^{\infty} d\varepsilon \delta\rho_i(\vec{p},\varepsilon)[f(\varepsilon) - f(\varepsilon_{\vec{p}+\vec{q}})] \times$$

$$\times \left(\frac{1}{\varepsilon - \varepsilon_{\vec{p}+\vec{q}} - \omega - i0} + \frac{1}{\varepsilon - \varepsilon_{\vec{p}+\vec{q}} + \omega + i0} \right). \tag{5.5}$$

From the formula (5.4) it is well seen that the function G has an additional peculiarities connected with the features of the scattering operator. The electron local energy levels on insulated impurity atoms correspond to them. A contribution of local levels in the spectral density of the mean Green function equals to

$$\delta\rho(\vec{p},\varepsilon) = |v_0| n_i (\varepsilon - \varepsilon_{\vec{p}})^{-2} \delta[1 - v_0 F(\varepsilon)], \tag{5.6}$$

where v_0 is a constant characterizing the intensity of short-range acting impurity potential; $F(\varepsilon)$ is the function of the Lifshits equation [10] $1 - v_0 F(\varepsilon) = 0$ for the local levels. From the formula (5.6) it is well seen that the spectral density has the delta-shaped maxima on the local levels:

$$\delta\rho(\vec{p},\varepsilon) = n_i \sum_l r_l (\varepsilon_p - \varepsilon_l)^{-2} \delta(\varepsilon - \varepsilon_l), \tag{5.7}$$

where ε_l is location of l-th local level;

$$r_l = \left| \frac{dF(\varepsilon)}{d\varepsilon} \right|_{\varepsilon=\varepsilon_l}^{-1}$$

is a residue of electron scattering amplitude by the impurity center in a pole ε_l. In the case of a shallow ($m|v_0| << 1$) local level it equals to

$$r = 2\pi|\varepsilon_l|/m\,.$$

From the formulae (5.5) and (5.7) we find the contribution of the local levels into the high-frequency spin susceptibility:

$$\delta\chi(\vec{q},\omega) = 2\mu^2 n_i \sum_{\vec{p}l} r_l(\varepsilon_{\vec{p}-\vec{q}} - \varepsilon_l)^{-2}[f(\varepsilon_{\vec{p}}) - f(\varepsilon_l)]\times$$

$$\times\left(\frac{1}{\varepsilon_l - \varepsilon_{\vec{p}} - \omega - i0} + \frac{1}{\varepsilon_l - \varepsilon_{\vec{p}} + \omega + i0}\right). \tag{5.8}$$

As it would be expected, the real part of this expression is an even function of the frequency and the imaginary one is an odd function.

If in the energy spectrum of the system there is only one local level located at the lower edge of two-dimensional conductivity zone from the formula (5.8) after the integration over vector $\vec{p}$ directions we obtain

$$\delta\chi(q,\omega) = 4\pi m\mu^2 r n_i \times$$

$$\times\int_0^\infty d\varepsilon[f(\varepsilon) - f(\varepsilon_l)]\left(\frac{1}{\varepsilon_l - \varepsilon + \omega + i0} + \frac{1}{\varepsilon_l - \varepsilon - \omega - i0}\right)\times$$

$$\times\left|\varepsilon - \varepsilon_l + \varepsilon_q\right|[(\varepsilon - \varepsilon_l + \varepsilon_q)^2 - 4\varepsilon\varepsilon_q]^{-3/2}. \tag{5.9}$$

In the case of the weak spatial dispersion ($\varepsilon_q << |\varepsilon_l|$) we can expand the real part of the function (5.9) in series over $\varepsilon_q/\varepsilon_l$ powers. Then with taking into account the terms of q^2 order for the degenerated electrons we find

$$\mathrm{Re}\,\delta\chi(q,\omega) = 4\pi m\mu^2 r n_i \omega^{-2}\times$$

$$\times\left\{\left[1 - \frac{4\varepsilon_q}{\omega}\left(1 - \frac{3}{2}\frac{\varepsilon_l}{\omega}\right)\right]\ln\left|\frac{\varepsilon_F - \varepsilon_l}{\omega + \varepsilon_F - \varepsilon_l}\right| + \frac{4\varepsilon_q}{\varepsilon_l}\left(1 - \frac{\varepsilon_F}{\varepsilon_l}\right)^{-1}\times\right.$$

$$\left.\left[1 - \frac{3}{4}\left(1 - \frac{\varepsilon_F}{\varepsilon_l}\right)^{-1}\right]\right\} + (\omega \to -\omega), \tag{5.10}$$

where ε_F is Fermi energy and by ($\omega \to -\omega$) the item obtained from the previous one by a change of the frequency sign is designated. The function (5.10) has a logarithm peculiarity at the threshold frequency $\varepsilon_F + |\varepsilon_l|$ of activation of electrons localized on impurities by the temporal magnetic field. Impurity absorption of temporal field energy has a threshold at this frequency.

The imaginary part of (5.9) at any degree of electron degeneracy and any q equals to

$$\mathrm{Im}\,\delta\chi(q,\omega) = 4\pi^2 m\mu^2 r n_i |\omega_+| \theta(\omega+\varepsilon_l)[f(\varepsilon_l) -$$

$$- f(\varepsilon_l+\omega)][\omega_+^2 - 4\varepsilon_q(\omega+\varepsilon_l)]^{-3/2} - (\omega\to-\omega), \qquad (5.11)$$

where $\omega_\pm = \omega \pm \varepsilon_q$; θ is Heaviside function. At finite temperature the expression (5.11) has a threshold on the frequency $\omega_g = |\varepsilon_l|$ of local level activation. As the temperature is reduced the threshold is displaced into the point $\varepsilon_F + |\varepsilon_l|$ in accordance with the Pauli principle. At transition through the threshold frequency the imaginary part of susceptibility (5.11) experiences a jump which is equal to (at $q=0$) $4\pi^2 m\mu^2 r n_i \omega_g^{-2}$. With increasing frequency the expression (5.11) decreases proportionally to ω^{-2}. Taking into account final width of the local level leads naturally to the jump disassembly.

The method of the local perturbations used above is also applicable in that case when two-dimensional electron gas is located in a quantizing magnetic field which is normal to $z=0$ plane in which electrons move. In this case electrons are located on the Landau levels and the local levels disjoined from them. For the calculation of spin susceptibility tensor of such a system it is convenient to use the Landau representation. In particular, the spatial Fourier component of the spin magnetization operator in this representation has a form

$$M_\alpha(\vec{q}) = -\mu \sum_{\nu\nu' ss'} \sigma^\alpha_{s's} I_{\nu'\nu}(-\vec{q}) a^+_{\nu' s'} a_{\nu s}\,,$$

where ν is a set of the electron orbital quantum numbers in a magnetic field; $I_{\nu\nu'}(\vec{q}) = \left\langle \nu \left| e^{i\vec{q}\vec{r}} \right| \nu' \right\rangle$ are matrix elements of the flat wave in the Landau basis.

As a result of the above described transformations we obtain the contribution of local levels in the tensor of high-frequency spin susceptibility of the two-dimensional electrons:

$$\delta\chi_{\alpha\beta}(q,\omega) = \frac{m\mu^2\omega_c n_i}{2\pi} \sum_{knn'ss'} r_{ks} \times$$

$$\times \frac{\varphi^2_{nn'}(q)}{(\varepsilon_{ns} - \varepsilon^l_{ks})^2} [f(\varepsilon_{n's'}) - f(\varepsilon^l_{ks})] \times$$

$$\times \left(\frac{\sigma^\alpha_{ss'}\sigma^\beta_{s's}}{\varepsilon^l_{ks} - \varepsilon_{n's'} + \omega + i0} + \frac{\sigma^\alpha_{s's}\sigma^\beta_{ss'}}{\varepsilon^l_{ks} - \varepsilon_{n's'} - \omega - i0} \right). \qquad (5.12)$$

Here ω_c is cyclotron frequency; ε_{ns} and ε^l_{ks} are a location of n-th Landau level and k-th local level;

$$\varphi_{n'n} = \left(\frac{n!}{n'!} \right)^{1/2} \xi^{\frac{1}{2}(n'-n)} \exp\left(-\frac{\xi}{2} \right) L_n^{n'-n}(\xi)\,,$$

$L_n^{n'-n}$ are generalized Laguerre polynomials; $\xi = q^2/(2m\omega_c)$; the wave vector $\vec{q}$ is parallel to y axis. If the distance ω_0 between the Landau level and the local level disjoined from it is small in comparison with ω_c the residue of electron scattering amplitude by the impurity atom in the pole ε_{ks}^l equals to

$$r = 2\pi\omega_0^2/(m\omega_c).$$

In that case when $\xi << 1$ one can neglect spatial dispersion of a tensor (5.12). Then the circular components of susceptibility equal to

$$\delta\chi_{\pm}(\omega) = \delta\chi_{xx}(\omega) \pm i\delta\chi_{yx}(\omega) =$$

$$= \frac{m\mu^2\omega_c n_i}{\pi} \sum_{kn} \frac{r_{k+}}{(\varepsilon_{n+} - \varepsilon_{k+}^l)^2} [f(\varepsilon_{n-}) - f(\varepsilon_{k+}^l)] \times$$

$$\times \frac{1}{\varepsilon_{k+}^l - \varepsilon_{n-} \pm \omega \pm i0} + (+ \leftrightarrow -), \qquad (5.13)$$

where indices $\pm$ at $r_{k\pm}$, $\varepsilon_{n\pm}$ and $\varepsilon_{k\pm}^l$ correspond to the electron spin orientation along and against a magnetic field; by $(+ \leftrightarrow -)$ the item (which is obtained from the previous one by the change of the sign of the electron spin projection and the sign of $\omega + i0$) is designated. From the formulae (5.12) and (5.13) it is well seen that spin susceptibility of the two-dimensional electron gas has the resonance features at the frequencies of the electron transitions between the Landau levels and the local levels accompanied by the spin flip. The resonance frequencies equal to $\left|\varepsilon_{n\pm} - \varepsilon_{k\mp}^l\right|$.

On fig. 13 the dependence of the real (*1*) and the imaginary (*2*) parts of the value

$$\delta Q = \frac{\omega_1\gamma}{2\mu^2 n_i}\delta\chi_- \qquad (5.14)$$

on $x = 1 - \omega/\omega_1$ near the frequency $\omega_1 = \omega_c - \omega_0$ of the resonance electron transitions between the Landau level and the local level with the electron spin flip $(- \to +)$ within one Landau subzone are listed. Here $\gamma = \Gamma/\omega_1$ where Γ is the summary width of the levels involved in the transitions. The calculations are performed for $\gamma = 0{,}1$. The ratio of the maximal value of $\mathrm{Re}\,\delta\chi_-$ to the Pauli susceptibility of the two-dimensional electron gas [98]

$$\chi_0 = e^2(4\pi mc^2)^{-1}$$

equals to

$$k = \frac{\pi n_i}{m\omega_1\gamma}.$$

Substituting the values $n_i = 10^{12}$ sm^{-2}, $H = 10^4$ Oe, i.e. the strength of constant magnetic field, $\omega_0 / \omega_c = 0{,}1$ are typical for the experiments with the inversion layer at the boundary Si – SiO_2 we will obtain $k = 218$.

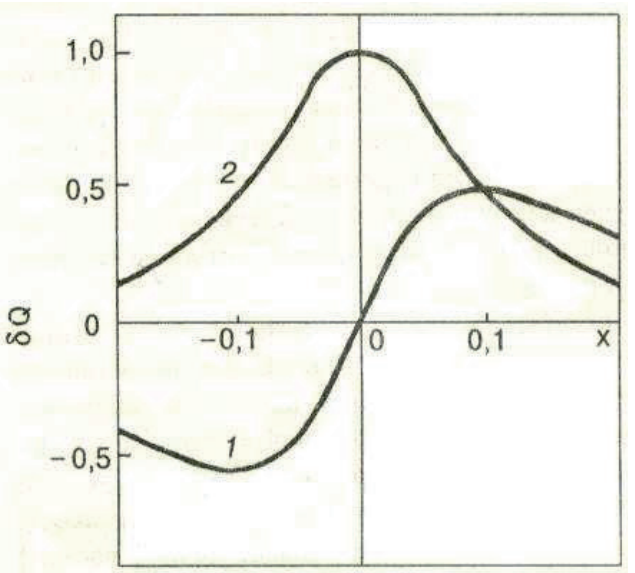


Fig. 13. The dependence of the real (*1*) and the imaginary (*2*) parts of susceptibility (5.14) on the frequency in the neighborhood of the resonance.

In this paragraph an influence of the electron localization in the field of the impurity atoms on tensor of high-frequency spin susceptibility of the two-dimensional electron gas is considered. The mean distance between the impurity atoms is supposed to be big in comparison with the radius of electron orbit in the magnetic field and the frequency of variable magnetic field is sufficiently higher than the electron collision frequency. It allows to use the expansion of susceptibility in series over powers of impurity atom density n_i and to select the contribution of the local levels which is proportional to n_i. The local levels are the poles of one-electron Green function averaged over the impurity configurations. They become apparent in the form of the delta-shaped maxima on the

dependence of the spectral density of the mean Green function on the electron energy. Taking into account these maxima allows to obtain the susceptibility contribution due to the electron transitions between the bound and zone states induced by the variable field. This contribution can be obtained both in appearance and absence of quantizing magnetic field which is normal to the electron layer.

In the absence of the magnetic field the real part of the dynamic spin susceptibility of the degenerated electrons has the logarithm feature on the threshold frequency of the localized electron transitions into the two-dimensional conductivity zone. The imaginary part of susceptibility has the threshold and experiences the jump at this frequency. Taking into account the finite width of the local level leads to the disassembly of the jump and to the maximum on the frequency dependence of susceptibility.

In the quantizing magnetic field susceptibility has the resonance features on the frequencies of the electron transitions between the Landau levels and the local levels alternating with them. The real part of susceptibility as a function of the frequency has the simple poles on the resonance frequencies and the imaginary one has the delta-shaped maxima. Let us notice that at the deduction of the formulae (5.10)-(5.13) only the fact of existence of the local levels in the electron energy spectrum is used. Their characteristics (locations of levels ε_{ks}^{l} and residues of scattering amplitude r_{ks}) are not defined precisely. Therefore, the formulae (5.10)-(5.13) can be used for obtaining these characteristics by comparison of the theory with the experiment.

The results obtained can be used at studying high-frequency magnetic properties of inversion layers at the boundary of semiconductors and dielectrics, heterotransitions, superlattices, two-dimensional and layered metals, thin metal films in the conditions when the electrons fill only the lower energy level due to the spatial quantization [97]. It is necessary to take into account the above obtained features of imaginary part of susceptibility in the dispersion equation for the spin wave spectrum in the two-dimensional non-ferromagnetic Fermi-liquid. Below we will see that they will lead to the reorganization of the wave spectrum in the neighborhood of the resonance frequencies. The maxima of the imaginary part of susceptibility must become apparent in the energy absorption of the high-frequency field and in the cross-section of inelastic neutron magnetic scattering by the two-dimensional electron gas.

5.2. The spin waves in the non-magnetic two-dimensional electron liquid

In par. 1.1 it was noticed that the spin waves in the non-ferromagnetic metals in a magnetic field are connected with the spin resonance of the conductivity electrons forming the degenerated electron liquid of the metal. The spin branch of the spin excitation spectrum of the system of interacting electrons corresponds to a dynamic spin susceptibility pole located outside the Stoner sectors [24,33]. Wave damping at low temperatures is due to the electron collisions with the impurity atoms and lattice defects. Usually they are taken into account by introducing collision frequency due to the relaxation of the electron momentum and spin [36].

In the presence of the impurity atoms attracting electrons and also the magnetic field in a sample the other types of the electron resonance transitions induced by the variable magnetic field are also possible. Those are the transitions with the spin flip between the quasi-local [10] and also magnetoimpurity [21,22] levels and the Landau levels. Near the frequencies of these transitions the new spectrum branches of the spin waves which were called magnetoimpurity waves [37,51] are located.

In the existence of an additional poles of dynamic spin susceptibility connected with the above mentioned electron resonance transitions can be proved on a basis of the simplest approximation, which takes into account the electron-electron interaction in random phase approximation [36]. In this approximation the electron exchange energy is taken into account and their mutual scattering is considered in the stair approximation [36]. The random phase approximation for the description of the spin waves in the non-ferromagnetic metals in the presence of the magnetic field was used in the work [75]. The review of the works in which an influence of impurity atoms on dynamic spin susceptibility without taking into account the electron impurity states is delivered in [76].

In connection with the heightened interest to the two-dimensional systems [97] it is advisable to ascertain how the impurity atoms influence the properties of the spin waves propagating in the two-dimensional electron liquid placed into a magnetic field. The actuality of this task is connected with the fact that in the two-dimensional electron system in a magnetic field an impurity which intensity is somehow small removes the degeneracy over a location of the electron “orbit” center and splits off the local levels from each Landau level [47,105]. In contrast to a three-dimensional case [22] in two-dimensional system the local levels exist in the field of both attracting and repulsing impurity atoms. The electron resonance transitions between the local levels and the Landau levels must be accompanied by appearance of new branches in a wave spectrum.

In the present paragraph the results of calculation of spin wave spectrum and damping in the two-dimensional electron liquid are listed with taking into account the electron local states on impurity atoms in a magnetic field [108]. Electron-electron interaction is taken into account at random phase approximation. Rare impurity atoms are supposed to be distributed at random.

Let us consider two-dimensional electron liquid in a plane z=0 which is normal to stable magnetic field $\vec{H}$. Electron dispersion law is supposed to be isotropic and quadratic and chaotically distributed impurity atoms in a small concentration are supposed to be short-range acting. In the random phase approximation a dispersion equation for the spin waves propagating in the two-dimensional electron liquid normally to the magnetic field has a form [75]

$$1-\frac{I}{2\mu^2}\chi_{\pm}(\vec{q},\omega)=0\,, \tag{5.15}$$

where μ is electron spin magnetic moment; $\chi_{\pm}=\chi_{xx}\pm i\chi_{yx}$ are circular components of dynamic spin susceptibility tensor (they depend on a wave vector $\vec{q}$ and the frequency ω); I is Fourier component of electron-electron interaction energy. The last one takes into account only s-th wave part of particle mutual scattering amplitude. The sheet z =0 occupied by electrons is submerged in a media whose magnetic susceptibility is taken to be equal to 1. The value I in quasi-classical approximation is connected with the parameter B_0 which appear in Fermi-liquid theory by a relationship

$$B_0 = mI/(2\pi\hbar^2)$$

(m is electron effective mass). The constant B_0 is proportional to zero item of spin part expansion of Landau interaction function over Legendre polynomials [36]. Its sign is opposite to the constant sign used in [36].

Let us use quasi-classical long-wavelength approximation for the components $\chi_{\pm}^{(0)}$ of pure sample susceptibility:

$$\chi_{\pm}^{(0)}(\vec{q},\omega)=\chi_0\frac{\Omega_0}{\Omega_0\pm\omega}\left[1+\frac{1}{2}\left(\frac{qv_F}{\Omega_0\pm\omega}\right)^2\right]. \tag{5.16}$$

Here v_F is Fermi velocity, Ω_0=2$\mu H/\hbar$ is frequency of electron paramagnetic resonance,

$$\chi_0 = m\mu^2/\pi\hbar^2$$

is Pauli susceptibility of two-dimensional electrons. Substituting the expression (5.16) in dispersion equation (5.15) we make sure that distribution of spin waves with polarization

corresponding to the "plus" sign in (5.15) and (5.16) is impossible in the electron system. The waves with the polarization "minus" are characterized by a dispersion law

$$\omega(q) = \Omega_0(1 - B_0)\left[1 - \frac{1}{2B_0}\left(\frac{qv_F}{\Omega_0}\right)^2\right]. \tag{5.17}$$

It differs from the dispersion law of the waves in three-dimensional sample [24] only in numerical factor before q^2. Damping decrement of the waves with the spectrum (5.17) equals to the frequency ν of electron collisions with impurity atoms. This frequency is due to momentum and spin relaxation [36].

In the previous paragraph it was shown that taking into account local levels in energy spectrum of two-dimensional electrons leads to appearance of resonance contributions $\delta\chi_{\pm}$ in components of high-frequency ($\omega >> \nu$) susceptibility tensor. It is necessary to take into account these contributions in dispersion equation (5.15). Near frequencies $\omega_s^{\pm}$ of the resonance electron transitions between Landau levels and local levels circular components of spin susceptibility besides (5.16) consist items

$$\delta\chi_{\pm}^{(s)} = \chi_0 \alpha_s^{\pm} \frac{\omega_s^{\pm}}{\omega_s^{\pm} - \omega - i\nu_0}, \tag{5.18}$$

where $\alpha_s^{\pm}$ is oscillator forces of the resonance transitions, $\hbar\nu_0$ is local level width. The values $\alpha_s^{\pm}$ depend on a wave vector. This dependence becomes apparent in terms of the order $(qR)^2$ (R is cyclotron radius), which lead to the weak renormalization of the group velocity of the waves and will not be taken into account later on.

In the case of electron transitions from the Landau level into the local level with a spin-flip $\pm \to \mp$ the resonance frequencies equal to

$$\omega_s^{\pm} = s\omega_c \mp \Omega_0 - \omega_0, \tag{5.19}$$

where ω_c is electron cyclotron frequency, $\hbar\omega_0$ is a distance between the Landau level and a local level splitted off from it, s is a resonance number. In the case involved the oscillator forces equal to

$$\alpha_s^{\pm} = \frac{\omega_c n_i}{\hbar^2 (s\omega_c - \omega_0)^2 \omega_s^{\pm}} \sum_k r_k^{\mp}\left[f(\varepsilon_{(k-s)\pm}) - f(\varepsilon_{k\mp}^l)\right], \tag{5.20}$$

where $\varepsilon_{n\sigma}$ and $\varepsilon_{k\sigma}^l$ are locations of n-th Landau level and k-th local one with a spin projection $\sigma = \pm$, f is Fermi function, $r_k^{\pm}$ is a residue of amplitude of electron-impurity scattering in a pole $\varepsilon_{k\pm}^l - i\hbar\nu_0$, n_i is impurity atom density. Summarizing in (5.20) is performed over couples

of the levels participating in the transitions at a frequency $\omega_s^{\pm}$. Function Fermi difference includes the Pauli principle. The number of items in (5.20) depends on location of Fermi energy ε_F of degenerated electrons.

Electron transition frequencies from the local level into the Landau level with a spin-flip $\pm \to \mp$ equal to

$$\omega_s^{\pm} = s\omega_c \mp \Omega_0 + \omega_0 . \tag{5.21}$$

The corresponding oscillator forces have a form

$$\alpha_s^{\pm} = \frac{\omega_c n_i}{\hbar^2 (s\omega_c + \omega_0)^2 \omega_s^{\pm}} \sum_k r_k^{\pm} \left[f(\varepsilon_{k\pm}^l) - f(\varepsilon_{(k+s)\mp}) \right]. \tag{5.22}$$

Let us consider a neighborhood of frequency $\omega_0^- = \Omega_0 - \omega_0$ of electron transitions from Landau level ε_{N-} into local level ε_{N+}^l. Since $\varepsilon_{N-} < \varepsilon_F < \varepsilon_{N+}^l$, in the sum over k incoming in the formula (5.20) only one item with $k = N$ remains. Other transitions at the frequency are forbidden by the Pauli principle. It is supposed that $\Omega_0 > \omega_0$. And if $\omega_0 > B_0\Omega_0$ the resonance frequency ω_0^- is less then a limit frequency of the wave with a spectrum (5.17). In this case a dispersion curve (5.17) of Silin wave intersects with a straight line $\omega = \omega_0^-$ and there is a cross-situation which is analogous to the discovered one in a crystalline lattice with the quasi-local oscillations [84]. The cross-situation is an intersection of dispersion law curves of two type waves or elementary excitations. If one takes into account the contributions (5.16) and (5.18) the dispersion equation (5.15) for limit ($q = 0$) frequencies in the spin wave spectrum takes a form

$$\frac{1 - B_0 - \omega/\Omega_0}{1 - \omega/\Omega_0} = B_0 \frac{\alpha}{1 - \omega/\omega_r}, \tag{5.23}$$

where $\omega_r = \omega_0^-$,

$$\alpha = \alpha_0^- = \frac{\omega_c r_N^+ n_i}{(\hbar\omega_0)^2 \omega_0^-} . \tag{5.24}$$

This equation has two solutions $\omega_{\pm}$ corresponding to low- and high-frequency branches of spin wave spectrum:

$$\omega_{\pm} = \frac{1}{2}\omega_r(1 - \alpha B_0) + \frac{1}{2}\Omega_0(1 - B_0) \pm$$

$$\pm \frac{1}{2}\left\{ \left[\omega_r(1 - \alpha B_0) - \Omega_0(1 - B_0) \right]^2 + 4\Omega_0\omega_r\alpha B_0^2 \right\}^{1/2} . \tag{5.25}$$

The limit frequency ω_- is located below ω_r and ω_+ is located in a gap $[\Omega_0(1-B_0),\ \Omega_0]$. The parameter defining branch (5.17) splitting into two branches is α. If $\alpha \to 0$ then the frequency ω_- approaches to ω_r, and ω_+ approaches to $\Omega_0(1-B_0)$. In the spin wave spectrum there is a gap $[\omega_-,\ \omega_r]$ within which wave propagation is impossible. Its width equals to

$$\delta\omega = \Omega_0 - \omega_0 - \omega_- . \tag{5.26}$$

The curve (5.17) crosses a line $\omega = \omega_r$ in a point

$$q_0 = \frac{\Omega_0}{v_F}\left[\frac{2B_0(\omega_0 - B_0\Omega_0)}{\Omega_0(1-B_0)}\right]^{1/2} .$$

If $q << q_0$ one may only take an expansion of the dispersion equation (5.15) in a series over q powers. In long-wavelength limit we obtain a dispersion law of considered spin wave branches:

$$\omega_\pm(q) = \omega_\pm - \frac{1}{2}\frac{(qv_F)^2}{\Omega_0 - \omega_\pm}\left[1 + \alpha\frac{\omega_r}{\Omega_0}\left(\frac{\Omega_0 - \omega_\pm}{\omega_r - \omega_\pm}\right)^2\right]^{-1} . \tag{5.27}$$

The dispersion of these waves is anomalous. They represent heterogeneous precession of magnetization around the direction of a constant magnetic field. This precession propagates in a plane z=0. Component ratio of spin magnetization vector $\vec{m}$ induced by a variable magnetic field in the waves with the spectrum (5.27) equals to

$$\frac{m_y}{m_x} = \frac{\chi_{yx}}{\chi_{xx}} = i\frac{\chi_- - \chi_+}{\chi_- + \chi_+} .$$

One can easily obtain cartesian coordinates of susceptibility tensor from (5.16) and (5.18).

The damping of spin waves propagating normally to the magnetic field is due to electron collisions with the impurity atoms. They are determined by the parameters ν and ν_0 characterizing the impurity broadening of Landau levels and local levels. Taking into account small imaginary additions in the expansion (5.16) and (5.18) we make sure that the solution of the dispersion equation (5.15) has a form

$$\omega = \omega_\pm(q) - i\gamma_\pm(q) ,$$

where $\omega_\pm(q)$ is the dispersion law of the waves (5.27) and $\gamma_\pm$ is damping decrement. It equals to

$$\gamma_{\pm} = \left[\nu + \nu_0 \alpha \frac{\omega_r}{\Omega_0} \left(\frac{\Omega_0 - \omega_{\pm}}{\omega_r - \omega_{\pm}} \right)^2 \right] \left[1 + \alpha \frac{\omega_r}{\Omega_0} \left(\frac{\Omega_0 - \omega_{\pm}}{\omega_r - \omega_{\pm}} \right)^2 \right]^{-1} . \tag{5.28}$$

Small values ν and ν_0 provide fulfillment of inequality $\gamma_{\pm} << \omega_{\pm}$. If $\alpha \to 0$ then from the formulae (5.27) and (5.28) we obtain spectrum and damping decrement of spin waves in the absence of electron localization.

Two solutions (5.27) of the dispersion equation also remain in the case $\omega_0 < B_0 \Omega_0$. But now

$$\omega_- < \Omega_0 (1 - B_0), \ \omega_0^- < \omega_+ < \Omega_0 .$$

Let us consider electron transitions from local level ε_{N-}^{l} into Landau level ε_{N+}. The transition frequency equals to $\omega_r = \Omega_0 + \omega_0$ and the oscillator force is

$$\alpha = \frac{\omega_c r_N^- n_i}{(\hbar \omega_0)^2 (\Omega_0 + \omega_0)} . \tag{5.29}$$

The dispersion equation for the limit frequencies in a considered case has a previous form (5.23) but now $\omega_r = \Omega_0 + \omega_0$ and oscillator force α is expressed by the formula (5.29). The limit frequencies are located in the regions

$$\omega_- < \Omega_0 (1 - B_0), \ \Omega_0 < \omega_+ < \omega_r .$$

Low-frequency spectrum branch is recovered with the band of Silin wave (5.17). High-frequency branch is located in a frequency region where propagation of quasi-classical Silin waves is impossible. The solutions of the dispersion equation differ from (5.25), (5.27) and (5.28) by the other values of resonance frequency and oscillator force. Spin waves with a dispersion law $\omega_+(q)$ slightly damp in a transparency band [ω_+, ω_r] with width

$$\Delta\omega = \Omega_0 + \omega_0 - \omega_+ . \tag{5.30}$$

Let us consider the waves with "plus" polarization. From the formula (5.16) it is well seen that in the absence of electron localization a slightly damping solution of the equation (5.15) for the spin waves with "plus" polarization can exist only in a case $B_0 > 1$. But electron liquid under this condition becomes instable [36]. The positive contribution (5.18) of local levels in real part of spin susceptibility in a region $\omega < \omega_s^+$ leads to the possibility of propagation of such waves. This situation reminds antihelicons in electron gas [39] whose propagation is possible due to the existence of subsystem of localized electrons with a direction of rotation which is determined not only by a magnetic field but also by impurity center.

Resonance frequency of transitions from Landau level $\varepsilon_{(N-1)+}$ into local level ε_{N-} with spin flip $+\to-$ equals to $\omega_1^+ = \omega_c - \Omega_0 - \omega_0$. Oscillator force

$$\alpha_1^+ = \frac{\omega_c r_N^- n_i}{\hbar^2(\omega_c - \omega_0)^2 \omega_1^+}[f(\varepsilon_{(N-1)+}) - f(\varepsilon_{N-}^l)]. \tag{5.31}$$

In this case dispersion equation for limit frequencies of spin waves has a form

$$\frac{\omega + \Omega_0(1-B_0)}{\omega + \Omega_0} = \alpha B_0 \frac{\omega_r}{\omega_r - \omega}, \tag{5.32}$$

where $\omega_r = \omega_1^+$, $\alpha = \alpha_1^+$. In a region $\omega < \omega_r$ two solutions of this equation exist

$$\omega_\pm = \frac{1}{2}\omega_r(1-\alpha B_0) - \frac{1}{2}\Omega_0(1-B_0) \pm$$

$$\pm\frac{1}{2}\{[\omega_r(1+\alpha B_0) + \Omega_0(1-B_0)]^2 -$$

$$-4\alpha B_0 \omega_r(\omega_r + \Omega_0)\}^{1/2}. \tag{5.33}$$

If $\alpha \to 0$ the upper branch (5.33) approaches the resonance frequency ω_r and a solution ω_- becomes negative.

In a long-wavelength limit the solutions of the equation (5.15) in a considered case are the following:

$$\omega_\pm(q) = \omega_\pm + \frac{1}{2}\frac{(qv_F)^2}{\Omega_0 + \omega_\pm}\left[1 - \alpha\frac{\omega_r}{\Omega_0}\left(\frac{\Omega_0 + \omega_\pm}{\omega_r - \omega_\pm}\right)^2\right]^{-1}, \tag{5.34}$$

where $\omega_\pm$ are the limit frequencies of (5.33). Dispersion of these waves is normal. They slightly damp due to electron collisions with impurity atoms in the transparency bands located between limit frequencies (5.33) and a resonance frequency ω_1^+.

The frequency of electron transitions from the local level ε_{N+}^l into Landau level $\varepsilon_{(N+1)-}$ equals to

$$\omega_r = \omega_c - \Omega_0 + \omega_0$$

and oscillator force is

$$\alpha = \frac{\omega_c r_N^+ n_i}{\hbar^2(\omega_c + \omega_0)^2 \omega_r}[f(\varepsilon_{N+}^l) - f(\varepsilon_{(N+1)-})]. \tag{5.35}$$

In this case two branches of spin waves are located in the interval $(0, \omega_r)$. The solutions of the dispersion equation are given by formulae (5.33) and (5.34) in which $\omega_r = \omega_c - \Omega_0 + \omega_0$ and the oscillator force is determined by (5.35).

5.3. Neutron magnetic scattering by two-dimensional electron gas with the magnetoimpurity electron states

The spin waves considered in the parts 5.1 and 5.2 can be detected in the experiments with slow neutrons. Differential cross-section of neutron magnetic scattering by two-dimensional electron liquid which is calculated on a square unit equals to [60]

$$\frac{d^2\sigma}{dO'd\varepsilon'} = \frac{1}{4\pi}\left(\frac{\gamma r_0}{\mu}\right)^2 \frac{k'}{k}(n_\omega + 1)\times$$

$$\times \sum_{ik}(\delta_{ik} - e_i e_k)\,\mathrm{Im}\,\chi^s_{ik}(\vec{q},\omega), \tag{5.36}$$

where χ^s_{ik} is symmetrized tensor of spin susceptibility, $r_0 = e^2/mc^2$ is classical electron radius, γ=1,91 is a neutron giromagnetic ratio, $\vec{q} = \vec{k} - \vec{k}'$ and $\hbar\omega = \varepsilon - \varepsilon'$ are changes of the neutron wave vector and energy at scattering in a solid angle dO', n_ω is a Planck distribution function, $\vec{e} = \vec{q}/q$.

Since scattering vector $\vec{q}$ is normal to magnetic field the sum incoming in (5.36) equals to

$$\frac{1}{2}(\chi_+ + \chi_-) + \chi_{zz}, \tag{5.37}$$

where components of spin susceptibility tensor are calculated in random phase approximation. In the absence of electron-electron interaction a local level contribution in longitudinal component of dynamic spin susceptibility equals to

$$\delta\chi_{zz}(\omega) = \frac{1}{2}\chi_0\hbar\omega_c n_i \sum_{nk\sigma}\frac{r_k^\sigma}{(\varepsilon_n - \varepsilon_k^l)^2}\times$$

$$\times[f(\varepsilon_{k\sigma}^l) - f(\varepsilon_{n\sigma})]\left(\frac{1}{\varepsilon_n - \varepsilon_k^l + \hbar\omega + io} + \frac{1}{\varepsilon_n - \varepsilon_k^l - \hbar\omega - io}\right).$$

This function has resonance singularities at frequencies $\left|\varepsilon_n - \varepsilon_k^l\right|/\hbar$ of electron transitions between Landau levels and local levels without spin flip.

From the formula (5.36) one can easily obtain a contribution of one-particle excitations of electrons localized on impurity atoms in a cross-section of inelastic neutron scattering. Items with $\chi_\pm$ in (5.3) give contribution in scattering cross-section with electron spin flip $\pm \to \mp$ and items with χ_{zz} give a contribution without spin flip. In particular, cross-section of the scattering accompanied by electron transitions from local level into Landau level with spin flip $\pm \to \mp$ near $\omega = \omega_s^\pm$ (5.21) equals to

$$\frac{d^2\sigma_{\pm}}{dO'd\varepsilon'} = \frac{1}{8\pi}\left(\frac{\gamma r_0}{\mu}\right)^2 \frac{k'}{k}\chi_0\omega_s^{\pm}\alpha_s^{\pm}\frac{\nu_0}{(\omega-\omega_s^{\pm})^2+\nu_0^2}. \qquad (5.38)$$

Here temperature is supposed to be small in comparison with the transition energy. In the energy spectrum of scattered neutrons symmetrical maxima (5.38) are present due to one-particle excitations of localized electrons. Such maxima connected with electron transitions from Landau levels into local levels must be observed at $\omega = \omega_s^{\pm}$ (5.19). Let us notice that in a three-dimensional case analogous maxima are asymmetrical [52]. It is connected with asymmetry of electron state density at the Landau levels.

Besides maxima described by the formula (5.38) in spectrum of scattered neutrons a series of Lorentz maxima is present due to the scattering on spin waves with the spectrum (5.27) and (5.34). Cross-section of a scattering with emission of spin wave quantum with a dispersion law ω_q equals to:

$$\frac{d^2\sigma_s}{dO'd\varepsilon'} = \frac{1}{2m\Omega_0}\left(\frac{\hbar\gamma r_0}{I}\right)^2 \frac{k'}{k}(\omega_q-\Omega_0)^2 \times$$

$$\times\left[1+\alpha\frac{\omega_r}{\Omega_0}\left(\frac{\omega_q-\Omega_0}{\omega_q-\omega_r}\right)^2\right]^{-1}\frac{\gamma_q}{(\omega-\omega_q)^2+\gamma_q^2}, \qquad (5.39)$$

where ω_r is a resonance frequency (5.19) or (5.21), α is an oscillator force (5.20) or (5.22), γ_q is a wave damping decrement. If $\alpha \to 0$ from the formula (5.39) we obtain neutron cross-section on the spin waves with the spectrum (5.17) [90].

Characteristics of local electron states (locations and widths of local levels, residues of electron scattering amplitude by impurity atoms) have not been given a precise expression so far. Only the fact of existence of a pole of electron-impurity scattering amplitude has been used. These characteristics can be obtained by the way of comparison of the theory with the experiment or they can be calculated on a basis of a certain model of impurity potential. In particular, in a case of short-range acting potential of impurity atom and slight splitting local level from Landau level off ($\omega_0 << \omega_c$) a residue of scattering amplitude equals to [82,107] $r = 2\pi\hbar^3\omega_0^2/m\omega_c$. This expression will be used for estimations.

For an estimation of values of differential cross-section maxima obtained in this part for neutron scattering we use values of parameters which are typical for thin films of semi-metals and inversion layer at a boundary of Silicon with Silica [97]: m =10^{-31} kg, n_e=10^{16} m^{-2}, i.e. two-dimensional electron liquid density, n_i/n_e=0,01, ω_0/Ω_0 = 0,2, B_0 = 0,1, $\nu = \nu_0$. Then in

the field with magnetic flux density 10 T we obtain $\Omega_0 = 1{,}9 \cdot 10^{12}$ s^{-1}, ratios of maximal values of cross-sections (5.38) and (5.39) to the cross-section maximum of the scattering on a Silin wave equal to 0,23 and 0,12. In this case ratios of widths of the gap (5.26) and of the transparency band (5.30) to Ω_0 equal to 0,74 and 0,02 respectively.

Resuming the results obtained in the parts 5.2 and 5.3 we notice that impurity atoms in two-dimensional electron systems exert important influence on a quasi-particle energy spectrum. In such systems a very weak impurity is able to form a local level at the edge of a conductivity zone. In a quantizing magnetic field which is normal to a plane of electron motion multiplication of local levels occurs. They are splitting off up or down from each Landau level in dependence on a sign of impurity potential. Such a structure of spectrum of two-dimensional electron system in magnetic field affects on its high-frequency characteristics. In particular, dynamic spin susceptibility has resonance singularities on frequencies of electron transitions between Landau levels and local level with spin flip. On these singularities new branches of spin wave spectrum are formed in non-ferromagnetic two-dimensional electron liquid.

Here it is shown that electron localization on impurity atoms competing with dissipation processes is conducive to spin wave propagation. New spectrum branches of collective oscillations of spin magnetization exist in those regions where Silin wave propagation is impossible. The spectrum and damping decrement of these waves have been calculated.

When the Silin wave frequency coincides with the frequency of electron resonance transitions between Landau levels and resonance levels the rearrangement of spin wave spectrum which is due to binding oscillations in the spin wave with oscillations at the resonance occurs. Dispersion curve of Silin wave in two-dimensional electron liquid splits into two branches: low- and high-frequency. They are divided by a gap within which the wave propagation is impossible.

The spin waves considered here can be detected in experiments over measurement of differential cross-section of inelastic neutron magnetic scattering by the current of spin magnetization of two-dimensional electrons. In energy spectrum of scattered neutrons there are maxima due to both one-particle excitations of electrons localized on impurities and spin waves. Symmetrical maxima due to one-particle excitations are located on resonance frequencies of transitions between Landau levels and local levels. The widths of these maxima are determined by the widths of levels participating in the transitions. Locations of

Lorentz maxima of scattering cross-section on the spin waves allow to obtain the wave spectrum and their widths allows to obtain damping decrement.

The results listed in this chapter can be used at studying two-dimensional metals, inversion layers on the boundary of semi-conductor with dielectric, layered systems, thin metal films under the conditions when electrons are located on a lower energy level which is due to dimensional quantization.

C O N C L U S I O N

Here we resume the main results listed in the monograph.

Dynamic spin susceptibility of conductivity electrons in non-ferromagnetic metals with quasi-local states of carriers on impurity atoms in a magnetic field has the resonance singularities on frequencies of electron transitions induced by a variable magnetic field between quasi-local levels and Landau levels.

Resonance contribution in a tensor of dynamic spin susceptibility of electrons with the quasi-local states in metals whose Fermi surface has a form of a revolution ellipsoid depends on orientation of strength vector of the magnetic field relatively to an axis of ellipsoid revolution.

In non-ferromagnetic metals with the quasi-local electron states in magnetic field new spectrum branches of transversal spin waves exist. Their frequencies lie in the transparency bands near the frequencies of the resonance transitions of localized electrons into Landau levels accompanied by a spin flip.

Characteristics of these waves (dispersion law, damping decrement, polarization) depends on parameters of the quasi-local states.

The quasi-local states of electrons in a field of impurity atoms influence spectrum and damping quantum spin waves in the non-ferromagnetic metals in a magnetic field. In the region of intersection of spin wave dispersion curve with the frequency of resonance electron transitions between the quasi-local levels and Landau levels the rearrangement of quantum spin wave spectrum which is well-known under a name “cross-situation” occurs. Instead of one branch in every transparency window two wave spectrum branches exist.

Inelastic neutron magnetic scattering in the non-ferromagnetic metals with the quasi-local electron states on the impurity atoms in magnetic field leads to excitation of spin waves which damp slightly in transparency bands near frequencies of resonance electron transitions between quasi-local levels and Landau levels. Calculation of differential cross-section of neutron magnetic scattering by these waves shows that in energy spectrum of scattered neutrons there is a series of Lorentz satellites located symmetrically relatively to the unshifted line.

High-frequency asymptotic of dynamic spin susceptibility of two-dimensional electron gas is calculated. Local states of electrons on impurity atoms and quantizing magnetic field are considered. Susceptibility has resonance singularities on frequencies of electron transitions between Landau levels and local levels. In the absence of a magnetic field the real part of

susceptibility has logarithm feature and the imaginary one has a maximum on threshold activation frequency of bound electrons by a variable electromagnetic field.

In a random phase approximation spin waves in non-ferromagnetic two-dimensional electron liquid are considered in a magnetic field. Bound electron states in a field of impurity atoms are considered. It was shown that electron localization provides spin wave propagation. New spectrum branches of the waves exist in frequency regions where propagation of Silin waves is impossible. Wave spectrum and damping decrement of the waves are found. When crossing a dispersion curve of Silin wave with electron resonance frequency between Landau levels and local levels a cross-situation which is typical for bound waves takes place.

Differential cross-section of neutron magnetic scattering by two-dimensional electron liquid in a magnetic field is calculated. In an energy spectrum of scattered neutrons additional maxima are present due to one-particle excitations of localized electrons and spin waves. Locations and widths of these maxima allow to obtain the data about a spectrum of electron impurity states and also the spectrum and the damping of the spin waves.

APPENDIX I

The formulae used in the text for the resonance contributions to circular components of a tensor of dynamic spin susceptibility of electrons with isotropic and quadratic dispersion law have a form:

$$\delta\chi_{+}^{(n)}(\vec{q},\omega;LL\to QL)=a_{+}^{n}(\vec{q};LL\to QL)i\left(\frac{\omega_{rn}^{-}}{\omega_{rn}^{-}-\omega-i\Gamma_{-}/\hbar}\right)^{1/2}, \qquad \text{(I.1,a)}$$

$$a_{+}^{n}(\vec{q};LL\to QL)=2\pi\mu^{2}n_{i}\left(\frac{m}{2}\right)^{3/2}\frac{\Omega r_{\downarrow}}{\pi^{2}\hbar^{2}(\hbar\omega_{rn}^{-})^{2}\sqrt{\hbar\omega_{rn}^{-}}}\times$$

$$\times\left[f(\varepsilon_{r\downarrow}-\hbar\omega_{rn}^{-})-f(\varepsilon_{r\downarrow})\right]\frac{1}{\left(1+\Omega_{0}/\omega_{rn}^{-}\right)^{2}}+\frac{\hbar^{2}q_{z}^{2}/m}{\hbar\omega_{rn}^{-}(1+\Omega_{0}/\omega_{rn}^{-})^{3}}+$$

$$+\frac{\bar{q}^{2}(n+1)}{2(1+\frac{\Omega_{0}-\Omega}{\omega_{rn}^{-}})^{2}}+\frac{\bar{q}^{2}n}{2(1+\frac{\Omega+\Omega_{0}}{\omega_{rn}^{-}})^{2}}-\frac{\bar{q}^{2}(n+1/2)}{(1+\Omega_{0}/\omega_{rn}^{-})^{2}}],$$

$$\hbar\omega_{rn}^{-}=\varepsilon_{r}-\varepsilon_{n}-2\mu H;$$

$$\delta\chi_{+}^{(n)}(\vec{q},\omega;QL\to LL)=a_{+}^{n}(\vec{q};QL\to LL)i\left(\frac{\omega_{rn}^{+}}{\omega-\omega_{rn}^{+}+i\Gamma_{+}/\hbar}\right)^{1/2}, \qquad \text{(I.1,b)}$$

$$a_{+}^{n}(\vec{q};QL\to LL)=2\pi\mu^{2}n_{i}(m/2)^{3/2}\times$$

$$\times\frac{\Omega r_{\uparrow}}{\pi^{2}\hbar^{2}(\hbar\omega_{rn}^{+})^{2}\sqrt{\hbar\omega_{rn}^{+}}}\left[f(\varepsilon_{r\uparrow})-f(\varepsilon_{r\uparrow}+\hbar\omega_{rn}^{+})\right]\times$$

$$\times\left[\frac{1}{(1-\Omega_{0}/\omega_{rn}^{+})^{2}}+\frac{\hbar^{2}q_{z}^{2}/m}{\hbar\omega_{rn}^{+}(1+\Omega_{0}/\omega_{rn}^{+})^{3}}+\frac{\bar{q}^{2}(n+1)}{2\left(1+\frac{\Omega+\Omega_{0}}{\omega_{rn}^{+}}\right)^{2}}+\right.$$

$$\left.+\frac{\bar{q}^{2}n}{2\left(1+\frac{\Omega_{0}-\Omega}{\omega_{rn}^{+}}\right)^{2}}-\frac{\bar{q}^{2}(n+1/2)}{(1+\Omega_{0}/\omega_{rn}^{+})^{2}}\right],$$

$$\hbar\omega_{rn}^{+}=\varepsilon_{n}-\varepsilon_{r}-2\mu H;$$

$$\delta\chi_{-}^{(n)}(\vec{q},\omega;LL\to QL)=a_{-}^{n}(\vec{q};LL\to QL)i\left(\frac{\omega_{rn}^{-}}{\omega_{rn}^{-}-\omega-i\Gamma_{+}/\hbar}\right)^{1/2}, \qquad \text{(I.2,a)}$$

$$a_n^-(\vec{q};LL\to QL)=2\pi\mu^2 n_i(m/2)^{3/2}\times$$

$$\times\frac{\Omega r_\uparrow}{\pi^2\hbar^2(\hbar\omega_{rn}^-)^2\sqrt{\hbar\omega_{rn}^-}}\left[f(\varepsilon_{r\uparrow}-\hbar\omega_{rn}^-)-f(\varepsilon_{r\uparrow})\right]\times$$

$$\times\left[\frac{1}{(1-\Omega_0/\omega_{rn}^-)^2}+\frac{\hbar^2q_z^2/m}{\hbar\omega_{rn}^-(1-\Omega_0/\omega_{rn}^-)^3}+\frac{\bar{q}^2(n+1)}{2(1-\frac{\Omega+\Omega_0}{\omega_{rn}^-})^2}+\right.$$

$$\left.+\frac{\bar{q}^2 n}{2(1+\frac{\Omega-\Omega_0}{\omega_{rn}^-})^2}-\frac{\bar{q}^2(n+1/2)}{(1-\Omega_0/\omega_{rn}^-)^2}\right],$$

$$\hbar\omega_{rn}^-=\varepsilon_r-\varepsilon_n+2\mu H;$$

$$\delta\chi_-^{(n)}(\vec{q},\omega;QL\to LL)=a_-^n(\vec{q};QL\to LL)i\left(\frac{\omega_{rn}^+}{\omega-\omega_{rn}^++i\Gamma_-/\hbar}\right)^{1/2},\qquad\text{(I.2,b)}$$

$$a_-^n(\vec{q};QL\to LL)=2\pi\mu^2 n_i(m/2)^{3/2}\frac{\Omega r_\downarrow}{\pi^2\hbar^2(\hbar\omega_{rn}^+)^2\sqrt{\hbar\omega_{rn}^+}}\times$$

$$\times[f(\varepsilon_{r\downarrow})-f(\varepsilon_{r\downarrow}+\hbar\omega_{rn}^+)]\left[\frac{1}{(1-\Omega_0/\omega_{rn}^+)^2}+\frac{\hbar^2q_z^2/m}{\hbar\omega_{rn}^+(1-\Omega_0/\omega_{rn}^+)^3}+\right.$$

$$\left.+\frac{\bar{q}^2(n+1)}{2(1+\frac{\Omega-\Omega_0}{\omega_{rn}^+})^2}+\frac{\bar{q}^2 n}{2(1-\frac{\Omega+\Omega_0}{\omega_{rn}^+})^2}-\frac{\bar{q}^2(n+1/2)}{(1-\Omega_0/\omega_{rn}^+)^2}\right],$$

$$\hbar\omega_{rn}^+=\varepsilon_n-\varepsilon_r+2\mu H;$$

$$\delta\chi_{zz}^{(n)}(\vec{q},\omega;LL\to QL)=\sum_\sigma a_{zz}^{n\sigma}(\vec{q};LL\to QL)\times$$

$$\times i\left(\frac{\omega_{rn}^-}{\omega_{rn}^--\omega-i\Gamma_\sigma/\hbar}\right)^{1/2},\qquad\text{(I.3, a)}$$

$$a_{zz}^{n\sigma}(\vec{q};LL\to QL)=\pi\mu^2 n_i(m/2)^{3/2}\times$$

$$\times\frac{\Omega r_\sigma}{\pi^2\hbar^2(\hbar\omega_{rn}^-)^2\sqrt{\hbar\omega_{rn}^-}}[f(\varepsilon_{r\sigma}-\hbar\omega_{rn}^-)-f(\varepsilon_{r\sigma})]\times$$

$$\times\left[1+\frac{\hbar^2q_z^2/m}{\hbar\omega_{rn}^-}+\frac{\bar{q}^2(n+1)}{2(1-\Omega/\omega_{rn}^-)^2}+\frac{\bar{q}^2 n}{2(1+\Omega/\omega_{rn}^-)^2}-\bar{q}^2(n+1/2)\right];$$

$$\hbar\omega_{rn}^-=\varepsilon_r-\varepsilon_n;$$

$$\delta\chi_{zz}^{(n)}(\vec{q},\omega;QL\to LL)=\sum_{\sigma}a_{zz}^{n\sigma}(\vec{q};QL\to LL)\times$$

$$\times i\left(\frac{\omega_{rn}^{+}}{\omega-\omega_{rn}^{+}+i\Gamma_{\sigma}/\hbar}\right)^{1/2}, \qquad \text{(I. 3, b)}$$

$$a_{zz}^{n\sigma}(\vec{q};QL\to LL)=\pi\mu^{2}n_{i}(m/2)^{3/2}\times$$

$$\times\frac{\Omega r_{\sigma}}{\pi^{2}\hbar^{2}(\hbar\omega_{rn}^{+})^{2}\sqrt{\hbar\omega_{rn}^{+}}}[f(\varepsilon_{r\sigma})-f(\varepsilon_{r\sigma}+\hbar\omega_{rn}^{+})]\times$$

$$\times\left[1-\frac{\hbar^{2}q_{z}^{2}/m}{\hbar\omega_{rn}^{+}}+\frac{\bar{q}^{2}(n+1)}{2(1+\Omega/\omega_{rn}^{+})^{2}}+\frac{\bar{q}^{2}n}{2(1-\Omega/\omega_{rn}^{+})^{2}}-\bar{q}^{2}(n+1/2)\right],$$

$$\hbar\omega_{rn}^{+}=\varepsilon_{n}-\varepsilon_{r}.$$

APPENDIX II

Formulae for resonance contributions to components of a tensor of dynamic spin susceptibility of electrons with anisotropic dispersion law

$$\delta\chi_{xx}^{(n)}(\omega, QL \to LL) = i\frac{m_\perp\sqrt{m_\parallel}\Omega\mu^2 n_i}{2^{3/2}\pi\hbar^{5/2}}\times$$

$$\times\frac{1}{|F'_\uparrow|}\cdot\frac{1}{(\varepsilon_n-\varepsilon_r)^2}\cdot\frac{1}{(\omega-\frac{\varepsilon_n-\varepsilon_r-2\mu H}{\hbar}+i\frac{\Gamma_\uparrow}{\hbar})^{1/2}}\times$$

$$\times[f(\varepsilon_r^\uparrow)-f(\varepsilon_n^\downarrow)]+ \qquad \text{(II. 1)}$$

$$+i\frac{m_\perp\sqrt{m_\parallel}\Omega\mu^2 n_i}{2^{3/2}\pi\hbar^{5/2}|F'_\downarrow|}\cdot\frac{f(\varepsilon_r^\downarrow)-f(\varepsilon_n^\uparrow)}{(\varepsilon_n-\varepsilon_r)^2\left(\omega-\frac{\varepsilon_n-\varepsilon_r+2\mu H}{\hbar}+\frac{i}{\hbar}\Gamma_\downarrow\right)^{1/2}},$$

$$\delta\chi_{xy}^{(n)}(\omega, QL \to LL) = -\frac{m_\perp\sqrt{m_\parallel}\Omega\mu^2 n_i}{2^{3/2}\pi\hbar^{5/2}}\times$$

$$\times\frac{1}{|F'_\uparrow|}\frac{1}{(\varepsilon_n-\varepsilon_r)^2}\frac{f(\varepsilon_r^\uparrow)-f(\varepsilon_n^\downarrow)}{\left(\omega+\frac{\varepsilon_r-\varepsilon_n+2\mu H}{\hbar}+i\frac{\Gamma_\uparrow}{\hbar}\right)^{1/2}}+$$

$$+\frac{m_\perp\sqrt{m_\parallel}\Omega\mu^2 n_i}{2^{3/2}\pi\hbar^{5/2}(\varepsilon_n-\varepsilon_r)^2}\cdot\frac{1}{|F'_\downarrow|}\times \qquad \text{(II. 2)}$$

$$\times\frac{f(\varepsilon_r^\downarrow)-f(\varepsilon_n^\uparrow)}{\left(\omega-\frac{\varepsilon_n-\varepsilon_r+2\mu H}{\hbar}+i\frac{\Gamma_\downarrow}{\hbar}\right)^{1/2}},$$

$$\delta\chi_{zz}^{(n)}(\omega, QL \to LL) = i\frac{m_\perp\sqrt{m_\parallel}\Omega\mu^2 n_i}{2^{3/2}\pi\hbar^{5/2}(\varepsilon_n-\varepsilon_r)^2}\times$$

$$\times\left\{\frac{f(\varepsilon_r^\uparrow)-f(\varepsilon_n^\uparrow)}{|F'_\uparrow|\left(\omega-\frac{\varepsilon_n-\varepsilon_r}{\hbar}+i\frac{\Gamma_\uparrow}{\hbar}\right)^{1/2}}+\right. \qquad \text{(II. 3)}$$

$$\left.+\frac{f(\varepsilon_r^\downarrow)-f(\varepsilon_n^\downarrow)}{|F'_\downarrow|\left(\omega-\frac{\varepsilon_n-\varepsilon_r}{\hbar}+i\frac{\Gamma_\downarrow}{\hbar}\right)^{1/2}}\right\}.$$

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