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POLYADIC SYSTEMS, REPRESENTATIONS AND QUANTUM GROUPS

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A review of polyadic systems and their representations is given. The classification of general polyadic systems is done. The multiplace generalization of homomorphisms, preserving associativity, is presented. The multiplace representations and multiactions are defined, concrete examples of matrix representations for some ternary groups are given. The ternary algebras and Hopf algebras are defined, their properties are studied. At the end some ternary generalizations of quantum groups and the Yang-Baxter equation are presented.
KEY WORDS: n-ary group, Post theorem, commutativity, homomorphism, group action, Yang-Baxter equation

ПОЛИАДИЧЕСКИЕ СИСТЕМЫ, ПРЕДСТАВЛЕНИЯ И КВАНТОВЫЕ ГРУППЫ

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Приведен обзор полиадических систем и их представлений, дана классификация общих полиадических систем. Построены многоместные обобщения гомоморфизмов, сохраняющие ассоциативность. Определены мультидействия и мультиместные представления. Приведены конкретные примеры матричных представлений для некоторых тернарных групп. Определены тернарные алгебры и Хопф алгебры, изучены их свойства. В заключение, предствлены некоторые тернарные обобщения квантовых групп и уравнения Янга-Бакстера.
КЛЮЧЕВЫЕ СЛОВА: n-арная группа, теорема Поста, коммутативность, гомоморфизм, групповое действие, уравнение Янга-Бакстера

ПОЛІАДИЧНІ СИСТЕМИ, ПРЕДСТАВЛЕННЯ І КВАНТОВІ ГРУПИ

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Зроблено огляд поліадичних систем та їх представлень, дана класифікація загальних поліадичних систем. Побудовані багатомісні узагальнення гомоморфізмив, що зберігають асоціативність. Визначені мультидії і мультимісні представлення. Наведені конкретні приклади матричних представлень для деяких тернарних груп. Визначені тернарна алгебра і алгебри Хопфа, вивчені їх властивості. На закінчення, предствлені деякі тернарні узагальнення квантових груп та рівняння Янга-Бакстера.
КЛЮЧОВІ СЛОВА: n-арна група, теорема Поста, комутативність, гомоморфізм, групова дія, рівняння Янга-Бакстера

One of the most promising steps in generalizing physical theories is consideration of higher arity algebras [1], in other words ternary and n-ary algebras, in which the binary composition law is substituted by ternary or n-ary one [2].

Firstly ternary algebraic operations (with the arity $n = 3$) were introduced already in the XIX-th century by A. Cayley in 1845 and later by J. J. Silvester in 1883. The notion of an n-ary group was introduced in 1928 by [3] (inspired by E. Nöther) which is a natural generalization of the notion of a group. Even before in 1924, a particular case, that is, ternary group of idempotents, was used in [4] to study infinite abelian groups. The important Post's coset theorem explained the connection between n-ary groups and their covering binary groups [5]. The next step in study of n-ary groups was the Gluskin-Hosszú theorem [6, 7]. Another definition of n-ary group can be given as a universal algebra with additional laws [8] or identities containing special elements [9].

The representation theory of (binary) groups [10, 11] plays an important role in their physical applications [12]. It is initially based on a matrix realization of group elements and abstract group action as a usual matrix multiplication [13, 14]. The cubic and n-ary generalizations of matrices and determinants were made is [15, 16], and their physical application appeared in [17, 18]. In general, particular questions of n-ary group representations were considered in and matrix representations were derived by the author [19], and some general theorems

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