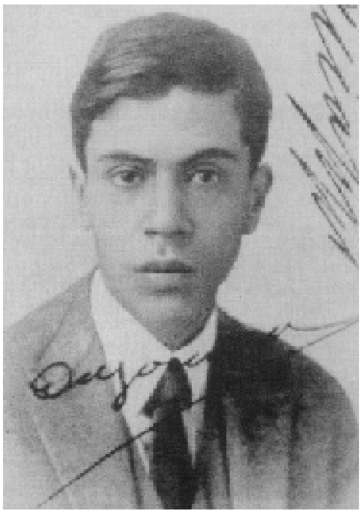
ETTORE MAJORANA AND MATVEI BRONSTEIN (1906-1938): MEN AND SCIENTISTS

YU.P. STEPANOVSKY
National Scientific Centre "Kharkov Institute of Physics and Technology"
1 Akademicheskaja Street, Kharkov — 61108, Ukraine



There are many categories of scientists in the world; people of second and third rank who do their best but do not go very far. There are people of the first class who make great discoveries, fundamental to the development of science. And then there are the geniuses like Galileo and Newton. Well, Ettore Majorana was one of these.

E. Fermi [1]

Men like Matvei Bronstein are born to beautify mankind and to highlight some outlines of the Universe.

V.Ya. Frenkel, G.E. Gorelik [2]

1. Introduction

The first meeting of two prominent physicists — Ettore Majorana and Matvei Bronstein — happened in Erice in 1991, more than 50 years after they died [2]. In July 1991 Ettore Majorana Centre for Scientific Culture organized 29th Course: *Physics at the Highest Energy and Luminosity: To Understand the Origin of Mass* at the International School of Subnuclear Physics. A well-known Russian physicist L.B. Okun' in his lecture discussed classification of physical theories based on fundamental constants c, G and $\hbar$. This classification was first introduced by the Soviet physicist Matvei Bronstein in 1933. L.B. Okun' told about the short life and tragic death of Matvei Petrovich Bronstein. Matvei Bronstein and Ettore Majorana had similar lives in a particular sense. They both were born in 1906 and died in 1938. The scientific works of the both were connected with each other and are of interest up to nowadays. They both possessed great personalities and were great scientists. And in this article they are meeting again.

2. The Life and Disappearance of Ettore Majorana

It is said that for over two-thousand years history only two great men of exact science were brought to mankind by wonderful land of Sicily. The first was Archimedes born in Syracuse in 290/280 BC and killed by the Romans in autumn of 212 or spring of 211 BC in the sack of the city. The second one was Ettore Majorana born in Catania, which lies on the east coast of Sicily 53 km to the north of Syracuse.

Ettore Majorana was born on August 5, 1906, of a well-known family of Catania.[1] The uncle of Ettore, Quirino Majorana, was a respected professor of experimental physics at the University of Bologna. Ettore was a fourth child in the family. He had two brothers, Salvatore and Luciano, and two sisters, Rosina and Maria. Ettore had begun to show signs of unusual ability for numerical calculation when he was just four. His favourite game was to multiply in his head in few seconds two three-figure numbers given to him by relatives or their friends. His elementary education Ettore obtained in the school directed by Jesuit fathers.

In 1921 the Ettore family moved to Rome, where in 1923 Ettore became a student of the School of Engineering at the University of Rome. At the end of 1927 Emilio Segrè introduced E. Majorana to the new circle of physicists, which had grown up around Enrico Fermi at the Institute on the via Panisperna. E. Majorana took up physics at the beginning of 1928 after a conversation with E. Fermi.

[1] Detailed account of life and the works of Ettore Majorana see in the biographical sketch by E. Amaldi [3], in the book and paper of E. Recami [1, 4] and in the book of L. Sciascia [5].

His first 6 papers (**M1** — **M6**) all deal with problems of atomic and molecular physics. In the paper **M6** the problem of non-adiabatic spin flip in a beam of polarized atoms was considered. The papers **M7** — **M9** were devoted to the theory of nuclear forces and elementary particles. And the posthumous paper **M10** concerned relations between physics and social sciences. In the papers **M6** — **M9** E. Majorana introduced such important notions as the sphere of Majorana, infinite component Majorana equation, Majorana exchange forces, Majorana representation of Dirac matrices and Majorana neutrino.

E. Majorana was very clever and penetrating man. He much sooner than others in the circle of E. Fermi understood that mysterious beryllium radiation consists of the neutrons, that atomic nuclei are built from protons and neutrons, and that nuclear forces have the exchange character. However, E. Majorana had published neither of these ideas. E. Fermi tried to persuade him to publish at least something, but all his efforts were futile. Nevertheless, E. Fermi succeeded in persuading E. Majorana to go abroad to Leipzig and Copenhagen, where E. Majorana meet W. Heisenberg and N. Bohr. In Leipzig E. Majorana made friends with W. Heisenberg. Before leaving Rome for Leipzig E. Majorana published the paper **M7** on the relativistic theory of particles with arbitrary intrinsic angular momentum. It was namely W. Heisenberg, who persuaded E. Majorana to publish his well-known paper on nuclear theory **M8**.

When E. Majorana returned to Rome in autumn of 1933 he was not in the good health. He rarely visited Institute on the via Panisperna and after some months no longer came at all. The attempts of E. Majorana friends to bring him back to living a normal life were unsuccessful. One of his friends E. Amaldi wrote [3]: “None of us succeeded in finding out whether he was still doing theoretical physics research; I believe he was, but I have no proof”.

At the beginning of 1937 the University of Palermo advertised the competition for three professor chairs in theoretical physics. The first three leading contenders were Gian Carlo Wick, Giulio Racah, and Giovanni Gentile Jr. Suddenly, E. Majorana also decided to take part in the competition. He had not published any scientific paper for some years. To enter the competition E. Majorana sent for publication in the Nuovo Cimento his paper on the symmetrical theory of the electrons and positrons, **M9**. The decision of E. Majorana to enter the competition was a scandal because the winners were predetermined definitely: E. Majorana, G.C. Wick, and G. Racah. The friend of E. Majorana G. Gentile Jr. would not have passed the competition. But G. Gentile Jr. was the son of G. Gentile Sr., the latter was a famous philosopher and fascist politician.[2] The solution of the problem

[2]Giovanni Gentile Sr. was the founder of Domus Galileana in Piza, where unpublished

was found: E. Majorana *in recognition of his special merit* without competition was appointed as ordinary professor of theoretical physics in the Royal University of Naples. On Thursday, January 13, 1938, E. Majorana had given his first lecture.

On March 26, 1938, he left Naples for Palermo, had arrived to Palermo, then, most probably, on March 27 returned to Naples and disappeared forever. The prominent Italian writer Leonardo Sciascia in his book "The disappearance of Majorana" [5] supported the point of view that E. Majorana decided to escape in religious life and entered an Italian monastery. L. Sciascia held a romantic view that E. Majorana disappearance was attributed to unwillingness to collaborate in the releasing of the destructive power of the atom. The famous researcher of E. Majorana life and works, the well-known physicist-theoretician Erasmo Recami published recently a new edition of his book "The Case of Majorana" [1]. In his another work [4] E. Recami outlines human and scientific personality of E. Majorana (on the basis of letters, documents, testimonies, collected by him in about twenty years). However, even Professor E. Recami does not know what happened to E. Majorana, and we have to agree with E. Fermi [3], who observed: "With his intelligence, once as he decided to disappear, Majorana would certainly have succeeded".

3. The Life and Death of Matvei Bronstein

Let us digress on Kharkov physics in the 1930s, which is directly relevant to our subject.

From 1918 to 1934 Kharkov was the capital of the Soviet Ukraine and in 1930 the Ukrainian Institute of Physics and Technology was founded there. It was known under the acronym 'UFTI' (now — NSC "KIPT", National Scientific Centre "Kharkov Institute of Physics and Technology"). One could say that Kharkov in 1930s played nowaday's role of Erice. Paul Ehrenfest, Niels Bohr, Yakov Frenkel, Igor Tamm, Boris Podolsky, Vladimir Fock, Wolfgang Pauli, Yurii Rumer, Paul Dirac, Pascual Jordan, Friedrich Houtermans, Walter Heitler, George Gamov, Dmitrii Ivanenko, Leon Rosenfeld, George Plachek, Rudolf Peierls, Laszlo Tisza, Lev Landau, Victor Weisskopf, [3] and many others lived and worked for some time in the city or took part in conferences on nuclear or theoretical physics that took place in Kharkov. One of the organizers of these conferences was Matvei Bronstein,

manuscripts of E. Majorana are kept. In the April of 1944 anti-fascist communists came across Giovanni Gentile Sr. going back to his home. "Are you Professor Giovanni Gentile?" — "Yes, I am." Giovanni Gentile Sr. was shot immediately... He was guilty of being the president of the Academy of Italy in the Fascist Social Republic established by the Germans.

[3] The names are ordered by the date of birth

a remarkable Leningrad physicist-theoretician, who had wide interests in astrophysics, cosmology, quantum gravitation, nuclear and semiconductor physics.

On October 1932 a group of Kharkov UFTI physicists repeated the Cambridge experiment of J.D. Cockroft and E.T.S. Walton, who in the April of the same year just broke-up the nucleus of ^7Li by bombarding it with the protons.

$$p + {}^7Li \rightarrow {}^4He + {}^4He.$$

The report about this achievement of Kharkov scientists was published in a telegram send to Moscow that looked approximately in the following way:

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                MOSCOW, KREMLIN,
           TO COMRADE JOSEPH STALIN

THE NUCLEUS OF LITHIUM
WAS DISINTEGRATED.
THE WORK IS CONTINUING.
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Working at Kharkov UFTI at the time Lev Landau mocked this message to J. Stalin and narrated that he and his students send to Moskow another one:

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                MOSCOW, KREMLIN,
           TO COMRADE JOSEPH STALIN

THE SINE WAS DIFFERENTIATED,
THE COSINE WAS OBTAINED.
THE WORK IS CONTINUING.
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L. Landau was wrong to consider the disintegration of ^7Li as unimportant experiment. Cambridge and Kharkov physicists for the first time experimentally demonstrated the validity of the famous Einstein formula $E = mc^2$. J.D. Cockroft and E.T.S. Walton received the 1951 Nobel Prize *for their pioneer work on the transmutation of atomic nuclei by artificially accelerated atomic particles.*[4]

And afterwards the time came that is known as the "Year 1937". In 1937 — 1938 Kharkov UFTI was smashed up [6, 7, 8]. Many noted Soviet physicists were arrested and executed. Among them — the head of cryogenic

[4]One should keep in mind that at the time Stalin and his assistants organized in the Ukraine genocidal artificial famine and about seven million people perished. People were dying straight in the streets of big cities. When P. Ehrenfest was invited during his short visit to hold a full-time position, live and work at Kharkov, he pointed at dead bodies lying at the streets and said: "Shall I really come here?" P. Ehrenfest left Kharkov on January 11, 1933. On September 25, 1933 he committed suicide.

laboratory of Kharkov UFTI and prominent scientist Lev Shubnikov, who was shot dead for "economic sabotage in the low temperature physics". Lev Landau, who moved to work at Moscow by then, was arrested and spent a year in prison. Under tortures he confessed to "counter-soviet activity in the theoretical physics" and only by a miracle was saved by Pyotr Kapitsa. Similarly, Matvei Bronstein was arrested and shot for fictitious "terrorist activity".

Matwei Bronstein was born on December 2, 1906 in Vinnitsa, Ukraine.[5] Matvei Bronstein was the second child in the family. He had the elder sister Mikhalina and the younger (on 10 minutes) twin-brother Isidor. The father of Matvej Bronstein was a physician, the family was poor, only the daughter Mikhalina attended the school, the brothers have had no such a possibility, they never were given a formal education. They were self-taught persons and both were highly educated men. Matvei Bronstein was a man-encyclopedia and all that he knew he got to know without any teacher only from numerous books he read. In 1915 the Matvei family moved to Kiev. The Bronstein's family was of Jewish origin but not religious. When brothers were twelve they decided to test whether the God exists. They carried out such a dangerous experiment — they both cried loudly: "The God is a fool!" This terrible crime was not followed any punishment. So it was tested that the God does not exist! In 1923 the brothers became students of a professional electro-technical school but in 1924 they abandoned it because of their family poverty and started working at a factory. In autumn 1925 M. Bronstein begun to visit "Section of Physics" of undergraduates study group at Kiev University. Later on he with sincere gratitude remembered the founder and head of that undergraduates study group Pyotr Tartakovsky. In 1926 M. Bronstein went to Leningrad and became a student of the Leningrad University. Previously to his enrolment the student M. Bronstein had published three scientific papers **B1** — **B3** on quantum mechanics (in 1925), two of them in Zeitschrift für Physik.

In spring of 1927 M. Bronstein made acquaintance with Lev Landau, George Gamov, and Dmitrii Ivanenko and made friends with them. They formed brilliant "quartet" that lived full life. They all loved physics devotedly and this love was mutual [9]. There was a somewhat curious story how M. Bronstein came to know Landau and Gamov [2]. This was because of Landau and Gamov close friend Evgeniya Kanegisser, later the lady Peierls. Lady Peierls told about her first meeting with M. Bronstein:

> There were pools, sparrows twittered, a warm wind blew... I have turned to short young person, wearing big spectacles, and told him: "Again gentle wind made drunk my heart...". Then he had immediately recited

[5] Detailed account on the life and works of Matvei Bronstein see in the book [2]

> the introduction to this poem of Gumilyov.[6] I joyfully screamed, and on the way to the university we were reciting our favourite poetry.

She was greatly impressed and immediately introduced this a great admirer and connoisseur of poetry to Landau and Gamov circle.

From 1930 to his tragical death M. Bronstein was the research assistant of headed by Yakov Frenkel Theoretical Department of the Leningrad Institute of Physics and Technology. In 1930 together with Ya. Frenkel M. Bronstein published a paper on "Quantization of Free Electrons in Magnetic Field" (**B4**). The Bronstein's book "The Structure of Matter" (1935, **B6**) contains the Part "The Relationship Between the Physical Theories and the Problem of Relativistic Quantum Theory", which he regarded as very important to his philosophical view of the natural world. In 1936 there was published the most significant M. Bronstein work "Quantization of Gravitational Waves" (**B7**) and in 1937 — the last scientific work of M. Bronstein "On Possibility of Spontaneous Splitting of the Photons" (**B9**). His scientific activity was impressive: though M. Bronstein's life was very brief, he published 35 scientific papers, 32 popular scientific articles and books, 6 articles in scientific encyclopedias; he translated and edited scientific books, delivered lectures to school pupils and undergraduates, postgraduates and research fellows.

M. Bronstein was the son-in-law of the famous Russian kids writer Kornej Chukovsky and a remarkable writer for children himself. He became a writer with the help of his wife Lidiya Chukovskaya (1907-1996), well-known as an eminent writer and fighter for human rights. M. Bronstein published three beautiful scientific popular books for children: "Solar Matter" (1934), "X-rays" (1936) and "Radiotelegraph Inventors" (1936); all of which were reprinted in 1990 (**B10**). M. Bronstein started to write one more book for children — about Galileo Galilei [10]. But alas! He was forced to leave this World without finishing the book about the great Italian scientist.

M. Bronstein was arrested on August 6, 1937 and was shot on February 18, 1938 when he was only thirty-one. Matvej Bronstein was guilty not only of fictitious "terrorist activity" but mainly of having the same surname as the original surname of Lev Trotskii. M. Bronstein was often been heard saying for fun, "...if Trotskii comes to power, I will call him my nephew..." [11]. But it was the old Russian tradition that a joke as much as one's life was worth.

[6] Nikolay Gumilyov was an eminent Russian poet. In August 1921 he was arrested and shot for fictitious counter-revolutionary activities.

4. The Works of Majorana and Bronstein and Related Topics

4.1. QUATERNION FORM OF MAXWELL EQUATIONS, DIRAC AND WEYL EQUATIONS AND UNPUBLISHED MAJORANA'S WORK ON PHOTONS

Let us consider Maxwell equations describing electromagnetic field in vacuum (with units adopted $c = G = \hbar = 1$).

$$\begin{aligned} \frac{\partial \vec{H}}{\partial t} &= -\mathrm{rot}\vec{E}, \qquad \mathrm{div}\vec{H} = 0, \\ \frac{\partial \vec{E}}{\partial t} &= \quad \mathrm{rot}\vec{H}, \qquad \mathrm{div}\vec{E} = 0, \end{aligned} \tag{1}$$

or equivalently

$$\begin{aligned} \frac{\partial}{\partial t} H_i &= -\varepsilon_{ikl}\frac{\partial}{\partial x_k} E_l, \qquad \frac{\partial}{\partial x_i} H_i = 0, \\ \frac{\partial}{\partial t} E_i &= \quad \varepsilon_{ikl}\frac{\partial}{\partial x_k} H_l, \qquad \frac{\partial}{\partial x_i} E_i = 0, \end{aligned} \tag{2}$$

were ε_{ikl} — Levi-Civita tensor, $\varepsilon_{123} = 1$. After introducing complex quantities

$$\begin{aligned} \vec{\psi}_R &= \vec{E} + i\vec{H}, \\ \vec{\psi}_L &= \vec{E} - i\vec{H}, \end{aligned} \tag{3}$$

we can rewrite Maxwell equations in the form:

$$\begin{aligned} \frac{\partial \vec{\psi}_R}{\partial t} &= \quad \mathrm{rot}\ \vec{\psi}_R, \qquad \mathrm{div}\ \vec{\psi}_R = 0, \\ \frac{\partial \vec{\psi}_L}{\partial t} &= -\mathrm{rot}\ \vec{\psi}_L, \qquad \mathrm{div}\ \vec{\psi}_L = 0, \end{aligned} \tag{4}$$

Let us make use of the Pauli matrices σ_i and their usual multiplication rules

$$\sigma_i \sigma_k = \delta_{ik} I + i\varepsilon_{ikl}\sigma_l. \tag{5}$$

Now we can rewrite the Maxwell equations (4) in the form:

$$\begin{aligned} \left(\frac{\partial}{\partial t} + \vec{\sigma}\frac{\partial}{\partial \vec{x}}\right)\vec{\sigma}\vec{\psi}_R &= 0, \\ \left(\frac{\partial}{\partial t} - \vec{\sigma}\frac{\partial}{\partial \vec{x}}\right)\vec{\sigma}\vec{\psi}_L &= 0. \end{aligned} \tag{6}$$

So, we have obtained the spinor form of the Maxwell equations that is equivalent to the quaternion form, which was originally considered by Maxwell [12].

In 1928 P.A.M. Dirac discovered his famous equation [13]. The first edition of the well-known H. Weyl's book [14] was published in the same year. In his book H. Weyl represented 4-component Dirac equation as a pair of two 2-component equations,

$$
\begin{aligned}
&(\tfrac{\partial}{\partial t} + \vec{\sigma}\tfrac{\partial}{\partial \vec{x}})\psi_R + im\psi_L = 0, \\
&(\tfrac{\partial}{\partial t} - \vec{\sigma}\tfrac{\partial}{\partial \vec{x}})\psi_L + im\psi_R = 0,
\end{aligned}
\tag{7}
$$

where ψ_R and ψ_L are 2-component spinors. In the limit $m \to 0$, the equations (7) transform into the Weyl equations [15] that are very similar to the Maxwell ones (6),

$$
\begin{aligned}
&(\tfrac{\partial}{\partial t} + \vec{\sigma}\tfrac{\partial}{\partial \vec{x}})\psi_R = 0, \\
&(\tfrac{\partial}{\partial t} - \vec{\sigma}\tfrac{\partial}{\partial \vec{x}})\psi_L = 0.
\end{aligned}
\tag{8}
$$

It was E. Majorana who for the first time in his unpublished manuscript [16] made clear the near resemblance of the Maxwell and Weyl equations. The manuscript was written between 1928 and 1932 and is kept in Domus Galileana in Piza. E. Majorana used the transformation law of vectors $\vec{\psi}_R$ and $\vec{\psi}_L$ under a rotation on infinitesimal angle $\vec{\varphi}$,

$$\vec{\psi}'_{R,L} = \vec{\psi}_{R,L} - [\vec{\varphi}, \vec{\psi}_{R,L}]. \tag{9}$$

Omitting labels R and L, we have

$$\psi'_k = \psi_k - \varepsilon_{kil}\varphi_i\psi_l. \tag{10}$$

Let us introduce a notation

$$(S_i)_{kl} = -i\varepsilon_{ikl}. \tag{11}$$

From (10) we obtain

$$\psi'_k = \psi_k + i(S_i)_{kl}\varphi_i\psi_l. \tag{12}$$

Therefore $(S_i)_{kl}$ are spin angular momentum operators, and by using them we can represent the Maxwell equations in the same form as the Weyl equations:

$$
\begin{aligned}
&(\tfrac{\partial}{\partial t} + \vec{S}\tfrac{\partial}{\partial \vec{x}})\psi_R = 0, \qquad && \mathrm{div}\vec{\psi}_R = 0, \\
&(\tfrac{\partial}{\partial t} - \vec{S}\tfrac{\partial}{\partial \vec{x}})\psi_L = 0, \qquad && \mathrm{div}\vec{\psi}_L = 0.
\end{aligned}
\tag{13}
$$

Thus we see that E. Majorana was also the first who understood a quantum mechanical nature of the Maxwell equations. He discovered that Maxwell equations may be represented as a Schrödinger equation with a very simple Hamiltonian. In the general case the wave equations for a massless field of arbitrary spin also may be represented in the Majorana form (of course those equations must be supplemented by some relations of the type div$\vec{\psi} = 0$).

4.2. BRONSTEIN'S WORK ON WEAK GRAVITATIONAL WAVES

Matwei Bronstein's work "The Quantization of Gravitational Waves" (**B7**) was the first deep investigation of quantum gravity. This is the summary of the article written by M. Bronstein

> A consequent quantum theory of weak gravitational field is constructed. The gravitational field in empty space is treated as a quantum-mechanical system, and relativistic-invariant commutation rules are introduced. The gravitational interaction of material bodies is established through the intermediate agency of gravitational quanta. Two physical applications are considered: 1) the loss of energy of material systems by means of the radiation of gravitational waves, and 2) the derivation of Newton's Law of Gravitation.

The Bronstein's derivation of Newton's Law of Gravitation was based on the work of Fock and Podolsky [17], who derived the Coulomb Law from Dirac's theory. M. Bronstein represented metric tensor for weak gravitational field in the form

$$g_{\mu\nu} = \Delta_{\mu\nu} + 2h_{\mu\nu}, \tag{14}$$

where $\Delta_{\mu\nu}$ — metric tensor with nonzero diagonal components (1, -1, -1, -1). We introduced factor 2 to the Bronstein's expression (14) for more similarity with the case of electromagnetic field. M. Bronstein obtained the following commutation relation for the components of $h_{\mu\nu}$:

$$[h_{\mu\nu}(x), h_{\rho\sigma}(x)] = -i(\Delta_{\mu\rho}\Delta_{\nu\sigma} + \Delta_{\mu\sigma}\Delta_{\nu\rho} - \Delta_{\mu\nu}\Delta_{\rho\sigma})D_0(x - x'), \tag{15}$$

where D_0 is the Green function,

$$D_0(x) = \frac{1}{(2\pi)^3} \int e^{ikx} \mathrm{sgn} k_0 \delta(k^2) d^4k. \tag{16}$$

The corresponding commutation relation for electromagnetic 4-potentials are

$$[A_\mu(x), A_\nu(x')] = i\Delta_{\mu\nu} D_0(x - x'). \tag{17}$$

It follows from (15) and (17) that

$$[h_{12}(x), h_{12}(x')] = -iD_0(x - x'), \tag{18}$$

and

$$[A_1(x), A_1(x')] = -iD_0(x - x'). \tag{19}$$

Let us put

$$h_{00} = \Phi, \quad A_0 = \phi,$$

where Φ and ϕ are the gravitational and electric potentials. From (15) and (17) we obtain

$$[h_{00}(x), h_{00}(x')] = [\Phi(x), \Phi(x')] = -iD_0(x - x'), \tag{20}$$

$$[A_0(x), A_0(x')] = [\phi(x), \phi(x')] = iD_0(x - x'). \tag{21}$$

As was shown by M. Bronstein, just this difference of commutators signs leads to the attraction in gravitation and repulsion in electricity.

The Bronstein's paper contains also the quantum-mechanical derivation of the Einstein's formula for the loss of energy by radiating of gravitational waves.

$$I = \frac{1}{80\pi}(\frac{d^3}{dt^3}Q_{ik})^2, \tag{22}$$

where Q is quadruple momentum of the system. M. Bronstein was right when he predicted that for first time the Einstein's formula will be confirmed by an observation of slowing rotation of binary stars [2]. The 1993 Nobel Prize was awarded jointly to: Russell A. Hulse and Joseph H. Taylor Jr. "for the discovery of a new type of pulsar, a discovery that has opened up new possibilities for the study of gravitation". R.A. Hulse and J.H. Teylor discovered binary pulsar PSR 1913 + 16 of very strange behaviour and investigated their characteristics with wonderful accuracy [18]. They determined the mass of visible component of pulsar

$$m_1 = 1,4411 \ m_\odot,$$

and the mass of nonvisible component of pulsar

$$m_2 = 1,3874 \ m_\odot,$$

the pulsar period

$$\tau = 0,059029997929613 \ s,$$

the orbital motion period

$$T = 27906,980895 \ s,$$

the rate of increasing of the pulsar period

$$\Delta\tau = 0,272246 \ s/year$$

and the rate of decreasing the orbital motion period

$$\Delta \ T = 76,0(3) \ \mu s/year.$$

The last value was found to be in good agreement with Einstein theory that gives for this quantity the following value,

$$\Delta\ T\ (Einstein\ theory) = 75,8\ \mu s/year.$$

In his paper M. Bronstein represented the linearized Einstein equations in the following form:

$$\begin{aligned} \frac{1}{c}\frac{\partial}{\partial t}H_{ij} &= -\varepsilon_{ikl}\frac{\partial}{\partial x_k}E_{lj}, \qquad \frac{\partial}{\partial x_i}H_{ij} = 0, \\ \frac{1}{c}\frac{\partial}{\partial t}E_{ij} &= \quad \varepsilon_{ikl}\frac{\partial}{\partial x_k}H_{lj}, \qquad \frac{\partial}{\partial x_i}E_{ij} = 0, \end{aligned} \tag{23}$$

where E_{ij} and H_{ij} are symmetrical and traceless tensors,

$$\begin{aligned} E_{ij} &= R_{4j4i} = \tfrac{1}{4}\varepsilon_{ikl}\varepsilon_{jmn}R_{klmn}, \\ H_{ij} &= \tfrac{i}{2}\varepsilon_{imn}R_{4jmn} \\ &= \tfrac{i}{2}\varepsilon_{imn}R_{mn4j}, \end{aligned} \tag{24}$$

$R_{\mu\nu\rho\sigma}$ is the curvature tensor.

The equations (23) are very similar to the Maxwell ones. We will make sure that they are very similar to the Weyl equations also. Let us introduce the complex fields

$$\begin{aligned} \psi_{Rij} &= E_{ij} + iH_{ij}, \\ \psi_{Lij} &= E_{ij} - iH_{ij}. \end{aligned} \tag{25}$$

Since rotation operators for the fields ψ_{ij} are

$$(S_k)_{ijlm} = -i\varepsilon_{kil}\delta_{jm} - i\varepsilon_{kjm}\delta_{il}, \tag{26}$$

we can rewrite the equations (23) in the form

$$\begin{aligned} (2\tfrac{\partial}{\partial t} + \vec{S}\tfrac{\partial}{\partial \vec{x}})\psi_R &= 0, \qquad \partial_i\psi_{Rij} = 0, \\ (2\tfrac{\partial}{\partial t} - \vec{S}\tfrac{\partial}{\partial \vec{x}})\psi_L &= 0, \qquad \partial_i\psi_{Lij} = 0. \end{aligned} \tag{27}$$

There is direct evidence of near resemblance of Bronstein (23), Maxwell (13) and Weyl (8) equations.

A very simple demonstration of impossibility to explain the cosmological red shift by splitting of photons was given by M. Bronstein in his last paper **B7**. It was shown by him that the splitting of photons leads to the dependence of the red shift on the wavelength, which is in contradiction with the experiment.

4.3. EXTENDED LITTLE LORENTZ GROUP AND THE WAVE EQUATIONS OF MASSLESS FIELDS

The great similarity of Bronstein, Maxwell and Weyl equations is not accidental because the arbitrary spin wave equations for the massless fields may be written in the following form [19, 20]:

$$(iS_{\mu\nu} + S\delta_{\mu\nu})\frac{\partial}{\partial x_\nu}\Psi(x) = 0, \tag{28}$$

where $S_{\mu\nu}$ are infinitesimal operators of the (s_1, s_2) — representation of the proper Lorentz group, $S = s_1 + s_2$. The well-known Weinberg theorem [21] states that the helicity of the massless field $\lambda = s_1 - s_2$.

The wave equations (28) follow from the existence of the *Extended Little Lorentz Group* [20, 22], i.e. *Little Lorentz Group* [23] (Lorentz transformations leaving invariant the 4-momentum p_μ) *enlarged by dilatations*, and a very simple assumption that *the state of an elementary particle with arbitrary spin may be completely described by its 4-momentum* [19].

For particular values of (s_1, s_2) one has the following correspondence:

(1,0) and (0,1)	J.C. Maxwell, 1864 (photons) [24],
(1/2,0) and (0,1/2)	H. Weyl, 1929 (neutrinos) [15],
(2,0) and (0,2)	M.P. Bronstein, 1936 (gravitons) [**B7**],
(1/2,1/2)	V.I. Ogievetsky, I.V. Polubarinov, 1966 (notophs) [25],
(-1/4,-1/4)	P.A.M. Dirac, 1971 [26],
(-1/4,-3/4) and (-3/4,-1/4)	L.P. Staunton, 1974 [27].

It is necessary to note that infinite component (-1/4,-1/4), (-1/4,-3/4) and (-3/4,-1/4) representations of the proper Lorentz group were for the first time introduced in the theory of relativistic wave equations by E. Majorana in his paper **M7** in 1932.

4.4. MAJORANA-PENROSE REPRESENTATION OF SPIN-TENSORS

In 1960 Roger Penrose [28] discovered the following remarkable property of symmetrical spin-tensor $\Psi_{\alpha_1\alpha_2\ldots\alpha_n}$, $\alpha_i = 1, 2$,

$$\Psi_{\alpha_1\alpha_2\ldots\alpha_n} = A\psi^1_{(\alpha_1}\psi^2_{\alpha_2}\ldots\psi^n_{\alpha_n)}, \tag{29}$$

where A is some constant, ψ^1, ψ^2, ... ψ^n are 2-component spinors, $_{(\ldots)}$ denotes symmetrization. The existence and uniqueness of (29) follows from the fundamental theorem of algebra. To get the representation (29), let us multiply the spin-tensor $\Psi_{\alpha_1\alpha_2\ldots\alpha_n}$ on spin-tensor $\chi^{\alpha_1}\chi^{\alpha_2}\ldots\chi^{\alpha_n}$, where χ^α is the spinor with the components $\chi^1 = z$, $\chi^2 = 1$,

$$\Psi_{\alpha_1\alpha_2\ldots\alpha_n}\chi^{\alpha_1}\chi^{\alpha_2}\ldots\chi^{\alpha_n} = P_n(z), \tag{30}$$

where

$$P_n(z) = \Psi_{111...1} z^n + n\Psi_{211...1} + \frac{n(n-1)}{2}\Psi_{221...1} + ... + \Psi_{222...2} \qquad (31)$$

is some polynomial of complex variable z. The factorization of polynomial $P_n(z)$ gives

$$P_n(z) = \Psi_{111...1}(z-a_1)(z-a_2)...(z-a_n). \qquad (32)$$

Let us inroduce the spinors ψ^i with components $\psi^i_1 = 1, \quad \psi^i_2 = -a_i$. Then we have

$$\Psi_{\alpha_1\alpha_2...\alpha_n}\chi^{\alpha_1}\chi^{\alpha_2}...\chi^{\alpha_n} = A(\psi^1_{\alpha_1}\chi^{\alpha^1})(\psi^2_{\alpha_2}\chi^{\alpha^2})...(\psi^n_{\alpha_n}\chi^{\alpha^n}), \qquad (33)$$

where $A = \Psi_{111...1}$. The relation (33) expresses the general binary multilinear form as a product of linear factors. These factors are essentially unique. Omitting the spinors χ's in (33) we obtain finally (29). Thus the validity of Penrose representation (29) is proved.

The spin-tensor of rank $2S$ describes the paricle with spin S. So, according to Penrose, a state of a particle with arbitrary spin S can be represented as a set of $2S$ spin 1/2 particles states:

$$\Psi_S = A\Big\{\psi^{(1)} \times \psi^{(2)} \times \psi^{(3)} \times \ldots \times \psi^{(2S)}\Big\}, \qquad (34)$$

For the above mentioned, we can attach to each normalized spinor some unit vector $\vec{n} = (\psi^\star\vec{\sigma}\psi)/(\psi^\star\psi)$ and the following visual picture arises for description of an arbitrary state of spin S particle (see Figure 1).

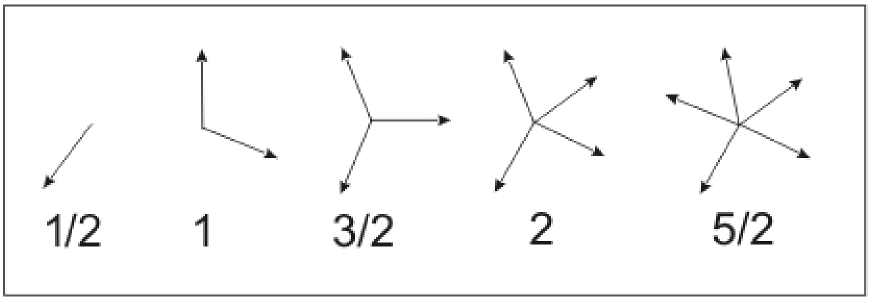

Figure 1. Majorana-Penrose visual description of spin S particle

Thus the state of spin S particle can be represented as a set of $2S$ unit vectors. R. Penrose did not know that he rediscovered the visual description of spin S particle that E. Majorana found in 1932 (**M6**) and that known as sphere of Majorana. Later on R. Penrose payed tribute to work of E. Majorana [29]. Majorana sphere is the unit sphere on which the ends of unit vectors $\vec{n}_i$ are lying. E. Majorana obtained the formulae (31) and (32), so his approach to the problem was identical to Penrose.

R. Penrose used the Majorana representation for elementary description of the gravity field classification by Petrov [28]. Thus, according to Penrose, the following pictures correspond to Petrov's types I, II, III, D, and N (Figure 2)

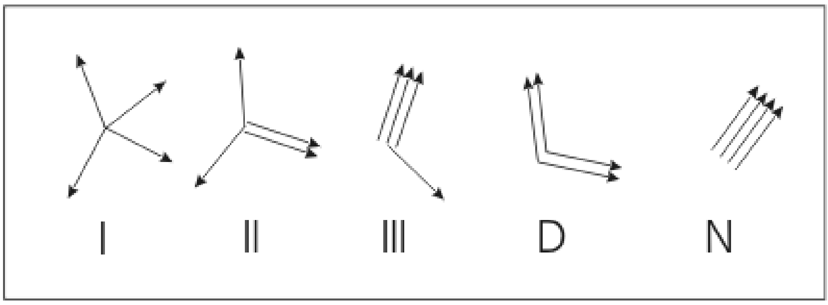

Figure 2. Gravity fields classification by Petrov according R. Penrose

Majorana-Penrose representation may be used also for description and calculating of the Maxwell axes in electrodynamics [30] and in the earth magnetism theory [31] and for simple treatment of conical refraction in crystal optics [32].

4.5. MAJORANA AND DIRAC INFINITE COMPONENT RELATIVISTIC WAVE EQUATIONS WITHOUT NEGATIVE ENERGY SOLUTIONS

Let us consider a bispinor A_α

$$A_\alpha == \begin{pmatrix} a_1 \\ a_2 \\ -a_2^+ \\ a_1^+ \end{pmatrix}, \tag{35}$$

where a_1^+, a_2^+ and a_1, a_2 are the usual creation and annihilation operators. The commutator of A_α's is

$$A_\alpha A_\beta - A_\beta A_\alpha = C_{\alpha\beta}, \tag{36}$$

where C is the charge conjugation matrix. If we define

$$S_{\mu\nu} = \frac{1}{8i}\left(\overline{A}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)A\right) \tag{37}$$

as the infinitesimal operators of the corresponding representation of the proper Lorentz group, then the quantity

$$\Phi_\alpha = \langle\psi|A_\alpha|\psi\rangle \tag{38}$$

is the bispinor and $|\psi\rangle$ is in a particular sense the square root of bispinor Φ_α.

Now we can write the following equations:

$$(\gamma_\mu \frac{\partial}{\partial x_\mu} - m)_\alpha^\beta A_\beta |\psi(x)\rangle = 0. \tag{39}$$

These equations, discovered by Dirac in 1971 [26], have only the positive energy plane wave solutions

$$|\psi(p,x)\rangle = exp(ipx)|u(p)\rangle. \tag{40}$$

Dirac didn't know, that the same representation of the Lorentz group was successfully used by E. Majorana in 1932 (**M7**) for the same purpose, namely, to obtain the equations with no negative energy solutions.

Unfortunately, no one can introduce interaction in the positive energy Dirac equations. We have four conditions on one function

$$K_\alpha |\psi(x)\rangle = 0 \tag{41}$$

and ten conditions of compatibility

$$(K_\alpha K_\beta - K_\beta K_\alpha)|\psi(x)\rangle = 0. \tag{42}$$

For free fields these ten conditions give

$$(\Box - m^2) C_{\alpha\beta} |\psi(x)\rangle = 0. \tag{43}$$

Unluckily, if we introduced an interaction those conditions would contradict one another.

The Majorana equation discovered in 1932 can be obtained from the positive energy Dirac equation (39) by multiplication on $A^\alpha = C^{\alpha\beta} A_\beta$,

$$A^\alpha (\gamma_\mu \frac{\partial}{\partial x_\mu} - m)_\alpha^\beta A_\beta |\psi(x)\rangle = 0. \tag{44}$$

Whereas the Dirac-1971 equation describes the particle with zero spin, the Majorana-1932 equation describes the particle with arbitrary spin S.

Positive energy Majorana-Dirac equation may be used for the treatment of squeezed states in quantum optics. Let Q_α be a spinor of the $SO(2,1)$ group,

$$Q_\alpha = \begin{pmatrix} a \\ a^+ \end{pmatrix}. \tag{45}$$

It is evident that

$$Q_\alpha Q_\beta - Q_\beta Q_\alpha = \varepsilon_{\alpha\beta}. \tag{46}$$

Let us consider the plane wave solution of the Majorana-Dirac equation

$$(\sigma_i \frac{\partial}{\partial x_i} - m)_\alpha^\beta Q_\beta |\psi\rangle = 0, \tag{47}$$

$$|\psi\rangle = e^{ipx}|u\rangle. \tag{48}$$

From (47) and (48) we have

$$(ip_i\sigma_i - m)_\alpha^\beta Q_\beta |u\rangle = 0. \tag{49}$$

For the $p_i = (0, p, i\varepsilon)$ we obtain

$$[(\varepsilon + m)a - pa^+]|u\rangle = 0, \tag{50}$$

i.e. the equation determining the squeezed vacuum state. If $p = 0$, we derive

$$a|u_0\rangle = 0, \tag{51}$$

so

$$|u_0\rangle = |0\rangle. \tag{52}$$

The plain wave solution for $p_i = (0, p, i\varepsilon)$ may be obtained by the Lorentz transformation

$$|u_p\rangle = \exp J_1 \mathrm{Arth}\frac{p}{\varepsilon}|0\rangle. \tag{53}$$

Let us introduce the Lorentz group generators [33]

$$J_i = \frac{1}{4}(\sigma_i)^{\alpha\beta} Q_\alpha Q_\beta, \qquad J_\pm = J_1 \pm iJ_2. \tag{54}$$

On using the relation

$$\begin{aligned} &\exp(J_1 \mathrm{Arth}\tfrac{p}{\varepsilon}) = \exp\tfrac{\varepsilon - m}{p} J_+ \times \\ &\exp(-\ln\tfrac{\varepsilon + m}{2m}) J_3 \times \exp\tfrac{\varepsilon - m}{p} J_-, \end{aligned} \tag{55}$$

we get

$$|u_p\rangle = \sqrt{\frac{2m}{\varepsilon + m}} \exp(-\frac{p}{\varepsilon + m})(a^+)^2|0\rangle. \tag{56}$$

As shown by S. Hawking in his classical paper, the black hole radiation is characterized by the Plank's spectrum with some temperature, and the state of radiation is a squeezed vacuum state. However, we can see from (56) that allowed photon numbers n for the state of squeezed vacuum are 0, 2, 4, 6, ..., which is in a sharp contradiction with photon numbers for the thermal radiation state 0, 1, 2, 3, ... So Hawking radiation is not thermal radiation [34].

4.6. ON MAJORANA REPRESENTATION OF DIRAC MATRICES AND $SO(3,3) \approx SL(4R)/Z_2$ ISOMORPHISM

May be, the last work of E. Majorana "Symmetric Theory of Electrons and Positrons", published in 1937, is the most popular one. In this work were introduced the new conceptions now well known as Majorana representation of Dirac matrices, Majorana neutrino, Majorana particles. E. Majorana discovered that irreducible two-valued representation of proper Lorentz Group may be realized by real 4×4-matrices. Then the wave function of antiparticle $\psi(antiparticle)$ is simply complex conjugate of the negative energy solution ψ_- of Dirac equation,

$$\psi(antiparticle) = \psi_-^\star, \tag{57}$$

and operator $\gamma_\mu \frac{\partial}{\partial x_\mu} + m$ that is a part of Dirac equation

$$(\gamma_\mu \frac{\partial}{\partial x_\mu} + m)\psi = 0 \tag{58}$$

is real operator. We will construct some simple generalization of Majorana representation. Let us consider arbitrary real antisymmetric matrix with components

$$R = \begin{pmatrix} 0 & x_3 + t_3 & -x_2 - t_2 & x_1 - t_1 \\ -x_3 - t_3 & 0 & x_1 + t_1 & x_2 - t_2 \\ x_2 + t_2 & -x_1 - t_1 & 0 & x_3 - t_3 \\ -x_1 + t_1 & -x_2 - t_2 & -x_3 + t_3 & 0 \end{pmatrix}. \tag{59}$$

The linear transformation

$$R_{\alpha\beta} \to R'_{\alpha'\beta'} = \Lambda^{\alpha}_{\alpha'} \Lambda^{\beta}_{\beta'} R_{\alpha\beta} \tag{60}$$

gives new matrix R' with components

$$R' = \begin{pmatrix} 0 & x'_3 + t'_3 & -x'_2 - t'_2 & x'_1 - t'_1 \\ -x'_3 - t'_3 & 0 & x'_1 + t'_1 & x'_2 - t'_2 \\ x'_2 + t'_2 & -x'_1 - t'_1 & 0 & x'_3 - t'_3 \\ -x'_1 + t'_1 & -x'_2 - t'_2 & -x'_3 + t'_3 & 0 \end{pmatrix}. \tag{61}$$

If the matrices Λ are real unimodular matrices, $\Lambda \in SL(4R)$, then the quantity

$$\varepsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta} R_{\gamma\delta} = x_1^2 + x_2^2 + x_3^2 - t_1^2 - t_2^2 - t_3^2, \tag{62}$$

where $\varepsilon^{\alpha\beta\gamma\delta}$ is Levi-Civita symbol, $\varepsilon^{1234} = 1$, is invariant. Thus we see that real unimodular 4×4-matrices correspond to Lorentz transformations $SO(3,3)$ of six-dimensional space that has three space and three time coordinates. The six-dimensional structure of 4×4 matrices was in details considered in the papers [35, 36, 37]

4.7. $CG\hbar$-CLASSIFICATION OF PHYSICAL THEORIES BY M. BRONSTEIN, QUANTUM HALL EFFECT AND SOME PROPERTIES OF THE NUMBER 137

M. Bronstein had great interest to fundamental constants of physics. In his popular scientific books he discussed the following $cG\hbar$-scheme (see Figure 3). For example this scheme may be found in the book **B6** in the Part "The Relationship Between the Physical Theories and the Problem of Relativistic Quantum Theory". The scheme on Figure 3 is given in some modification [2].

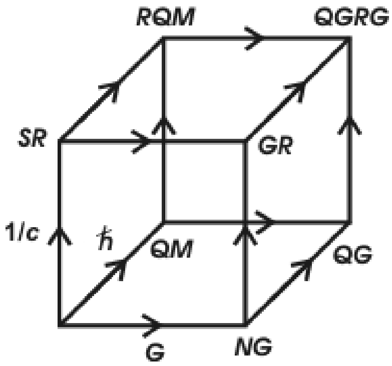

Figure 3. cGh-classification of physical theories by M. Bronstein

There are following abbreviations on the Figure 3:

NG	**Newtonian Gravity**
QM	**Quantum Mechanics**
SR	**Special Relativity**
GR	**General Relativity**
QG	**Quantum Gravity**
RQM	**Relativistic Quantum Mechanics**
QGRG	**Quantum General Relativistic Gravity**

From 35 articles of M. Bronstein only two were written with coauthors. The first coauthor was Ya. Frenkel (**B4**) and the second one was L. Landau (**B5**). In their paper Ya. Frenkel and M. Bronstein considered the quantum mechanical motion of an electron in magnetic field. It was the actual problem at the time. In the same 1930 L. Landau published his famous paper on this subject [38]. However Landau, Frenkel, and Bronstein were not the firsts who solved the problem. The firsts who solved in 1928 the problem on quantum mechanical motion of electron in magnetic field were V.Fock [39] and I.Rabi [40], one of the founders of Ettore Majorana Centre for Scientific Culture. V. Fock at the first time considered in his paper the object that now is well known as the “Quantum Dot”. I. Rabi solved the problem on the quantum motion of a relativistic electron in magnetic field using recently discovered Dirac equation. With connection with all these works let us briefly consider the remarkable phenomenon Quantum Hall Effect.

In 1980 Klaus von Klitzing [41] (the 1985 Nobel Prize) discovered experimentally that the Hall resistance of a thin semi-conductor film is quantized with great accuracy according to the formula

$$R_H = \frac{h}{e^2}\frac{1}{N}, \qquad N = 1, 2, 3, ..., \tag{63}$$

In the paper [41] was proposed new method of very precise establishment of the value of the fine structure constant. To make clear the connection of the Hall resistance and of the fine structure constant

$$\frac{e^2}{\hbar c} = \frac{1}{137,0359895(61)}$$

we have to make more precise the well-known connection of light velocity and resistance unit, $1/c = 30\ ohm$. Indeed,

$$ohm = \frac{volt}{ampere} = \frac{1}{9 \cdot 10^9} \cdot \frac{\frac{coulomb}{metr}}{\frac{coulomb}{second}} = \frac{1}{30 \cdot 3 \cdot 10^8 \frac{metr}{second}}.$$

To obtain more precise connection we have to make the substitution $9 \to (2,99792458)^2$ that finally gives

$$\frac{1}{c} = 29,9792458\ ohm.$$

Now we can express h/e^2 through ohm:

$$\frac{h}{e^2} = 2\pi \cdot \frac{\hbar c}{e^2} \cdot \frac{1}{c} = 2\pi \cdot 137,0359895(61) \cdot 29,9792458\ ohm = 25812,8056(11)\ ohm.$$

The resistance 25812,806 ohm is measured with grate accuracy. Thus we see that measurement of “resistance quantum” $\frac{h}{e^2}$ leads to precise establishment of the fine structure constant value.

Let us talk over now two curious properties of fine structure constant or more precisely of the number 137, which is approximately equal to the inverse value of this remarkable constant. The first property was announced by V. Weisskopf [42]. Once V. Weisskopf has talk about CABALAH[7] with one of great connoisseur in this matter Gershom Shalom. Gershom Shalom was explaining to V. Weisskopf how to put in correspondence the numbers to the words in Hebrew and suddenly asked: “Say me, please, have you some numbers of interest in physics?” V. Weisskopf answered: “Of course, we have, the number 137, for example!” Gershom Shalom was shocked. “O, — he exclaimed — but CABALAH is just 137!”

קבלה.

Figure 4. The number that corresponds to the world CABALAH is 137

Indeed, (see Figure 4) there are four letters that make up the world CABALAH: ‘kof’=100, ‘bet’=2, ‘lamed’=30, ‘hay’=5, 100+2+30+5=137.

And another one fanny property of the number 137 (it is a modest observation of the author of these lines): if everyone will write twice the number of the year he (or she) was born, then obtained eight-figure number will be divided on 137 without remainder. For example, Ettore Majorana and Matwei Bronstein were born in 1906. It is easily to verify that 19061906:137=139138.

5. Acknowledgements

Ettore Majorana and Matvei Bronstein are heros of mine for over than 40 years since I have read about E. Majorana in Laura Fermi’s book “Atoms in the Family” and for the first time have gotten to know about the remarkable works and tragic lot of M. Bronstein. I am very grateful to Professor Venzo De Sabbata for the opportunity to visit the Ettore Majorana Centre for Scientific Culture and for giving me the possibility to deliver the lectures about E. Majorana and M. Bronstein. I appreciate deeply the people at the Ettore Majorana Centre for their great hospitality. I am very thankful to Alexander Zheltukhin, who introduced me to Professor De Sabbata; to Dmitrii Sorokin for communicating me two inaccessible in Ukraine Majorana’s manuscripts; and to Kostyantin Ilyenko for essential help in writing this text.

[7] CABALAH (Hebrew: ”Tradition”) is the secret mystical knowledge (divine revelation) that was communicated by God to Moses and Adam and that provides a means of approaching God directly.

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