AIP | Low Temperature Physics

Determination of coordinate dependence of a pinning potential from a microwave experiment with vortices

V. A. Shklovskij and O. V. Dobrovolskiy

Citation: Low Temp. Phys. **39**, 120 (2013); doi: 10.1063/1.4791773
View online: http://dx.doi.org/10.1063/1.4791773
View Table of Contents: http://ltp.aip.org/resource/1/LTPHEG/v39/i2
Published by the American Institute of Physics.

Related Articles

Stripe order related in-plane fourfold symmetric superconductivity in La1.45Nd0.4Sr0.15CuO4 single crystal
J. Appl. Phys. 113, 053912 (2013)

Oscillating modes of a massive single vortex line in an anisotropic superconductor: The role of temperature
Low Temp. Phys. 39, 102 (2013)

Pinning features of the magnetic flux trapped by YBCO single crystals in weak constant magnetic fields
Low Temp. Phys. 39, 107 (2013)

Temperature- and field-dependent critical currents in [(Bi,Pb)2Sr2Ca2Cu3Ox]0.07(La0.7Sr0.3MnO3)0.03 thick films grown on LaAlO3 substrates
J. Appl. Phys. 113, 043916 (2013)

Vortex core size in unconventional superconductors
J. Appl. Phys. 113, 013906 (2013)

Additional information on Low Temp. Phys.

Journal Homepage: http://ltp.aip.org/

Journal Information: http://ltp.aip.org/about/about_the_journal

Top downloads: http://ltp.aip.org/features/most_downloaded

Information for Authors: http://ltp.aip.org/authors

ADVERTISEMENT

Downloaded 27 Feb 2013 to 141.2.253.163. Redistribution subject to AIP license or copyright; see http://ltp.aip.org/about/rights_and_permissions

Determination of coordinate dependence of a pinning potential from a microwave experiment with vortices

V. A. Shklovskij

Institute of Theoretical Physics, NSC-KIPT, Kharkiv 61108, Ukraine and Physical Department, Kharkiv National University, Kharkiv 61077, Ukraine

O. V. Dobrovolskiy[a)]

Physikalisches Institut, Goethe-University, Frankfurt am Main 60438, Germany

(Submitted July 22, 2012; revised August 30, 2012)
Fiz. Nizk. Temp. **39**, 162–167 (February 2013)

The measurement of the complex impedance response accompanied by power absorption $\mathcal{P}(\omega)$ in the radiofrequency and microwave ranges represents a most popular experimental method for the investigation of pinning mechanisms and vortex dynamics in type-II superconductors. In the theory, the pinning potential (PP) well for a vortex must be *a priori* specified in order to subsequently analyze the measured data. We have theoretically solved the inverse problem at $T = 0$ K and exemplify how the coordinate dependence of a PP can be determined from a set of experimental curves $\mathcal{P}(\omega|j_0)$ measured at subcritical dc currents $0 < j_0 < j_c$ under a small microwave excitation $j_1 \ll j_c$ with frequency ω. We furthermore elucidate how and why the depinning frequency ω_p, which separates the non-dissipative (quasi-adiabatic) and the dissipative (high-frequency) regimes of small vortex oscillations in the PP, is reduced with increasing j_0. The results can be directly applied to a wide range of conventional superconductors with a PP subjected to superimposed dc and small microwave ac currents at $T \ll T_c$. *© 2013 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4791773]

1. Introduction

One of the most popular experimental methods for the investigation of vortex dynamics in type-II superconductors is the measurement of the complex ac response in the radiofrequency and microwave ranges.[1] The reason for this is that at frequencies substantially smaller than those invoking the breakdown of the energy gap, the high-frequency and microwave impedance measurements of a mixed state contain information about flux pinning mechanisms and vortex dynamics accompanied by dissipative processes in a superconductor. It should be noted that this information cannot be extracted from the dc resistivity data obtained in the steady state regime when pinning in the sample is strong. In fact, in the latter, when the critical current densities j_c are rather large, the realization of the dissipative mode, in which the flux-flow resistivity ρ_f can be measured, requires that $j_0 \gtrsim j_c$. This is commonly accompanied by non-negligible electron overheating in the sample,[2,3] which changes the desired value of ρ_f. At the same time, measurements of power absorbed by the vortices from an ac current with the amplitude $j_1 \ll j_c$ allow one to determine ρ_f at a dissipative power $\mathcal{P}_1 \sim \rho_f j_1^2$, which can be many orders of magnitude less than $\mathcal{P}_0 \sim \rho_f j_0^2$. Consequently, measurements of the complex ac response versus frequency ω practically probe the pinning forces in the absence of overheating effects, otherwise unavoidable at overcritical steady-state dc current densities.

The appearance of experimental works utilizing the usual four-point scheme,[4] strip-line coplanar waveguides (CPWs),[5] Corbino geometry,[6,7] or the cavity method[8] to investigate the microwave vortex response in as-grown thin-film superconductors (or in those containing some nano-tailored pinning potential (PP) landscape) in the recent years reflects the explosively growing interest in the subject. In fact, such artificially fabricated pinning nanostructures provide a PP of unknown shape that requires certain assumptions concerning its coordinate dependence in order to fit the measured data. At the same time, in a real sample a certain amount of disorder is always present, acting as pinning sites for vortices as well. Therefore, an approach to experimentally reconstructing the form of the PP ensued in the sample is of great demand for both application-related and fundamental reasons. An early scheme to reconstructing the coordinate dependence of the pinning force from measurements implying a small ripple magnetic field superposed on a larger dc magnetic field was reported in Ref. 9. Similar problems in the reconstruction of a specific form of the potential subjected to a superimposed constant and small alternating signals arise not only in vortex physics, but also in a number of other fields. Mainly as the closest mathematical analogy, we would like to mention the Josephson junction problem, wherein plenty of non-sine forms of the current-phase relation are known to occur,[11] and which could in turn benefit from the results reported here.

Turning back to the development of the theory of our problem, the very early model describing the power absorbed by vortices refers to the work of Gittleman and Rosenblum (GR),[12] where a small ac excitation of vortices in the absence of dc current is considered. The GR results were obtained at $T = 0$ K in the linear approximation for the pinning force. We will briefly present their results in the present work since subsequent description of our new results requires these as essential background. The theory, accounting also for the vortex creep at non-zero temperature in a

 © 2013 American Institute of Physics

Downloaded 27 Feb 2013 to 141.2.253.163. Redistribution subject to AIP license or copyright; see http://ltp.aip.org/about/rights_and_permissions

one-dimensional cosine PP, was extended by Coffey and Clem (CC) later.[13] However, this theory was developed for a small microwave current in the absence of a dc current. We have recently substantially generalized the CC results[14,15] for a two-dimensional cosine washboard pinning potential (WPP). The washboard form of the PP enables exact theoretical description of two-dimensional anisotropic nonlinear vortex dynamics for any arbitrary values of ac and dc amplitudes, temperature, the Hall constant, and the angle between the direction of the transport current and the guiding direction of the WPP. Among other nontrivial results obtained, an enhancement[14] and a sign change[15] in the power absorption for $j_0 \gtrsim j_c$ have been predicted. Whereas the general solution of the problem in Refs. 14 and 15 has been obtained in terms of a matrix continued fraction and is suitable for the analysis mainly in the form of a data figure due to a large number of variable parameters, an analytical implementation of the solution at $T=0\,\mathrm{K}$, $j_0 < j_c$, and $j_1 \to 0$ has been performed in Refs. 16 and 17, also taking into account the anisotropy of the vortex viscosity and an arbitrary Hall constant.

In the present work, we report on the possibility of reconstructing the coordinate dependence of a PP if a set of $\mathcal{P}_0(\omega)$ curves is measured at different dc current amplitudes in the entire range $0 \le j_0 \lesssim j_c$ at a small microwave amplitude $j_1 \to 0$. Whereas a preliminary communication on this matter can be found in Ref. 18, here we provide a detailed description of the PP reconstruction procedure. Geometry of the problem implies a standard four-point thin-film superconductor microstrip bridge placed in a small perpendicular magnetic field with a magnitude $B \ll B_{c2}$ at $T \ll T_c$. The sample is assumed to have at least one pinning site, and dc and ac currents are directed collinearly. Theoretical treatment of the problem is described in detail below.

2. Dynamics of pinned vortices in a small microwave current

The GR model[12] considers oscillations of damped vortices in a parabolic PP. Absorption of power by vortices in PbIn and NbTa films was measured over a wide range of frequencies ω, and the data was successfully analyzed on the basis of a simple equation for a vortex moving with velocity $\upsilon(t)$ along the x-axis

$$\eta\dot{x} + k_p x = f_L, \tag{1}$$

where x is vortex displacement, η is vortex viscosity, and k_p is the constant that characterizes the restoring force f_p in the PP well $U_p(x)=(1/2)k_p x^2$ and $f_p=-dU_p/dx=-k_p x$. In Eq. (1) $f_L=(\Phi_0/c)\,j_1(t)$ is the Lorentz force acting on the vortex, Φ_0 is the magnetic flux quantum, c is the speed of light, and $j_1(t)=j_1\exp(i\omega t)$ is the density of a small microwave current with the amplitude j_1. Looking for a solution of Eq. (1) in the form $x(t)=x\exp(i\omega t)$, where x is the complex amplitude of vortex displacement, one immediately gets $\dot{x}(t)=i\omega x(t)$ and

$$x = \frac{(\Phi_0/\eta c) j_1}{i\omega + \omega_p}, \tag{2}$$

where $\omega_p \equiv k_p/\eta$ is the depinning frequency. To calculate the magnitude of the complex electric field arising due to the vortex on the move, one takes $E=B\dot{x}/c$. Then

$$E(\omega) = \frac{\rho_f j_1}{1 - i\omega_p/\omega} \equiv Z(\omega) j_1. \tag{3}$$

Here, $\rho_f = B\Phi_0/\eta c^2$ is the flux-flow resistivity, and $Z(\omega) \equiv \rho_f/(1 - i\omega_p/\omega)$ is the microwave impedance of the sample.

In order to calculate the power $\mathcal{P}$ absorbed per unit volume and averaged over a period of an ac cycle, the standard relation $\mathcal{P}=(1/2)\mathrm{Re}(EJ^*)$ is used, where E and J are the complex amplitudes of the ac electric field and the current density, respectively. The asterisk denotes the complex conjugate. Then from Eq. (3) it follows that

$$\mathcal{P}(\omega) = \frac{1}{2}\mathrm{Re}\,Z(\omega) j_1^2 = \frac{1}{2}\frac{\rho_f j_1^2}{1+(\omega_p/\omega)^2}. \tag{4}$$

For subsequent analysis, it is convenient to write out the real and imaginary parts of the impedance $Z=\mathrm{Re}Z + i\mathrm{Im}Z$, namely

$$\mathrm{Re}Z(\omega) = \frac{\rho_f}{1+(\omega_p/\omega)^2}, \quad \mathrm{Im}Z(\omega) = \frac{\rho_f(\omega/\omega_p)}{1+(\omega/\omega_p)^2}. \tag{5}$$

The frequency dependences (5) are plotted in dimensionless units Z/ρ_f and ω/ω_p in Fig. 1 (see the curve for $j_0=0$). From Eqs. (1), (2), and (4) it follows that pinning forces dominate at low frequencies ($\omega \ll \omega_p$), where $Z(\omega)$ is non-dissipative with $\mathrm{Re}Z(\omega) \approx (\omega/\omega_p)^2$, whereas at higher frequencies ($\omega \gg \omega_p$) frictional forces dominate, and $Z(\omega)$ is dissipative with $\mathrm{Re}Z(\omega) \approx \rho_f\,[1-(\omega_p/\omega)^2]$. In other words, due to the reduction of the amplitude of vortex displacement with the increase in ac frequency, the pinning force does not influence the vortex. This can be seen from Eq. (2), where $x \sim 1/\omega$ for $\omega \gg \omega_p$; however, this is accompanied by the independence of vortex velocity of ω in this regime, in accordance with Eq. (3).

3. Influence of dc current on depinning frequency

When an arbitrary dc current is superimposed on a small microwave signal, the GR model can be generalized for an arbitrary PP. For definiteness we consider a subcritical dc current with the density $j_0 < j_c$, where j_c is the critical current

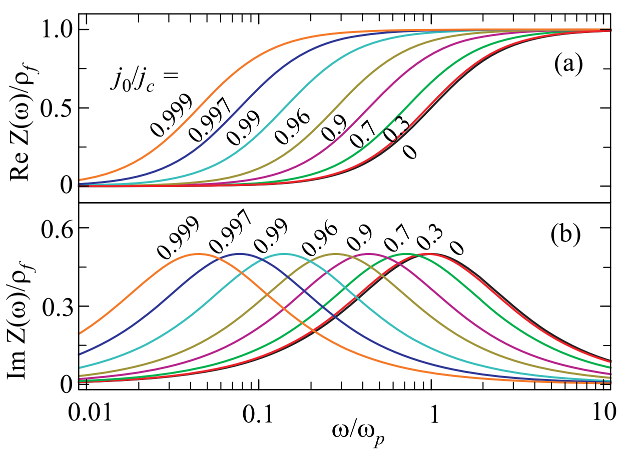

FIG. 1. Frequency dependences of real (a) and imaginary (b) parts of the ac impedance calculated for a cosine pinning potential $U_p(x)=(U_p/2)(1-\cos kx)$ at a series of dc current densities, as indicated. In the absence of a dc current, the GR results are revealed in accordance with Eqs. (5).

Downloaded 27 Feb 2013 to 141.2.253.163. Redistribution subject to AIP license or copyright; see http://ltp.aip.org/about/rights_and_permissions

density in the absence of a microwave current. Our goal now is to determine the changes in PP parameters of the superimposition of dc current leads. In the presence of $j_0 \neq 0$, the effective PP becomes $\tilde{U}(x) \equiv U_p(x) - xf_0$, where $U_p(x)$ is the x-coordinate dependence of the PP when $j_0 = 0$. Note also that $f_0 < f_c$, where f_0 and f_c are the Lorentz forces corresponding to the current densities j_0 and j_c, respectively.

In the presence of a dc current, the equation of motion for a vortex has the form

$$\eta \upsilon(t) = f(t) + f_p, \tag{6}$$

where $f(t) = (\Phi_0/c)\ j(t)$ is the Lorentz force with $j(t) = j_0 + j_1(t)$, where $j_1(t) = j_1 \exp(i\omega t)$, and j_1 is the amplitude of a small microwave current. Due to the fact that $f(t) = f_0 + f_1(t)$, where $f_0 = (\Phi_0/c)\ j_0$ and $f_1(t) = (\Phi_0/c) j_1(t)$ are the Lorentz forces for the subcritical dc and microwave currents, respectively, one can naturally assume that $\upsilon(t) = \upsilon_0 + \upsilon_1(t)$, where υ_0 does not depend on the time, whereas $\upsilon_1(t) = \upsilon_1 \exp(i\omega t)$. In Eq. (6) the pinning force is $f_p = -dU_p\,(x)/dx$, where $U_p(x)$ is a PP of some form. Our goal is to determine $\upsilon(t)$ from Eq. (6) which, taking into account the considerations above, acquires the following form:

$$\eta[\upsilon_0 + \upsilon_1(t)] = f_0 + f_p + f_1(t). \tag{7}$$

Let us consider the case when $j_1 = 0$. If $j_0 < j_c$, i.e., $f_0 < f_c$, where f_c is the maximal value of the pinning force, then $\upsilon_0 = 0$, i.e., the vortex is at rest. As seen in Fig. 2, the rest coordinate x_0 of the vortex in this case depends on f_0 and is determined from the condition of equality to zero of the effective pinning force $\tilde{f}(x) = -d\tilde{U}(x)/dx = f_p(x) + f_0$, which reduces to the equation $f_p(x_0) + f_0 = 0$, or

$$f_0 = \left.\frac{dU_p(x)}{dx}\right|_{x=x_0}, \tag{8}$$

the solution of which is the function $x_0(f_0)$.

Now add a small oscillation of the vortex in the vicinity of x_0 under the action of a small external alternating force $f_1(t)$ with the frequency ω. For this we expand the effective pinning force $\tilde{f}(x)$ in the vicinity of $x = x_0$ into a series of small displacements $u \equiv x - x_0$, which gives

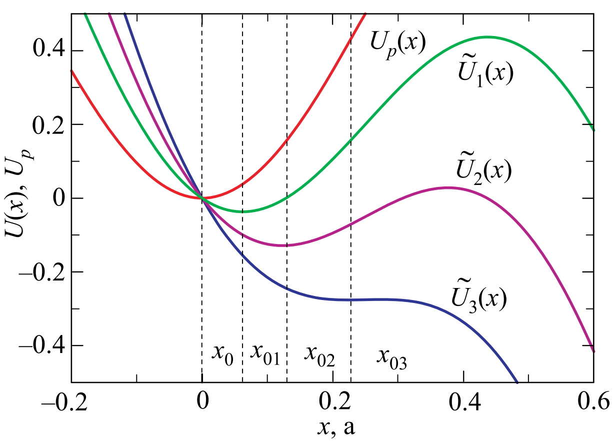

FIG. 2. Modification of the effective PP $\tilde{U}_i(x) \equiv U_p(x) - f_0 i x$, where $U_p(x) = (U_p/2)(1 - \cos kx)$ is the WPP, with the gradual increase of f_0 such as $0 = f_0 < f_{01} < f_{02} \lesssim f_{03} = f_c$, i.e., a vortex is oscillating in the gradually tilting pinning potential well in the vicinity of the rest coordinate x_{0i}.

$$\tilde{f}(x - x_0) \simeq \tilde{f}(x_0) + \tilde{f}'(x_0)u + \cdots \tag{9}$$

Then, taking into account that $\tilde{f}(x_0) = 0$ and $\tilde{f}'(x_0) = U_p{}^*(x_0)$, Eq. (7) takes on the form

$$\eta\dot{u}_1 + \tilde{k}_p u = f_1, \tag{10}$$

where $\tilde{k}_p(x_0) = U_p{}''(x_0)$ is the effective constant characterizing the restoring force $\tilde{f}(u)$ at small oscillations of the vortex in the effective PP $\tilde{U}(x)$ near $x_0(f_0)$, and $\upsilon_1 = \dot{u} = i\omega u$. Equation (10) used to determine υ_1 is physically equivalent to the GR Eq. (1) with the only distinction that the vortex depinning frequency $\tilde{\omega}_p \equiv \tilde{k}_p/\eta$ now depends on f_0 through Eq. (8), i.e., on the dc transport current density j_0. Thereby, all the results of the previous section [see Eqs. (2)–(5)] can be repeated here with the changes $x \to u$ and $\omega_p \to \tilde{\omega}_p$.

In order to discuss the changes in the dependences $\mathrm{Re}Z(\omega)$ and $\mathrm{Im}Z(\omega)$ caused by the dc current, the PP must be specified. As usual,[13–15] we take a cosine WPP of the form $U_p(x) = (U_p/2)(1 - \cos kx)$, where $k = 2\pi/a$, and a is the period; though any other non-periodic PP can also be used. Then, as it has been previously shown for the cosine WPP,[16] $\tilde{\omega}_p(j_0/j_c) = \omega_p\sqrt{1 - (j_0/j_c)^2}$ and the appropriate series of curves $\mathcal{P}(\omega|j_0)$ are plotted in Fig. 1. As evident from the figure, the curves shift to the left with increasing j_0. The reason for this is that with the increase of j_0 the PP well broadens while tilted, as is evident from Fig. 2. Thus, for the times shorter than $\tau_p = 1/\omega_p$ (i.e., for $\omega > \omega_p$) a vortex can no longer non-dissipatively oscillate in the PP well. As a consequence, the enhancement of $\mathrm{Re}Z(\omega)$ occurs at lower frequencies. At the same time, the curves in Fig. 1 maintain their original shape. Thus, the only universal parameter to be found experimentally is the depinning frequency ω_p. For a fixed frequency and variable j_0, the real part of $Z(\omega)$ always acquires larger values for larger j_0, whereas the maximum in the imaginary part of $Z(\omega)$ corresponds precisely to the middle point of the nonlinear transition in $\mathrm{Re}Z(\omega)$. It should be noted that even for $T = 0$ K the dissipation, though small, is non-zero even at very low frequencies.

4. Reconstruction of a pinning potential from microwave absorption data

We now turn to a detailed analytical description of the process of reconstruction of the coordinate dependence of a PP experimentally ensued in the sample, on the basis of microwave power absorption data in the presence of a subcritical dc transport current. It will be shown that from the dependence of the depinning frequency $\tilde{\omega}_p(j_0)$ as a function of dc transport current j_0 one can determine the coordinate dependence of the PP $U_p(x)$. The physical basis for the possibility of solving this problem is Eq. (8), which gives the ratio of the vortex rest coordinate x_0 to the value of the static force f_0 acting on the vortex and arising due to the dc current j_0.

4.1. General scheme of the reconstruction

From Eq. (8) it follows that when increasing f_0 from zero to its critical value f_c one in fact "probes" all the points of the dependence $U_p(x)$. Taking the x_0-coordinate derivative in Eq. (8), one obtains

Downloaded 27 Feb 2013 to 141.2.253.163. Redistribution subject to AIP license or copyright; see http://ltp.aip.org/about/rights_and_permissions

$$\frac{dx_0}{df_0} = \frac{1}{U_p'''(x_0)} = \frac{1}{\tilde{k}_p(x_0)}, \tag{11}$$

where the relation $U''(x_0) = \tilde{k}_p(x_0)$ is used [see Eq. (10) and the text below]. By substituting $x_0 = x_0(f_0)$, Eq. (11) can be rewritten as $dx_0/df_0 = 1/\tilde{k}_p[x_0(f_0)]$, and thus,

$$\frac{dx_0}{df_0} = \frac{1}{\eta\tilde{\omega}_p(f_0)}. \tag{12}$$

If the dependence $\tilde{\omega}(f_0)$ has been deduced from the experimental data, i.e., fitted by a known function, then Eq. (12) allows one to derive $x_0(f)$ by integrating

$$x_0(f_0) = \frac{1}{\eta}\int_0^{f_0} \frac{df}{\tilde{\omega}_p(f)}. \tag{13}$$

Then, having calculated the function $f_0(x_0)$, inverse to $x_0(f_0)$, and using the relation $f_0(x_0) = U_p'(x_0)$, i.e., Eq. (8), one finally obtains

$$U_p(x) = \int_0^x dx_0 f_0(x_0). \tag{14}$$

4.2. Sample WPP reconstruction procedure

Here we would like to support the above-mentioned considerations by giving an example of the reconstruction procedure for a WPP. Suppose that a series of power absorption curves $\mathcal{P}(\omega)$ was measured for a set of subcritical dc currents j_0. Then, to be specific, let's say that each i-curve of $\mathcal{P}(\omega|j_0)$, like those shown in Fig. 1, has been fitted with its fitting parameter $\tilde{\omega}_p$ so that one could map the points $[(\tilde{\omega}_p/\omega_p)_i, (j_0/j_c)_i]$, as shown by triangles in Fig. 3.

We fit the data in Fig. 3 to the function $\tilde{\omega}_p/\omega_p = \sqrt{1-(j_0/j_c)^2}$ and then substitute it into Eq. (13), from which one can calculate $x_0(f_0)$. In this case, the function has a simple analytical form, namely, $x_0(f_0) = (f_c/k_p)\arcsin(f_0/f_c)$.

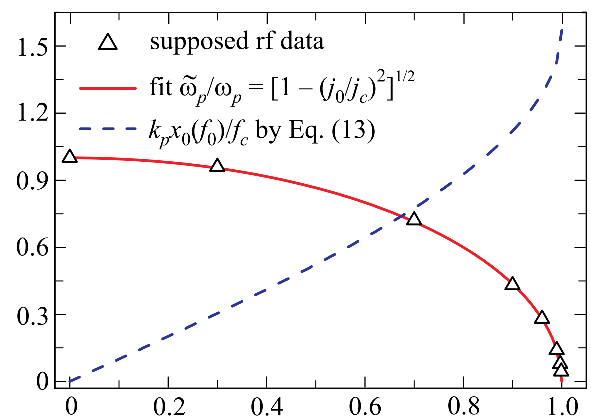

FIG. 3. The pinning potential reconstruction procedure: step 1. A set of $[(\tilde{\omega}_p/\omega_p)_i, (j_0/j_c)_i]$ points ($\triangle$) has been deduced from the supposed measured data and fitted as $\tilde{\omega}_p/\omega_p = \sqrt{1-(j_0/j_c)^2}$ (solid line). Then according to Eq. (13) $x_0(f_0) = (f_c/k_p)\arcsin(f_0/f_c)$ (dashed line).

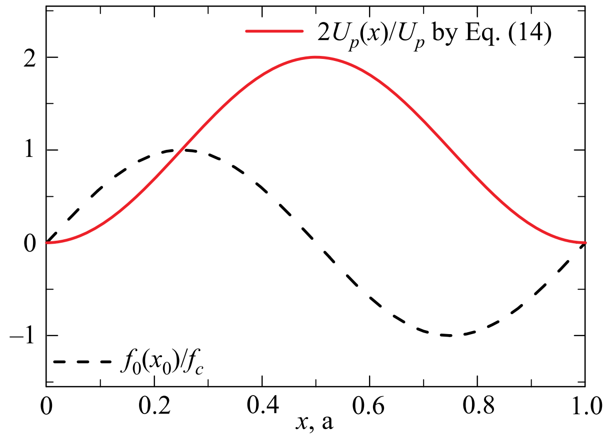

FIG. 4. The pinning potential reconstruction procedure: step 2. The inverse function to $x_0(f_0)$ is $f_0(x_0) = f_c\sin(x_0k_p/f_c)$ (dashed line). Then according to Eq. (14) $U_p(x) = (U_p/2)(1-\cos kx)$ is the PP sought (solid line).

Evidently, the inverse function is $f_0(x_0) = f_c\sin(x_0k_p/f_c)$ with the period $a = 2\pi f_c/k_p$ (see also Fig. 4). Taking the integral of Eq. (14) one finally gets $U_p(x) = (U_p/2)(1-\cos kx)$, where $k = 2\pi/a$ and $U_p = 2f_c^2/k_p$.

5. Conclusion

In this paper we show how coordinate dependence of the PP in the sample can be determined from the data on microwave power absorption by vortices in the presence of a subcritical dc transport current. The proposed procedure can be used at $T \ll T_c$ and implies a small microwave current density $j_1 \ll j_c$. In order to keep the transport current distribution in the sample as homogeneous as possible, the pinning potential is assumed to be not very "strong" in the sense that vortex pinning is caused, for example, by the reduction of vortex length rather than by suppression of the superconducting order parameter. Although the potential reconstruction scheme has been exemplified for a cosine WPP, that is, for the periodic and symmetric PP, the elucidated procedure in general does not require periodicity of the potential and can also account for an asymmetric potential. If this is the case, one has to perform the reconstruction under dc current reversal, i.e., two times: for $+j_0$ and $-j_0$. The scheme of reconstruction of the WPP $U_p(x)$ from the experimental data on $\tilde{\omega}_p(j_0)$ can be briefly summarized as follows: (a) from the data on $P(\omega/\tilde{\omega}_p(j_0))$ find $\tilde{\omega}_p(j_0)$; (b) take the integral (13) to calculate $x_0(f_0)$; (c) then from $x_0(f_0)$ find the inverse function $f_0(x_0)$; and finally (d) integrate $f_0(x_0)$, and by using Eq. (14) recover the PP $U_p(x)$.

Theoretically, we have limited our analysis to $T = 0$ K, $j_0 < j_c$, and $j_1 \to 0$, as this allowed us to provide a clear reconstruction procedure in terms of elementary functions accompanied by a simple physical interpretation. Experimentally, adequate measurements can be performed at $T \ll T_c$, i.e., on conventional thin-film superconductors (e.g., Nb, NbN). These are suitable due to significantly lower temperatures of the superconducting state, and relatively strong pinning in these materials allows one to neglect thermal fluctuations of the vortex due to the PP depth $U_p \simeq 1000$–5000 K.[19,20] It should be emphasized that due to the universal form of the dependence $\mathcal{P}(\omega|j_0)$, the depinning

Downloaded 27 Feb 2013 to 141.2.253.163. Redistribution subject to AIP license or copyright; see http://ltp.aip.org/about/rights_and_permissions

frequency ω_p plays a role of the only fitting parameter for each of the curves $\mathcal{P}(\omega|j_0)$, thus fitting the measured data seems simple. However, for the experiment it is crucial to adequately superimpose the applied currents and then to uncouple the picked-up dc and microwave signals maintaining that the line and the sample impedances be matching. Quantitatively, experimentally estimated values of the depinning frequency in the absence of a dc current and at a temperature of about $0.6T_c$ are $\omega_p \approx 7$ GHz for a 20 nm-thick[7] and a 40 nm-thick[8] Nb films. This value is strongly suppressed with the increase of both the field magnitude and the film thickness.

Concerning the general validity of the results obtained, three remarks should be given. First, though the data figure has been provided here for a cosine WPP as for the most commonly used potential, the coordinate dependence can be reconstructed not only for periodic potentials. In fact, single PP wells, like the one used in Ref. 5, can also be proven to be in accordance with the provided approach. Secondly, it should be noted that if a PP is periodic then the theoretical consideration here has been performed in the single-vortex approximation, i.e., is valid only at small magnetic fields $B \ll B_{c2}$, when the distance between two neighboring vortices, i.e., the period of a PP, is larger than the effective magnetic field penetration depth.

Finally, the results can be directly verified, for example, in the microstrip geometry for combined microwave and dc electrical transport measurements. While the experimental work on the subject has begun to appear,[4–7] we hope to have stimulated further developments in the field. Furthermore, due to the mathematical analogy between the equation of motion for a vortex used in this work and the equation for the phase difference in the Josephson junction problem, we believe that the proposed scheme of reconstruction can also be adopted for that case.

V.A.S. thanks partial financial support from the STCU project BNL-T2-368-UA through Grant No. P-424. O.V.D. gratefully acknowledges financial support by the Deutsche Forschungsgemeinschaft (DFG) through Grant No. DO 1511/2-1.

[a)]Email: Dobrovolskiy@physik.uni-frankfurt.de

[1]E. Silva, N. Pompeo, S. Sarti, and C. Amabile, in *Recent Developments in Superconductivity Research*, edited by B. P. Martins (Nova Science, Hauppauge, New York, 2006), p. 201.
[2]V. A. Shklovskij, J. Low Temp. Phys. **41**, 375 (1980).
[3]A. I. Bezuglyj and V. A. Shklovskij, Physica C **202**, 234 (1992).
[4]B. B. Jin, B. Y. Zhu, R. Wördenweber, C. C. de Souza Silva, P. H. Wu, and V. V. Moshchalkov, Phys. Rev. B **81**, 174505 (2010).
[5]C. Song, M. P. DeFeo, K. Yu, and B. L. T. Plourde, Appl. Phys. Lett. **95**, 232501 (2009).
[6]N. S. Lin, T. W. Heitmann, K. Yu, B. L. T. Plourde, and V. R. Misko, Phys. Rev. B **84**, 144511 (2011).
[7]N. Pompeo, E. Silva, S. Sarti, C. Attanasio, and C. Cirillo, Physica C **470**, 901 (2010).
[8]O. Janušević, M. C. Grbić, M. Požek, A. Dulčić, D. Paar, B. Nebendahl, and T. Wagner, Phys. Rev. B **74**, 104501 (2006).
[9]A. M. Campbell and J. E. Evetts, Adv. Phys. **21**, 294 (1972).
[10]N. Kokubo, R. Besseling, and P. H. Kes, Phys. Rev. B **69**, 064504 (2004) and Refs. 18 and 26 therein.
[11]A. A. Golubov, M. Yu. Kupriyanov, and E. Ilichev, Rev. Mod. Phys. **76**, 411 (2004).
[12]J. I. Gittleman and B. Rosenblum, Phys. Rev. Lett. **16**, 734 (1966).
[13]M. W. Coffey and J. R. Clem, Phys. Rev. Lett. **67**, 386 (1991).
[14]V. A. Shklovskij and O. V. Dobrovolskiy, Phys. Rev. B **78**, 104526 (2008).
[15]V. A. Shklovskij and O. V. Dobrovolskiy, Phys. Rev. B **84**, 054515 (2011).
[16]V. A. Shklovskij and D. T. B. Hop, Fiz. Nizk. Temp. **35**, 469 (2009) [Low Temp. Phys. **35**, 365 (2009)].
[17]V. A. Shklovskij and D. T. B. Hop, Fiz. Nizk. Temp. **36**, 89 (2010) [Low Temp. Phys. **36**, 71 (2010)].
[18]V. A. Shklovskij, in *Proceedings of Fifth International Conference on Mathematical Modeling and Computer Simulation of Materials Technologies, MMT-2008* (Ariel University, Ariel, Israel, 2008).
[19]O. K. Soroka, V. A. Shklovskij, and M. Huth, Phys. Rev. B **76**, 014504 (2007).
[20]O. V. Dobrovolskiy, E. Begun, M. Huth, and V. A. Shklovskij, New J. Phys. **14**, 113027 (2012).

This article was published in English in the original Russian journal. Reproduced here with stylistic changes by AIP.

Downloaded 27 Feb 2013 to 141.2.253.163. Redistribution subject to AIP license or copyright; see http://ltp.aip.org/about/rights_and_permissions